

**BED FORM GEOMETRY AND FRICTION FACTOR
OF FLOW OVER A BED COVERED BY DUNES**

by
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A Thesis

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ABSTRACT

An in-depth analysis of existing methods for prediction of dune steepness is presented. In particular, the expressions by Yalin and Scheuerlein (1988) and van Rijn (1982) are investigated. It is revealed that the expressions by these authors, which were developed on the basis mainly of laboratory data, do not adequately reflect the behavior of dunes occurring in rivers (characterized by large values of relative flow depth).

A new expression for dune steepness is introduced with the intention of extending the applicability of previous expressions to river data. This expression is a generalization of the expression by Yalin and Scheuerlein (1988).

The validity of the dune steepness expression introduced in this thesis is tested by using it in the determination of the friction factor. It is shown that the use of the dune steepness expression introduced herein yields an improved prediction of the friction factor.

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LIST OF SYMBOLS

1. General

A	cross-sectional area
B	flow width (wide channel)
$c = v/v_*$	dimensionless Chézy friction factor
c_0	flat bed value of c
C	(dimensional) Chézy friction factor
C_0	(dimensional) flat bed value of C
D	representative grain size
g	acceleration due to gravity
h	flow depth
k_s	granular roughness of bed surface
K_s	equivalent roughness
P	wetted parameter
Q	flow rate
q	flow rate per unit width (specific flow rate)
R	hydraulic radius
S	slope of the uniform flow
T	transport stage parameter
v	average flow velocity
$v_* = \sqrt{\tau_0/\rho} = \sqrt{gSh}$	shear velocity
τ_0	shear stress acting on bed surface
ρ	density of fluid
γ	specific weight of fluid
ν	kinematic viscosity
ρ_s	grain density
γ_s	specific weight of grains in fluid
Δ	developed bed form height
Λ	developed bed form length
$\kappa \approx 0.4$	Von Karman constant

2. Dimensionless Combinations

$$Fr = \frac{v^2}{gh}$$

Froude number

$$Re_* = \frac{v_* k_s}{\nu}$$

roughness Reynolds number

$$X = \frac{v_* D}{\nu}$$

grain size Reynolds number

$$Y = \frac{\rho v_*^2}{\gamma_s D}$$

mobility number

$$Z = \frac{h}{D}$$

dimensionless flow depth

$$W = \frac{\rho_s}{\rho}$$

density ratio

$$\xi^3 = \frac{X^2}{Y} = \frac{\gamma_s D^3}{\rho v_*^2}$$

material number

$$\eta = \frac{Y}{Y_{cr}}$$

relative flow intensity

$$\bar{\eta}_d$$

η corresponding to the largest Δ/Λ

3. Subscripts:

cr

signifies the value corresponding to the initiation of sediment transport

1.0 INTRODUCTION

1.1 General

In an alluvial stream, the mobile bed formed by cohesionless alluvium (sand) is seldom flat, rather, it is covered by periodic bed deformations, known as bed forms. These bed forms change in type and size depending on the flow conditions. They constitute an important obstacle to the flow, and thus, the resistance of alluvial channels changes as bed forms change.

An important aspect of river engineering, as emphatically pointed out by Chang (1988) is “the determination of the flow-induced resistance associated with the bed forms”. An adequate prediction of the resistance to flow is required in projects involving flood protection, navigation, stream restoration/stabilization, design of hydraulic structures, etc. For these reasons, much effort has been devoted in the past to the investigation of alluvial channel resistance, as well as to the geometric characteristics of bed forms. However, because of the complexity of the problem, no satisfactory formulation for alluvial channel flow resistance has been reached yet.

In the practice of river engineering, the resistance to flow is expressed by means of resistance equations, i.e. relations relating the average flow velocity to flow quantities such as the flow depth and channel slope. The most commonly used resistance equation is Chézy's equation which, for the case of a straight, uniform, two-dimensional flow is expressed as

$$v = c\sqrt{gSh} . \quad (1.1)$$

Here v is the average flow velocity, c is the dimensionless Chézy friction factor, g is acceleration due to gravity, S is the bed slope and h is the flow depth. Since $v_* = \sqrt{\tau_0/\rho} = \sqrt{gSh}$, the friction factor in Eq. (1.1) can be expressed as

$$c = \frac{v}{v_*} = \frac{v}{\sqrt{\tau_0/\rho}} . \quad (1.2)$$

[It should be mentioned that in practice it is often assumed that the flow can be treated as straight, uniform and two-dimensional. If, however, the channel is not

“wide”, Eq. (1.1) is still used, by replacing h by the hydraulic radius $R = A/P$ (where A is the cross-sectional area and P is the wetted perimeter). If the flow is gradually converging or diverging, then Eq. (1.1) is used, but S is to be interpreted as the energy gradient.]

The friction factor c is thus a coefficient which reflects the roughness of the solid boundary. For the case of an open channel flow over a rigid boundary, the dependency of the friction factor on the roughness of the boundary is well established. However, in the case of a river, as follows from previous explanation, the friction factor c depends on the type and size of bed forms. If the flow is sub-critical, the two most common types of bed forms are dunes and ripples. These bed forms occur as periodic bed deformations of the length Λ and the height Δ , as shown in Fig. 1.1.

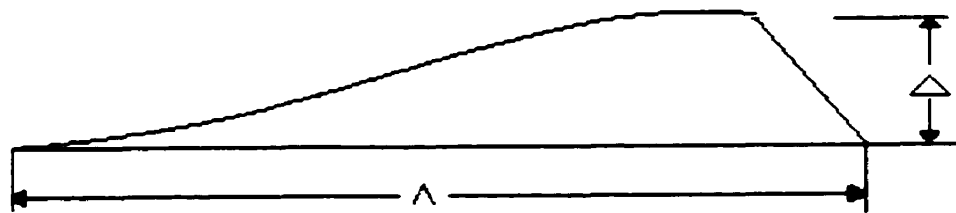


Figure 1.1: Illustration of bed form dimensions

The ratio Δ/Λ is called the bed form steepness. The length of dunes is proportional to the flow depth h (Yalin (1977), Yalin (1992)), whereas the length of ripples is proportional to the grain size D (Yalin (1977), Yalin (1992)). The presence of bed forms, whose geometric properties are themselves highly dependent on the flow conditions, makes the determination of friction factor rather difficult.

In fact, and in spite of extensive work on the topic over the last fifty years or so, there is still no agreement on how to determine the friction factor of an alluvial stream whose bed is covered by bed forms. Following the earlier works of Einstein and Barbarossa (1952), Engelund (1966), Yalin (1964), the current methods of determination of friction factor of flow in sand-bed channels covered by bed forms rest on the consideration of the bed shear stress by means of two components: one due

to skin friction, and another due to the drag force caused by bed forms. Since the latter component is usually much larger than the former, the adequate prediction of friction factor lies on the adequate prediction of bed form geometry.

The present thesis concerns the geometric properties of dunes, in particular their steepness Δ/Λ , and its impact on the determination of friction factor. Probably the most popular expressions for bed form steepness are those due to van Rijn (1982) and Yalin and his co-workers (see Yalin and Karahan (1979), Yalin and Scheuerlein (1988), Yalin (1992)). Given the scarcity of field data, the expressions by these authors rest mainly on the analysis of laboratory data, and thus it is doubtful whether they are applicable to river data. This fact was first realized and emphatically pointed out by Julien (1992) and Julien and Klaassen(1995).

Following the works by these authors, it became clear that there is a need to evaluate the applicability of existing expressions for dune steepness for the case of river data; and eventually to extend their applicability to river data ranges. The present thesis stems from the realization of these needs.

1.2 Objectives

With regard to distinguishing laboratory data from river data, the relevant parameter is $Z = h/D$ (see Section 1.4): laboratory data is characterized by “small” Z -values, whereas river data is characterized by “large” Z -values. With this in mind, the objectives of the present thesis can be formulated as follows:

1- To provide a literature review of existing methods of determination of dune geometry and friction factor of flow over a bed covered by dunes.

2- To evaluate the applicability of the dune steepness expressions of Yalin and his co-workers, as well as the expression of van Rijn, to river data.

3- To introduce a new expression for dune steepness, extending the applicability of existing methods of determination of dune steepness to the entire spectrum of possible Z -values.

4- To evaluate existing methods of determination of friction factor; and to establish whether the author's dune steepness expression yields somewhat improved results where the determination of friction factor is concerned.

1.3 Layout of thesis

Chapter 2 concerns the literature survey. Expressions for dune geometric characteristics Δ , Λ , and Δ/Λ are presented: various methods of determination of friction factor are explained.

Chapter 3 provides a detailed comparison between the expression for dune steepness by Yalin and his co-workers and the expression of van Rijn. Advantages and disadvantages of each method are discussed.

Chapter 4 presents the author's expression for dune steepness. The data and methods used to develop this expression are described in detail.

Chapter 5 presents a discussion of existing methods of determination of friction factor. The applicability of the author's dune steepness expression is tested, by comparing measured and computed values of the friction factor.

Chapter 6 concerns the conclusions. Some suggestions for future research are included.

1.4 Pertinent variables: dimensionless variables of the two-phase motion

The simultaneous motion of steady flow and transported sediment constitutes a mechanical totality which is referred to as the two-phase motion. Following Yalin (1972), (1992), the steady uniform two-dimensional two-phase motion is determined by seven characteristic parameters:

$$A = f_A(\rho, \nu, \rho_s, D, h, v_*, \gamma_s) \quad , \quad (1.3)$$

where ρ and ν are the fluid density and kinematic viscosity, respectively; ρ_s is the grain density and D is the representative grain size; h is the flow depth; v_* is the shear velocity; γ_s is the grain submerged specific weight (see also “list of symbols”).

Using π -theorem (with ρ , D and v_* as “repeaters”), it can be shown (see Yalin (1972)) that the dimensionless counterparts of A , viz Π_A , is a function of four dimensionless variables:

$$\Pi_A = \bar{\Phi}_A(X, Y, Z, W) \quad (1.4)$$

where

$$X = \frac{v_* D}{\nu} \quad (\text{grain size Reynolds number}) \quad (1.5)$$

$$Y = \frac{\rho v_*^2}{\gamma_s D} \quad (\text{mobility number}) \quad (1.6)$$

$$Z = \frac{h}{D} \quad (\text{dimensionless flow depth}) \quad (1.7)$$

$$W = \frac{\rho_s}{\rho} \quad (\text{density ratio}) . \quad (1.8)$$

For the case of motion *en masse*, ρ_s and thus W are of no importance, and can thus be left out. Therefore, any dimensionless property Π_A of the two-phase motion can be expressed as a function of the variables X , Y and Z :

$$\Pi_A = \phi_A(X, Y, Z) . \quad (1.9)$$

However, it is not necessary to express a property Π_A literally in terms of X , Y and Z . Any one of X , Y and Z can be replaced by another variable which is a function of that variable and of one or both of the other two (Yalin (1992)). Variables ξ and η , where

$$\xi^3 = \frac{X^2}{Y} = \frac{\gamma_s D^3}{\rho \nu^2} \quad (\text{material number}) \quad (1.10)$$

$$\eta = \frac{Y}{Y_{cr}} \quad (\text{relative flow intensity}) \quad (1.11)$$

are commonly used for this purpose (here Y_{cr} means the value of Y at the stage of initiation of sediment transport, as given by the Shields curve). Thus, one can write equivalently:

$$\Pi_A = \phi_{A_1}(X, Y, Z) = \phi_{A_2}(\xi, Y, Z) = \phi_{A_3}(\xi, \eta, Z) . \quad (1.12)$$

2.0 LITERATURE REVIEW

2.1 Conditions for dune formation

As mentioned in the Introduction, provided the flow is subcritical, the two most frequent bed forms are ripples and dunes. Ripples are known to occur for “small” values of the grain size Reynolds number $X = v_* D / \nu$ ($X < \approx 5.5$, say); dunes, on the other hand, are known to occur for “large” values of X ($X > \approx 30$, say). For “intermediate” values of X ($5.5 \approx < X < \approx 30$), both ripples and dunes occur, in the form of ripples superimposed on dunes. Fig. 2.1, which is due to M. S. Yalin (unpublished), shows the existence regions of ripples and dunes, as determined by X .

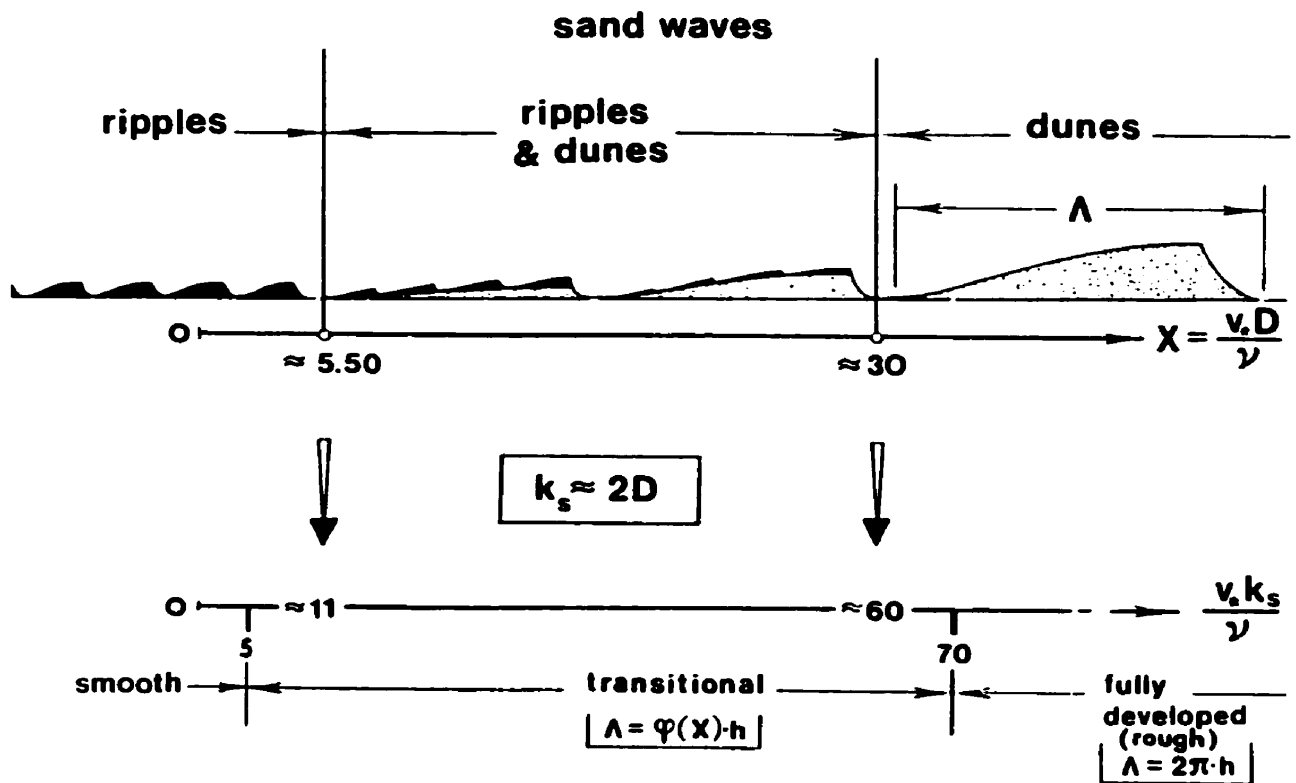


Figure 2.1: Regimes of turbulent flow past the flat initial bed

The consideration of grain roughness $k_s = 2D$ (Yalin (1992), Kamphuis (1974)), yields

$$Re_* = \frac{v_* k_s}{\nu} = 2 \frac{v_* D}{\nu} = 2X \quad (2.1)$$

As is well known, $Re_* < \approx 5$ defines the hydraulically smooth regime of turbulent flows, and $Re_* > \approx 70$ defines the rough turbulent regime of flows (see e.g. Schlichting (1968), Yalin (1972), (1992)). Thus, as indicated in Fig. 2.1, ripples are associated with hydraulically smooth flows and dunes with rough turbulent flows. In the region of transitional flows, ripples occur superimposed on dunes.

2.2 Dune length, height and steepness

2.2.1 Expression of Fredsoe (1975)

One of the earlier attempts to produce an expression for the dune steepness is due to Fredsoe (1975). According to this author,

$$\frac{\Delta}{\Lambda} = \frac{1}{8.4} \left(1 - \frac{0.06}{Y} - 0.4Y \right)^2. \quad (2.2)$$

2.2.2 Expressions of Yalin and Karahan (1979), Yalin and Scheuerlein (1988) and Yalin (1992)

i) Dune steepness

Yalin and Karahan (1979): Following Yalin and Karahan (1979), if $X > \approx 25$ (dunes only - see Section 2.1), then Δ/Λ is completely determined by the dimensionless variables η and Z , i.e.

$$\frac{\Delta}{\Lambda} = \Phi(\eta, Z). \quad (2.3)$$

According to these authors, “the influence of Z on the dune steepness progressively decreases with its increasing value”. For this reason, in their paper, Yalin and Karahan point out that the earlier expression of Fredsoe (Eq. (2.2)), which does not contain Z , should be regarded as applicable only for large values of Z .

On the basis of experimental data plotted in Fig. 2.2 (where the abscissa is η , and Δ/Λ is the ordinate), Yalin and Karahan suggested that the influence of Z on Δ/Λ becomes unnoticeable “for Z larger than, e.g., ≈ 100 ”. In order to determine the

variation of Δ/Λ with Z for $Z \lesssim 100$, these authors carried out a series of special experiments in a 21 m long, 0.16 m wide flume. Three different granular materials were used: bakelite ($D = 1$ mm), sand ($D = 1.1$ mm) and polystyrene ($D = 1.54$ mm). A total of 123 runs were carried out (only one run with polystyrene). The grain size Reynolds number X was larger than 20 so that only dunes were produced. Z values were in the range $20 \leq Z \leq 84$.

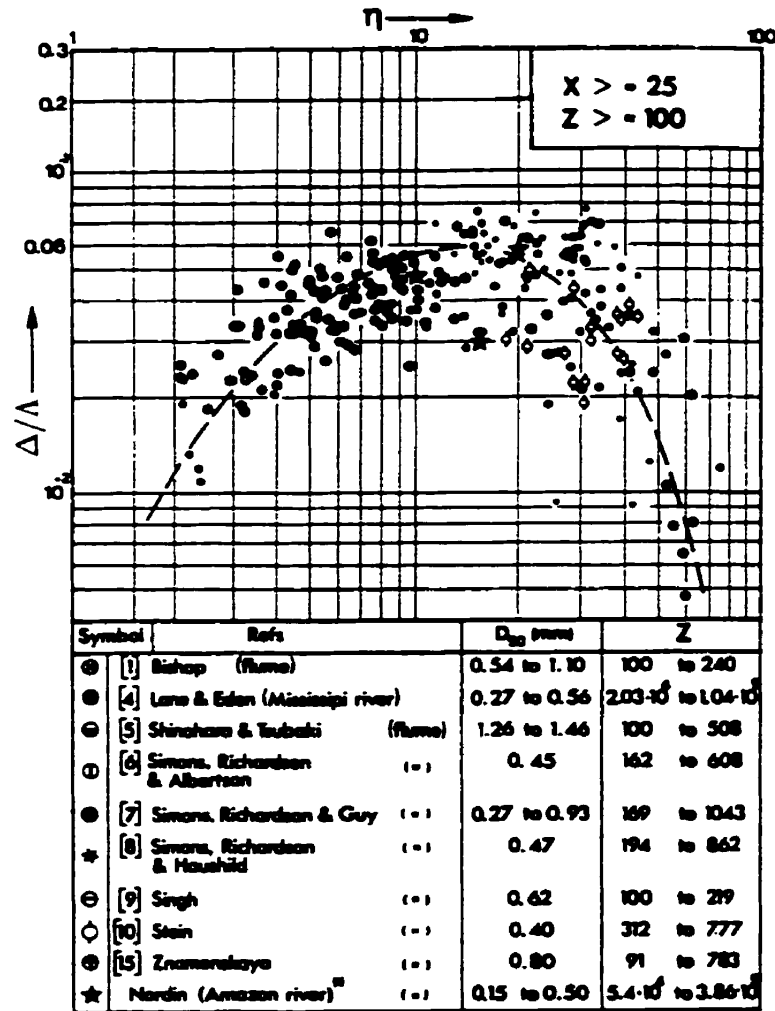


Figure 2.2: Plot of dune steepness versus η for $Z > 100$ (from Yalin and Karahan (1979))

The resulting experimental values of Δ/Λ were plotted versus η in Fig. 2.3. The data clearly sorts out according to Z -value.

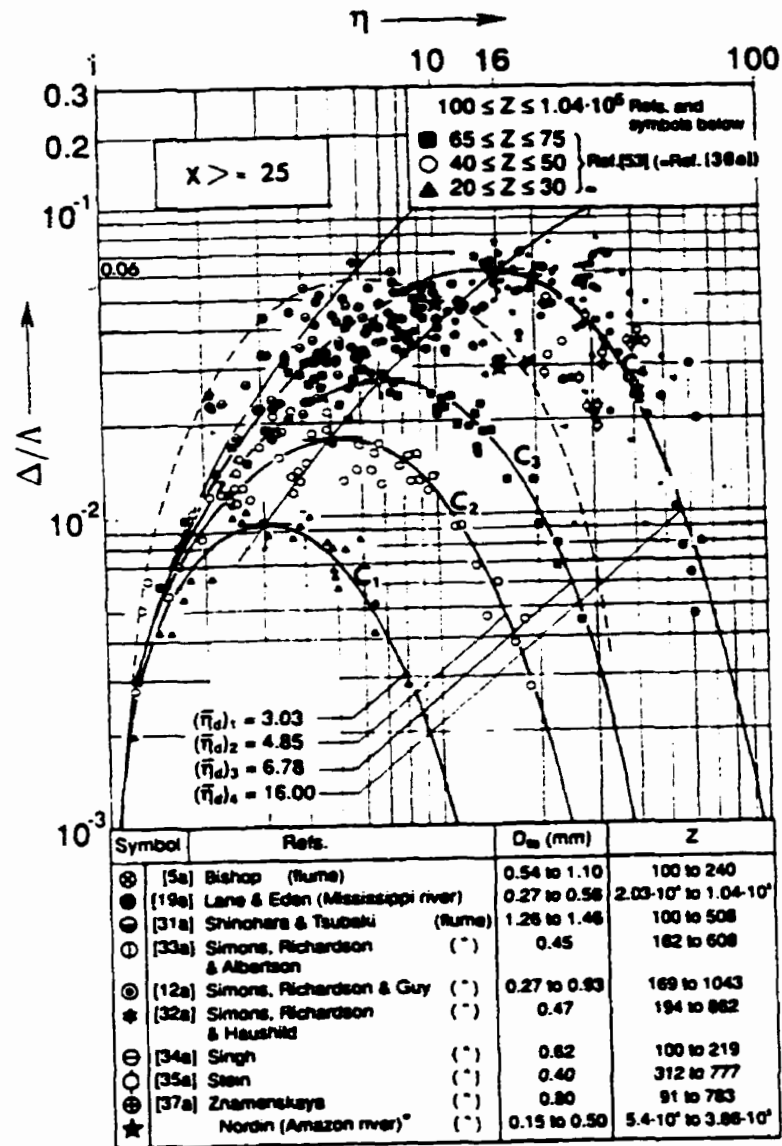


Figure 2.3: Plots of Δ/Λ versus η for all Z -values (from Yalin (1992))

According to Yalin and Karahan, the left-hand side of the patterns C_i (Fig. 2.3) merge into a common curve represented by a linear form, which can be expressed as

$$\lim_{\eta \rightarrow 0} \frac{\Delta}{\Lambda} = 0.0127(\eta - 1) \quad \text{for any } Z . \quad (2.4)$$

On the other hand, when η is very "large", the function should yield the "flat bed at advanced stages":

$$\lim_{\eta \rightarrow \infty} \frac{\Delta}{\Lambda} = 0 \quad \text{for any } Z . \quad (2.5)$$

In order to satisfy Eqs. (2.4) and (2.5), Yalin and Karahan (1979) suggested the following equation for the dune steepness:

$$\frac{\Delta}{\Lambda} = 0.0127(\eta - 1)e^{[-(\eta-1)f(Z)]} . \quad (2.6)$$

Differentiation of Eq. (2.6) with respect to $(\eta - 1)$ yields:

$$\frac{\partial(\Delta/\Lambda)}{\partial(\eta - 1)} = 0.0127e^{[-(\eta-1)f(Z)]} + 0.0127(\eta - 1)[-f(Z)]e^{[-(\eta-1)f(Z)]} \quad (2.7)$$

i.e.

$$\frac{\partial(\Delta/\Lambda)}{\partial(\eta - 1)} = 0.0127(\eta - 1)e^{[-(\eta-1)f(Z)]} \left\{ \frac{1}{\eta - 1} + [-f(Z)] \right\} . \quad (2.8)$$

Equating Eq. (2.8) to zero, and denoting the abscissa corresponding to the largest Δ/Λ (i.e. $(\Delta/\Lambda)_{max}$) of a pattern representing a certain value of Z by $\bar{\eta}_d$, yields

$$\left\{ \frac{1}{\bar{\eta}_d - 1} - f(Z) \right\} = 0 ,$$

i.e.

$$\frac{1}{\bar{\eta}_d - 1} = f(Z) . \quad (2.9)$$

Therefore, Eq. (2.6) can be written as

$$\frac{\Delta}{\Lambda} = 0.0127(\eta - 1)e^{-\frac{\eta-1}{\bar{\eta}_d-1}} . \quad (2.10)$$

Eq. (2.10) can be used in practice to determine Δ/Λ only if the function $1/(\bar{\eta}_d - 1) = f(Z)$ is known. However, in their work, these authors have only indicated that

- if $20 \leq Z \leq 30$, $\bar{\eta}_d - 1 = 2.03$
- if $40 \leq Z \leq 50$, $\bar{\eta}_d - 1 = 3.85$
- if $65 \leq Z \leq 75$, $\bar{\eta}_d - 1 = 5.78$
- if $100 \leq Z$, $\bar{\eta}_d - 1 = 12.84$

Yalin and Scheuerlein (1988): The first attempt to determine $1/(\bar{\eta}_d - 1) = f(Z)$ can be found in Yalin and Scheuerlein (1988) where

$$\frac{1}{\bar{\eta}_d - 1} = \frac{0.0778}{1 - e^{-0.01Z}} \quad (2.11)$$

By substituting Eq. (2.11) into Eq. (2.10), Yalin and Scheuerlein have expressed Δ/Λ as

$$\frac{\Delta}{\Lambda} = 0.0127(\eta - 1)e^{-0.0778(\eta-1)/(1-e^{-0.01Z})} \quad (2.12)$$

Yalin (1992): Consider now that at $\eta = \bar{\eta}_d$, we have $\Delta/\Lambda = (\Delta/\Lambda)_{max}$. Therefore, using Eq. (2.10):

$$(\Delta/\Lambda)_{max} = 0.0127(\bar{\eta}_d - 1)e^{-1} . \quad (2.13)$$

The division of Eq. (2.10) by Eq. (2.13) yields:

$$\frac{\Delta/\Lambda}{(\Delta/\Lambda)_{max}} = \frac{\eta - 1}{\bar{\eta}_d - 1} e^{1 - \frac{\eta-1}{\bar{\eta}_d-1}} . \quad (2.14)$$

Introducing

$$\zeta = \frac{\eta - 1}{\bar{\eta}_d - 1} , \quad (2.15)$$

Eq. (2.14) can be written as

$$\Delta/\Lambda = (\Delta/\Lambda)_{max} \zeta e^{1-\zeta} , \quad (2.16)$$

where $(\Delta/\Lambda)_{max} = f_1(Z)$, and $\zeta = f_2(Z)$.

Eq. (2.16) can be regarded as a generalized form of previous equations by the authors. In his most recent work, Yalin (1992) presents the following expressions for $(\Delta/\Lambda)_{max} = f_1(Z)$ and $\bar{\eta}_d = f_2(Z)$:

$$(\Delta/\Lambda)_{max} = 0.06(1 - e^{-0.008Z}) \quad (2.17)$$

$$\bar{\eta}_d = 14(1 - e^{-0.003Z}) + 2 \quad (2.18)$$

(see Figs. 2.4 and 2.5).

ii) Dune length

According to Yalin (1992), Yalin (1977), if the flow past the initial flat bed is rough turbulent, the dimensionless dune length is given by

$$\frac{\Lambda}{h} \approx 6 . \quad (2.19)$$

If the flow is not rough turbulent, then the ratio Λ/h is also a function of X and Z , and Eq. (2.19) must be generalized into

$$\frac{\Lambda}{h} = 6\Phi_{\Lambda}(X, Z) . \quad (2.20)$$

The family of Λ/h -curves implied by Eq. (2.20) is shown in Fig. 2.6. Yalin (1992) has expressed $\Lambda/h = 6\Phi_{\Lambda}(X, Z)$ as

$$\Lambda/h = 6 \left[1 + 0.01 \frac{(Z - 40)(Z - 400)}{Z} e^{-m} \right] \quad (\text{where } m = 0.055\sqrt{Z} + 0.04X) . \quad (2.21)$$

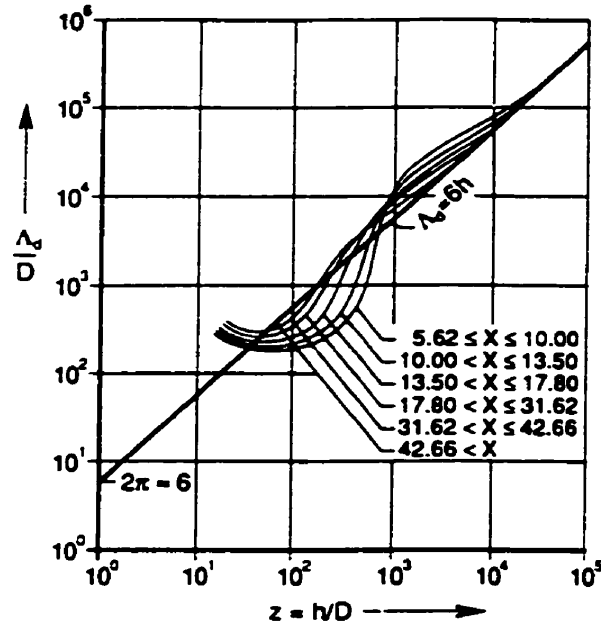


Figure 2.6: Plot of Λ/D versus Z (from Yalin (1992))

2.2.3 Expressions of van Rijn (1982)

i) Dune steepness

van Rijn expressed Δ/Λ as a function of Z and of the following dimensionless

variable:

$$T = \frac{(v'_*)^2 - (v_{*,cr})^2}{(v_{*,cr})^2} \quad (2.22)$$

which was termed “transport stage parameter”. In Eq. (2.22), v'_* is shear velocity related to grains:

$$v'_* = v \frac{g}{C_0}, \quad (2.23)$$

where v is the average flow velocity and C_0 is the dimensional flat bed value of the (dimensional) Chézy friction factor C . Based on 84 data points from flume experiments with grain size D in the range 0.19 mm - 2.3 mm and 22 field data points with grain size D in the range 0.49 mm - 3.6 mm (both with dune-type bed forms), van Rijn performed regression analysis and presented the following expression for dimensionless dune steepness:

$$\frac{\Delta}{\Lambda} = 0.015 \left(\frac{1}{Z} \right)^{0.3} (1 - e^{-0.5T})(25 - T) . \quad (2.24)$$

Fig. 2.7 is the original plot of van Rijn, which illustrates the agreement between Eq. (2.24) and the experimental data used by that author.

ii) Dune height

For dune height, the best agreement with the data was obtained for:

$$\frac{\Delta}{h} = 0.11 \left(\frac{1}{Z} \right)^{0.3} (1 - e^{-0.5T})(25 - T) \quad (2.25)$$

The agreement between the data used by van Rijn and Eq. (2.25) is shown in Fig. 2.8.

iii) Dune length

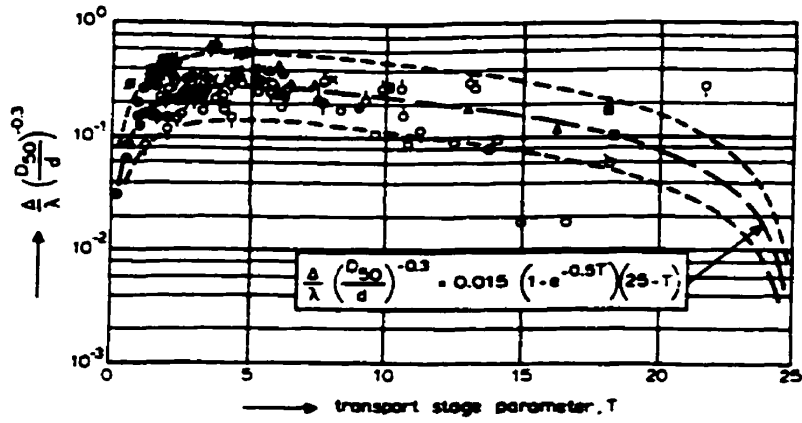
By combining Eqs. (2.24) and (2.25), van Rijn has determined for the dune length

$$\Lambda/h = 7.3 . \quad (2.26)$$

2.2.4 Expression of Adams (1990)

Adams (1990) presents the following expression for dune steepness:

$$\frac{\Delta}{\Lambda} = 0.025(\eta - 1)^2 e^{[3.25/\Phi(\eta) - 0.107(\eta - 1) - 3.25]/\Psi(Z)} \quad (2.27)$$



	source	flow velocity \bar{u} (m/s)	flow depth d (m)	particle size D_{50} (μ m)	temperature T_e ($^{\circ}$ C)
flume data	o Guy et al	0.34-1.17	0.18-0.32	180	8-34
	x Guy et al	0.41-0.85	0.14-0.34	270	8-34
	Δ Guy et al	0.47-1.15	0.16-0.32	280	8-34
	b Guy et al	0.77-0.88	0.16	330	8-34
	o Guy et al	0.48-1.00	0.10-0.25	450	8-34
	o Guy et al	0.53-1.15	0.12-0.34	930	8-34
	e Williams	0.54-1.06	0.15-0.22	1350	25-28
	# Delft Hydr. Lab.	0.45-0.87	0.28-0.48	780	12-18
	Δ Stein	0.52-0.85	0.24-0.31	400	20-28
	d Znamenskaya	0.53-0.80	0.11-0.21	800	-
field data	o Dutch Rivers	0.85-1.55	4.4-8.5	480-3800	5-20
	# Rio Parana	1.0	12.7	400	-
	Δ Japanese Channels	0.53-0.89	0.25-0.89	1100-2300	-
	o Mississippi River	1.35-1.45	6-16	350-550	-

Figure 2.7: Relation between Δ/λ and T (from van Rijn (1982))

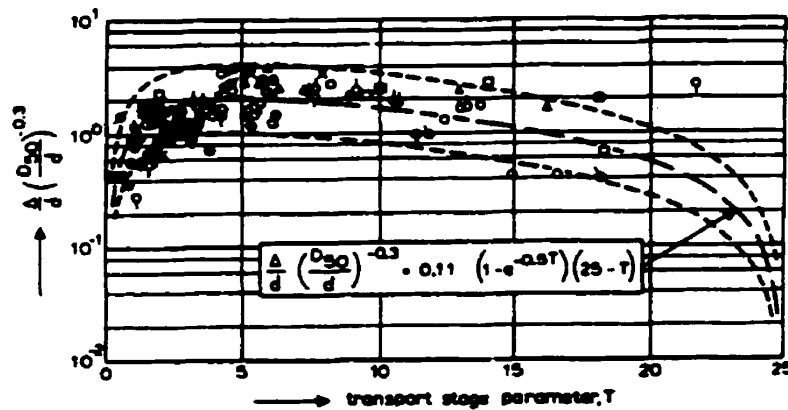


Figure 2.8: Relation between Δ/h and T (from van Rijn (1982))

where

$$\Phi(\eta) = [(\eta - 1)/12.5 + 1]^3, \quad \Psi(Z) = 1 - e^{-0.025Z}. \quad (2.28)$$

2.3 Determination of Friction Factor

Methods of determination of friction factor fall into two different approaches: those that divide resistance into grain resistance and form resistance (henceforth referred to as *divided approach* and those that do not (referred to as *undivided approach*). Some methods deviate slightly from the divided approaches in that the flow resistance is not exactly divided into two components (due to surface and form roughness), but that bed forms are treated as if they were large-scale grains, or “equivalent roughness”. Grain resistance is the “part of resistance contributed by the surface drag, and form resistance is caused by the pressure difference between the front and back surfaces of the bed forms” (Chang (1988)).

2.3.1 Undivided approaches

2.3.1.1 Expressions of Garde and Raju (1966)

Simons and Richardson (1961), based essentially on similarity in form, resistance to flow and modes of sediment transport, divided the bed forms into categories of lower flow regime, transition zone, and upper flow regime in order of increasing velocity as follows:

Lower flow regime: ripples, dunes with ripples superimposed, dunes.

Transition: bed changes from dunes to plane bed or standing waves.

Upper flow regime: plane bed, antidunes, chutes and pools.

(see also Chang (1988)).

Garde and Raju proposed friction factor relations for the different flow regimes (as defined by Simons and Richardson (1961)) as follows:

Lower regime ($G_1 Fr \leq 0.33$): $c = 3.194(R/k_s)^{1/6}$

Transition zone ($0.33 \leq G_1 Fr \leq 1$): $c = 5.976(R/k_s)^{1/6}$

Upper regime ($G_1Fr \geq 1$): $c = 5.976(R/k_s)^{1/6}$

These equations are valid for $0.011 \text{ mm} < D_{50} < 5.2 \text{ mm}$. The (dimensionless) bed form regime classifying parameter G_1Fr is given by

$$G_1Fr = \frac{Q}{B\sqrt{(W-1)gR^3}} \quad , \quad (2.29)$$

where Q is the flow rate, B is the flow width, and R is the hydraulic radius.

2.3.1.2 Method of Raudkivi (1967)

Raudkivi's method for the determination of friction factor consists of a graph of $\frac{v}{\sqrt{v_*^2 - v_{*cr}^2}} = \frac{c}{\sqrt{1-1/\eta}}$ versus mobility number Y . Here v_{*cr} is the value of the shear velocity v_* at the stage of incipient motion. This author plotted both laboratory and field data, and obtained the point-patterns shown in Fig. 2.9 (points shown in this figure are classified according to grain size D). Raudkivi suggested that the curves shown in Fig. 2.9 be used to determine the value of c . The author himself points out that the method appears satisfactory for small values of the mobility number Y ; however, for large values of Y , there is a large amount of scatter.

2.3.1.3 Expressions of Kishi (1980)

Kishi (1980) presented friction factor relations similar to those of Garde and Raju (1966). Different regimes are reflected by different constants and powers in a general-form formula as follows:

Dunes of type I ($Y < 0.02Z^{1/2}$) : $c = 13.868(W-1)(Fr)^{-1}(R/k_s)^{-1/2}$

Dunes of type II ($Y = 0.02Z^{1/2}$) : $c = 8.909$

Transition I ($0.02Z^{1/2} < Y < 0.02Z^{5/9}$) : $c = 7.293(W-1)^{-3/7}(Fr)^{3/7}(R/k_s)^{3/14}$

Plane bed ($0.02Z^{5/9} < Y < 0.07Z^{2/5}$) : $c = 6.901(R/k_s)^{1/6}$

Antidune ($Y > 0.07Z^{2/5}$) : $c = 21.822(W-1)(Fr)^{-1}(R/k_s)^{-1/10}$

These expressions are valid for $0.375 \text{ mm} < D_{50} < 3.6 \text{ mm}$.

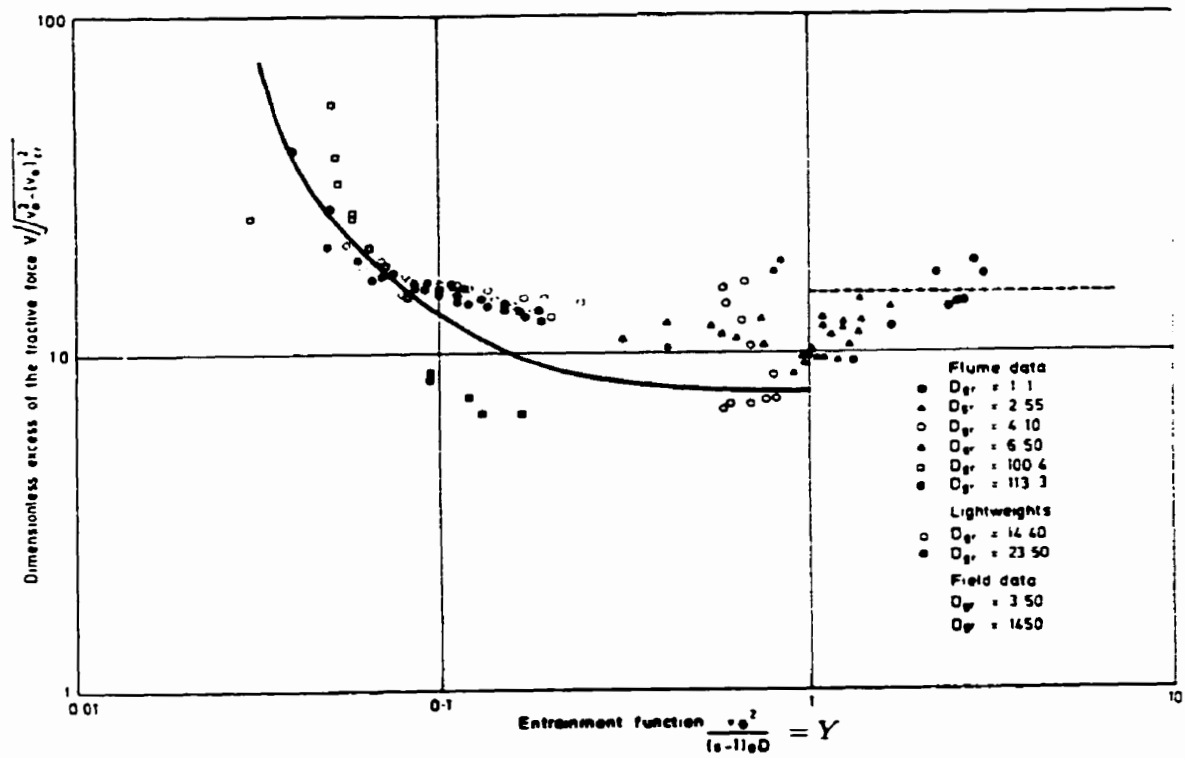


Figure 2.9: Resistance in an alluvial channel as a function of the entrainment function (from Raudkivi (1967))

2.3.1.4 Method of White, Paris and Bettess (1979)

These authors have attempted to take into account different behavior of the alluvial streams depending on the grain size of the material. Fine and coarse sediment were defined in terms of the material number $\xi = (\gamma_s D^3 / \rho \nu^2)^{1/3}$. According to these authors, sediments became truly 'fine' at $\xi = 1$ ($\simeq 0.04$ mm for sand) and 'coarse' when ξ exceeded 60 ($\simeq 2.5$ mm for sand). Intermediate sizes were termed 'transitional'. In addition to the dimensionless variable ξ , the friction factor depends on the following form of the mobility number:

$$F_{gr} = \frac{v_*^n}{\sqrt{(\gamma_s/\rho)D}} \left\{ \frac{v}{\sqrt{32} \log(10h/D)} \right\}^{1-n} \quad (2.30)$$

Eq. (2.30) can be written as

$$F_{gr} = \left\{ \sqrt{32} \log(10Z) \right\}^{n-1} \sqrt{Y} c^{1-n} \quad (2.31)$$

Here n is an exponent which varies from 1.0 for truly 'fine' sediments ($\xi = 1.0$) to 0 for coarse sediments ($\xi \geq 60$). Thus for fine sediments

$$F_{fg} = \sqrt{Y} \quad (2.32)$$

and for coarse sediments

$$F_{cg} = \left\{ \sqrt{32} \log(10Z) \right\}^{-1} \sqrt{Y} c \quad (2.33)$$

By using a selection of flume data covering a wide range of particle sizes, it was found that data could be fitted by the following functions:

$$\frac{F_{gr} - A}{F_{fg} - A} = 1 - 0.76 \left\{ 1 - \frac{1}{e^{(\log \xi)^{1.7}}} \right\} \quad \text{if } D = D_{35} \text{ (bed material)} \quad (2.34)$$

$$\frac{F_{gr} - A}{F_{fg} - A} = \phi(\xi) = 1 - 0.70 \left\{ 1 - \frac{1}{e^{1.4(\log \xi)^{2.65}}} \right\} \quad \text{if } D = D_{65} \text{ (surface material)} \quad (2.35)$$

The parameters A and n in the expressions above are given by

$$\left. \begin{array}{l} n = 0.0 \\ A = 0.17 \end{array} \right\} \quad \text{if } \xi \geq 60 \quad (2.36)$$

$$\left. \begin{aligned} n &= 1.0 - 0.56 \log \xi \\ A &= \frac{0.23}{\sqrt{\xi}} + 0.14 \end{aligned} \right\} \text{if } 1 \leq \xi < 60 \quad (2.37)$$

The following procedure for the determination of friction factor c and of the flow rate Q has been suggested by the authors. Given D, h, S :

1. Determine $v_* = \sqrt{gSh}$.
2. Determine $\xi = (\gamma_s D^3 / \rho \nu^2)^{1/3}$, and then calculate the parameters n and A from Eqs. (2.36) and (2.37).
3. Compute $F_{fg} = \sqrt{Y}$ (Eq. (2.32)).
4. Compute F_{gr} from Eq. (2.34) or (2.35).
5. Calculate c from Eq. (2.31), viz

$$F_{gr} = \left\{ \sqrt{32} \log(10Z) \right\}^{n-1} \sqrt{Y} c^{1-n}$$

6. Calculate v from $c = v/v_*$, and $Q = vBh$.

2.3.2 Approaches based on equivalent roughness concept

2.3.2.1 Method of Brownlie (1983)

In Brownlie's approach, the friction factor for (rough turbulent) flow over a bed covered by dunes is defined by the well-known power-law equation:

$$c = \frac{v}{\sqrt{gRS}} = a \left(\frac{R}{k_d} \right)^{1/6} \quad (2.38)$$

where the roughness height k_s in the original equation has been replaced by a measure of the dune height k_d , and a is a coefficient of proportionality. Introducing

$$q_* = \frac{q}{(gD^3)^{1/2}} \quad (2.39)$$

(where $q = vh$ is the specific flow rate) and solving v from Eq. (2.39), the following expression for v is obtained

$$v = \frac{(gD^3)^{1/2} q_*}{h} \quad (2.40)$$

Substitution of Eq. (2.40) into Eq. (2.38), after rearrangement, yields

$$\frac{RS}{D} = a^{-0.6} \left(\frac{k_d S}{D} \right)^{0.1} (q_* S)^{0.6} . \quad (2.41)$$

Arguing that the ratio of dune height and sediment size is a function of the power expenditure represented by $q_* S$, Brownlie assumed that k_d/D is proportional to $q_* S$ raised to some undetermined power. The effect of non-uniform sediment materials is considered by using the geometric standard deviation σ_g . With these assumptions, Eq. (2.41) becomes

$$\frac{RS}{D} = w(q_* S)^x S^y \sigma_g^z , \quad (2.42)$$

where w , x , y , and z are constants to be fitted empirically. Through multiple regression of large amounts of data, the best fit for the lower flow regime was found to be

$$\frac{R}{D} = 0.3724(q_*)^{0.6539} S^{-0.2542} (\sigma_g)^{0.1050} , \quad (2.43)$$

and for the upper regime

$$\frac{R}{D} = 0.2836(q_*)^{0.6248} S^{-0.2877} (\sigma_g)^{0.08013} . \quad (2.44)$$

For a given set of q , S , D , σ_g , two values of the hydraulic radius R (and thus two values of the flow depth) can be found from these two equations, one for upper regime and the other for lower regime. According to Brownlie, the flow regime is determined on the basis of the following three dimensionless parameters

$$F_g = \frac{v}{\sqrt{\gamma_s g D}}, \quad \frac{D}{\delta}, \quad S . \quad (2.45)$$

Here F_g is the grain size Froude number, D/δ is the ratio of grain size to the viscous sublayer thickness, where $\delta = 11.6\nu/v'_*$, v'_* being the shear velocity corresponding to the upper flow regime given by Eq. (2.44). Brownlie defined the upper limit of the lower flow regime by the following relation

$$\log \left(\frac{F_g}{F'_g} \right) = \begin{cases} -0.2026 + 0.07026 \log \left(\frac{D}{\delta} \right) + 0.9330 \left(\log \left(\frac{D}{\delta} \right) \right)^2 & \text{for } D/\delta < 2 \\ \log(0.8) & \text{for } D/\delta \geq 2 \end{cases} \quad (2.46)$$

and the lower limit of the upper flow regime by

$$\log \left(\frac{F_g}{F'_g} \right) = \begin{cases} -0.02469 + 0.1517 \log \left(\frac{D}{\delta} \right) + 0.8381 \left(\log \left(\frac{D}{\delta} \right) \right)^2 & \text{for } D/\delta < 2 \\ \log(1.25) & \text{for } D/\delta \geq 2 \end{cases} \quad (2.47)$$

where

$$F'_g = 1.74S^{-1/3} \quad (2.48)$$

Between these two limits, there is a transition region. The relations above are shown in Fig. 2.10.

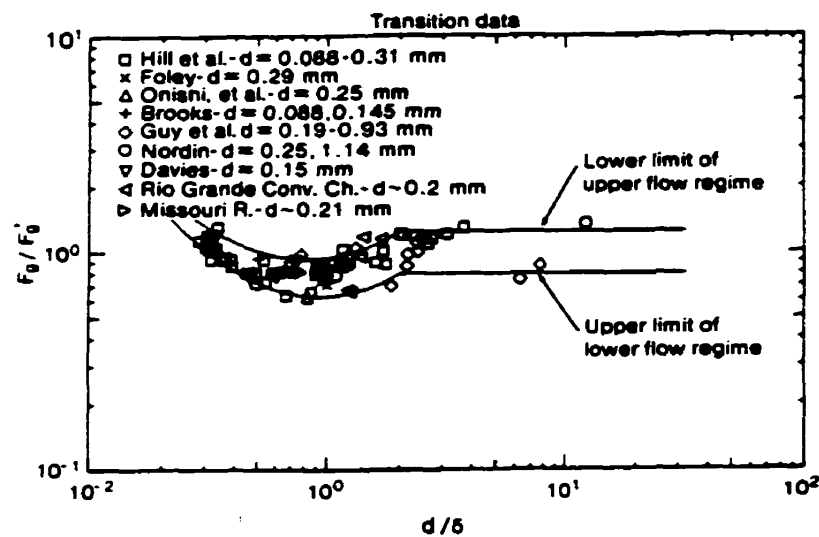


Figure 2.10: Transition from lower flow regime to upper flow regime (from Brownlie (1983))

The following procedure can be adopted for determination of flow depth and friction factor (see Chang (1988)):

Given $q, D, \sigma_g, S,$

1. Determine R for lower and upper regime, from Eqs. (2.43) and (2.44) respectively.
2. Determine $v = q/R$ and substitute v into the first relation of (2.45) to determine F_g .

3. Determine $D/\delta = 11.6\nu/v_*'$ and F_g' (from Eq. (2.48)).
4. Compute F_g/F_g' . In conjunction with the value of D/δ calculated in Step 3, use Fig. 2.10 to determine the valid flow regime.
5. Use the proper hydraulic radius R and the flow velocity v to calculate the friction factor $c = v/\sqrt{gSR}$.

2.3.2.2 Method of van Rijn (1982)

According to van Rijn, the friction factor c is given by

$$c = \frac{v}{v_*} = 2.5 \ln \left(11 \frac{R}{K_s} \right) , \quad (2.49)$$

where K_s is the equivalent roughness. By regression analysis of a large number of flume and field data with $B/h > 5$ and $h/K_s > 10$, the following expression for K_s was presented by this author:

$$K_s = 3D + 1.1\Delta(1 - e^{-25\Delta/\Lambda}) \quad (0.01 \leq \Delta/\Lambda \leq 0.2) \quad (2.50)$$

In the case of dunes, Eqs. (2.24) and (2.25) (of this author) for Δ/Λ and Δ/h are to be used.

2.3.3 Divided approaches

The division of total flow resistance into grain resistance and form resistance was first introduced by Einstein and Barbarossa (1952), and was followed by Engelund (1966). Yalin (1964), etc.

The divided resistance approach considers the total bed shear stress to be the sum of the bed shear stress due to the bed surface roughness (τ_0') and the bed shear stress due to the bed forms (τ_0''):

$$\tau_0 = \tau_0' + \tau_0'' . \quad (2.51)$$

Since

$$\frac{\tau_0}{\rho} = gSR , \quad (2.52)$$

Eq. (2.51) can also be expressed as

$$R = R' + R'' \quad (2.53)$$

or as

$$S = S' + S'' \quad (2.54)$$

2.3.3.1 Method of Einstein and Barbarossa (1952)

According to Einstein and Barbarossa, the friction factor contributed by bed form roughness is assumed to be of the form

$$\frac{v}{v_*''} = F(\Psi') \quad (2.55)$$

where Ψ' is given by

$$\Psi' = \frac{\gamma_s D_{35}}{R' S} \quad (2.56)$$

The power-law equation is used for calculation of average velocity:

$$\frac{v}{v_*'} = 7.66 \left(\frac{R'}{k_s'} \right)^{1/6} \quad (2.57)$$

where $k_s' = D_{65}$.

The functional relationship $F(\Psi')$ was developed based on field data as shown in Fig. 2.11. For a given set of q, S, D, h , the friction factor c can be computed following the procedure given below:

1. Assume a value for the flow velocity v .
2. Compute R' from Eq. (2.57).
3. Compute Ψ' from Eq. (2.56).
4. Compute v_*'' from Eq. (2.55) with the aid of the graph in Fig. 2.11, and determine R'' (from $v_*'' = \sqrt{gSR''}$).
5. Since $R = R' + R''$, the hydraulic radius R and thus h can be found.

6. Substitute h into $v = q/h$, and calculate a new value for v . Compare it with the assumed v (in Step 1), adjust the value, and repeat above steps until satisfactory solution is obtained.
7. Determine friction factor $c = v/v_*$.

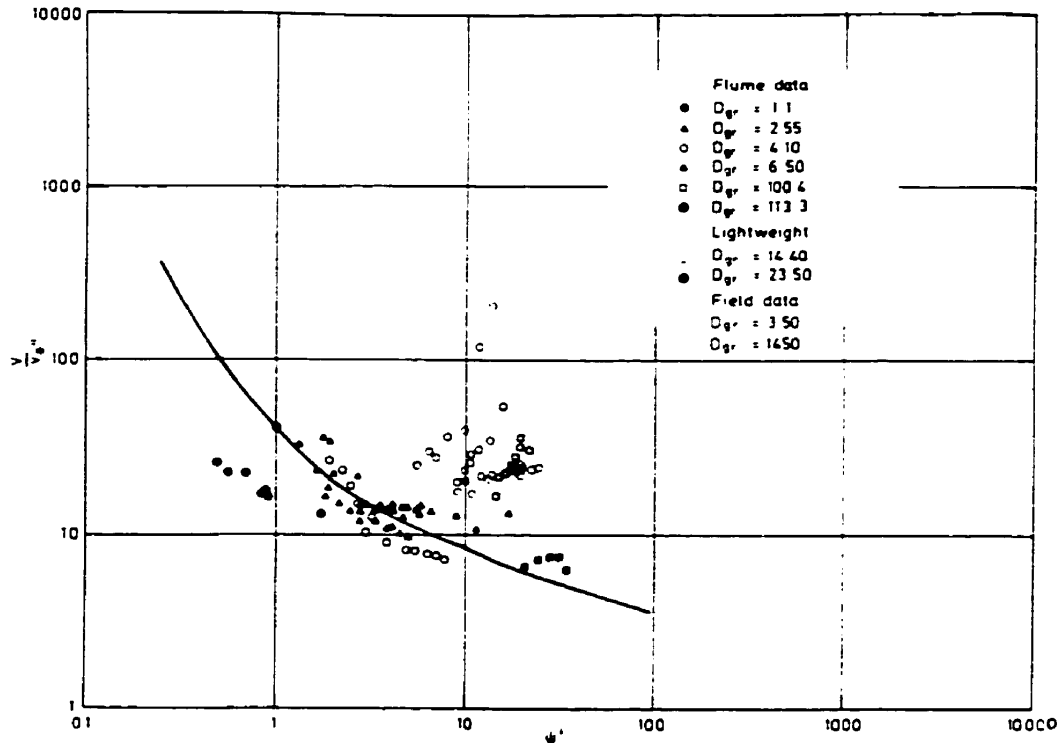


Figure 2.11: Friction loss due to bed form roughness as a function of sediment transport (from Einstein (1952))

2.3.3.2 Method of Engelund (1966)

Engelund assumed that energy gradient S can be divided into two parts $S = S' + S''$ (Eq. (2.54)), where S' is due to the bed surface roughness and S'' is due to the expansion losses associated with flow separation downstream of the dune crest. The head loss $\Delta H''$ may be estimated from

$$\Delta H'' = \alpha \frac{(v_1 - v_2)^2}{2g} = \frac{\alpha}{2g} \left(\frac{q}{h - \frac{\Delta}{2}} - \frac{q}{h + \frac{\Delta}{2}} \right)^2 \approx \alpha \frac{v^2}{2g} \left(\frac{\Delta}{h} \right)^2 \quad (2.58)$$

where α is loss coefficient, v_1 is the mean velocity above the crest, v_2 is the velocity over the trough. The energy gradient S'' is the head loss $\Delta H''$ divided by the distance

of one wavelength Λ , and thus

$$S'' = \frac{\alpha \Delta^2}{2 \Lambda h} Fr^2 \quad (2.59)$$

where Fr is the Froude number. Substituting Eq. (2.59) into (2.54) yields

$$S = S' + \frac{\alpha \Delta^2}{2 \Lambda h} Fr^2 \quad (2.60)$$

Replacing the flow depth h with hydraulic radius R , and multiplying both sides by $\gamma_s R/D$ yields

$$\frac{\gamma_s RS}{D} = \frac{\gamma_s RS'}{D} + \frac{\alpha \gamma_s \Delta^2}{2 \Lambda D} Fr^2 \quad (2.61)$$

With the assumption that $\tau'_0 = \gamma S' R = \gamma S R'$, Engelund derives

$$\tau_* = \tau'_* + \tau''_*$$

where τ_* , τ'_* and τ''_* are dimensionless total shear stress, dimensionless shear stress due to grain roughness, and dimensionless shear stress due to bed-form roughness respectively:

$$\tau_* = \frac{RS}{(W-1)D} \quad (2.62)$$

$$\tau'_* = \frac{R'S}{(W-1)D} \quad (2.63)$$

$$\tau''_* = \frac{\alpha \Delta^2}{2 (W-1)\Lambda D} Fr^2 \quad (2.64)$$

The following logarithmic resistance formula is adopted for grain roughness:

$$\frac{v}{(gR'S)^{1/2}} = 6 + 2.5 \ln \left(\frac{R'}{2.5D} \right) . \quad (2.65)$$

Although Engelund was the first to suggest resistance equation where the geometric properties of bed forms are present, he did not treat Δ and Λ explicitly and eventually bypassed their use.

Engelund and Hansen (1967) proposed the following relationship for lower flow regime :

$$\tau'_* = 0.06 + 0.4\tau_*^2 \quad (\tau'_* < 0.55) \quad (2.66)$$

For upper flow regime ($0.55 < \tau'_* < 1$),

$$\tau'_* = \tau_* \quad (2.67)$$

For a given set of q, D, S , the flow depth can be computed iteratively using Engelund's method. The steps are as follows:

1. Assume a trial value for R' , compute τ'_* and v from Eqs. (2.63) and (2.65).
2. Compute τ_* from Eqs. (2.66) and (2.67).
3. Compute flow depth h from Eq. (2.62).
4. Compute discharge $q = vh$. Compare it with given q and repeat the steps until the computed value matches the given q .

2.3.3.3 Method of Yalin (1964)

Yalin (1964), independently from Engelund, has derived a friction factor relation similar to that of Engelund. Just like Engelund, Yalin suggested the division of the energy gradient S into two parts: one part associated with the expansion from b to c in Fig. 2.12; the other part due to the skin friction along ab . The head loss due to the expansion from b to c can be expressed as:

$$\zeta_{bc} = \alpha \frac{(v_b - v_c)^2}{2g} \quad (2.68)$$

Substituting the equation of continuity, viz

$$v_b \left(h - \frac{\Delta}{2} \right) = v_c \left(h + \frac{\Delta}{2} \right) = vh \quad (2.69)$$

into Eq. (2.68) yields:

$$\zeta_{bc} = \left(\frac{\Delta/h}{1 - 1/4(\Delta/h)^2} \right)^2 \cdot \alpha \cdot \frac{v^2}{2g} \quad (2.70)$$

If $\Delta/h \leq 1/10$, then $1 - 1/4(\Delta/h)^2 \approx 1$. Also for usual values of angle of repose ψ , $\alpha \approx 1$. Thus,

$$\zeta_{bc} \approx \left(\frac{\Delta}{h} \right)^2 \frac{v^2}{2g}, \quad (2.71)$$

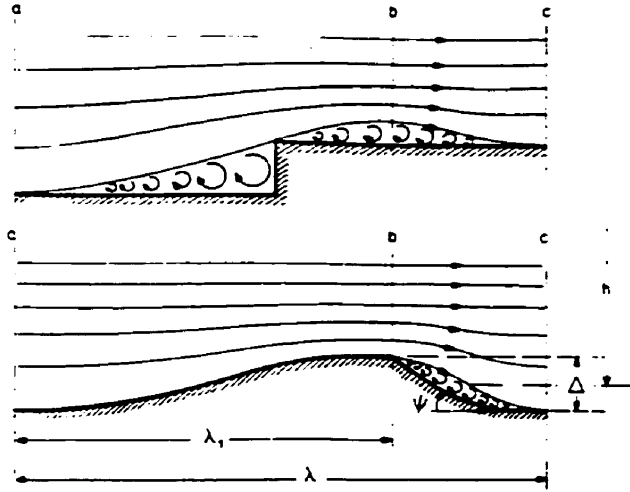


Figure 2.12: Friction loss due to expansion and skin friction (from Yalin (1964))

the corresponding energy gradient being:

$$S'' = \frac{\zeta_{bc}}{\Lambda} = \frac{1}{2} \cdot \left(\frac{\Delta}{h}\right) \cdot \left(\frac{\Delta}{\Lambda}\right) \cdot \frac{v^2}{gh} \quad (2.72)$$

where Δ is the bed form height and Λ is the bed form length. The energy gradient due to skin friction along ab can be expressed as:

$$S' = \frac{\Lambda_1}{\Lambda} \cdot \frac{\kappa^2}{[\ln(a \frac{h}{k_s})]^2} \cdot \frac{v^2}{gh} \approx \frac{\kappa^2}{[\ln(a \frac{h}{k_s})]^2} \cdot \frac{v^2}{gh} \quad (2.73)$$

where $\kappa = 0.4$, and $a = 11$. Thus:

$$S = S' + S'' = \left[\frac{1}{[\frac{1}{\kappa} \ln(a \frac{h}{k_s})]^2} + \frac{1}{2} \left(\frac{\Delta}{\Lambda}\right)^2 \frac{\lambda}{h} \right] \frac{v^2}{gh} \quad (2.74)$$

Therefore, the friction factor c can be expressed as:

$$c = \frac{v}{v_*} = \frac{v}{\sqrt{gSh}} = \frac{1}{\sqrt{\frac{1}{[\frac{1}{\kappa} \ln(a \frac{h}{k_s})]^2} + \frac{1}{2} \left(\frac{\Delta}{\Lambda}\right)^2 \frac{\lambda}{h}}} \quad (2.75)$$

If there are no bed forms, $\Delta = 0$, and Eq. (2.75) reduces to:

$$c_0 = \frac{1}{\kappa} \ln \left(a \frac{h}{k_s} \right) \quad (2.76)$$

From Eqs. (2.75) and (2.76), the following relationship between friction factors for plane bed and a bed covered with bed forms is obtained:

$$\frac{1}{c^2} = \frac{1}{c_0^2} + \frac{1}{2} \left(\frac{\Delta}{\Lambda} \right)^2 \frac{\Lambda}{h} \quad (2.77)$$

(see Yalin (1992)).

In some cases, more than one mode of bed forms can be present simultaneously. For example, ripples can be superimposed on dunes. Since energy losses are additive, the losses due to the bed different forms can therefore be added:

$$\frac{1}{c^2} = \frac{1}{c_0^2} + \frac{1}{2} \sum_{i=1}^2 \left(\frac{\Delta}{\Lambda} \right)_i^2 \frac{\Lambda_i}{h} \quad (2.78)$$

3.0 ANALYSIS OF EXISTING METHODS FOR PREDICTION OF DUNE STEEPNESS

3.1 General

As mentioned in Section 1.4, any dimensionless property Π_A of the two-phase motion under consideration is a function of, at most, three dimensionless variables X, Y, Z , or of their equivalent forms:

$$\Pi_A = \phi_{A_1}(X, Y, Z) = \phi_{A_2}(\xi, Y, Z) = \phi_{A_3}(\xi, \eta, Z) . \quad (3.1)$$

(see Section 1.4 and/or “List of Symbols” for definition of dimensionless variables above).

The dune steepness Δ/Λ and the dune length Λ/h are dimensionless properties of the two-phase motion. Therefore, their mathematical forms must be of the following types:

$$\frac{\Delta}{\Lambda} = \phi_{\Delta_1}(X, Y, Z) = \phi_{\Delta_2}(\xi, Y, Z) = \phi_{\Delta_3}(\xi, \eta, Z) \quad (3.2)$$

$$\frac{\Lambda}{h} = \phi_{\Lambda_1}(X, Y, Z) = \phi_{\Lambda_2}(\xi, Y, Z) = \phi_{\Lambda_3}(\xi, \eta, Z) . \quad (3.3)$$

As yet another dimensionless property of the two-phase flow, the friction factor c must also be given by:

$$c = \phi_{c_1}(X, Y, Z) = \phi_{c_2}(\xi, Y, Z) = \phi_{c_3}(\xi, \eta, Z) . \quad (3.4)$$

For the case of rough turbulent flows, ξ (which reflects the influence of viscosity) is no longer a parameter and Δ/Λ , Λ/h and c are functions of only two variables (Y and Z ; or η and Z).

3.2 Analysis of dune steepness expressions

3.2.1 Expressions of Yalin and Karahan (1979), Yalin and Scheuerlein (1988), and Yalin (1992)

As mentioned in the Literature Review, and in agreement with the considerations in Section 3.1, these authors give Δ/Λ as a function of two variables

$$\frac{\Delta}{\Lambda} = \Phi(\eta, Z) = \left(\frac{\Delta}{\Lambda}\right)_{max} \zeta e^{1-\zeta}, \quad \text{with } \zeta = \frac{\eta - 1}{\bar{\eta}_d - 1} \quad (3.5)$$

where $(\Delta/\Lambda)_{max} = f_1(Z)$ and $\bar{\eta}_d = f_2(Z)$ ($\bar{\eta}_d$ is the value of η which corresponds to $(\Delta/\Lambda)_{max}$ of a Δ/Λ -curve representing a certain value of Z).

Eq. (3.5) implies that one can compute a family of Δ/Λ -curves for the $(\Delta/\Lambda; \eta)$ -plane, each individual curve corresponding to $Z = (const)_i$ (see Fig. 2.3). Each Δ/Λ -curve in the $(\Delta/\Lambda; \eta)$ -plane is such that Δ/Λ first increases as η increases (from unity onwards), then it reaches a maximum, and finally it decreases for “large” values of η yielding the “flat bed at advanced stages”.

The forms of the functions $(\Delta/\Lambda)_{max} = f_1(Z)$ and $\bar{\eta}_d = f_2(Z)$ are given in Yalin and Scheuerlein (1988) as

$$(\Delta/\Lambda)_{max} = 0.163e^{-1}(1 - e^{-0.01Z}) \quad (3.6)$$

$$\bar{\eta}_d = \frac{1 - e^{-0.01Z}}{0.0778} + 1 \quad (3.7)$$

In his most recent work, Yalin (1992) proposed the equations below for $(\Delta/\Lambda)_{max}$ and $\bar{\eta}_d$:

$$(\Delta/\Lambda)_{max} = 0.06(1 - e^{-0.008Z}) \quad (3.8)$$

$$\bar{\eta}_d = 14(1 - e^{-0.003Z}) + 2 \quad (3.9)$$

Eqs. (3.6) and (3.8) for $(\Delta/\Lambda)_{max}$ are plotted in Fig. 3.1. Eqs. (3.7) and (3.9) for $\bar{\eta}_d$ are plotted in Fig. 3.2.

It is interesting to notice that Eqs. (3.6) and (3.8) for $(\Delta/\Lambda)_{max}$ yield very similar results. However, Eqs. (3.7) and (3.9) yield somewhat different values for $\bar{\eta}_d$ if $Z < \approx 600$.

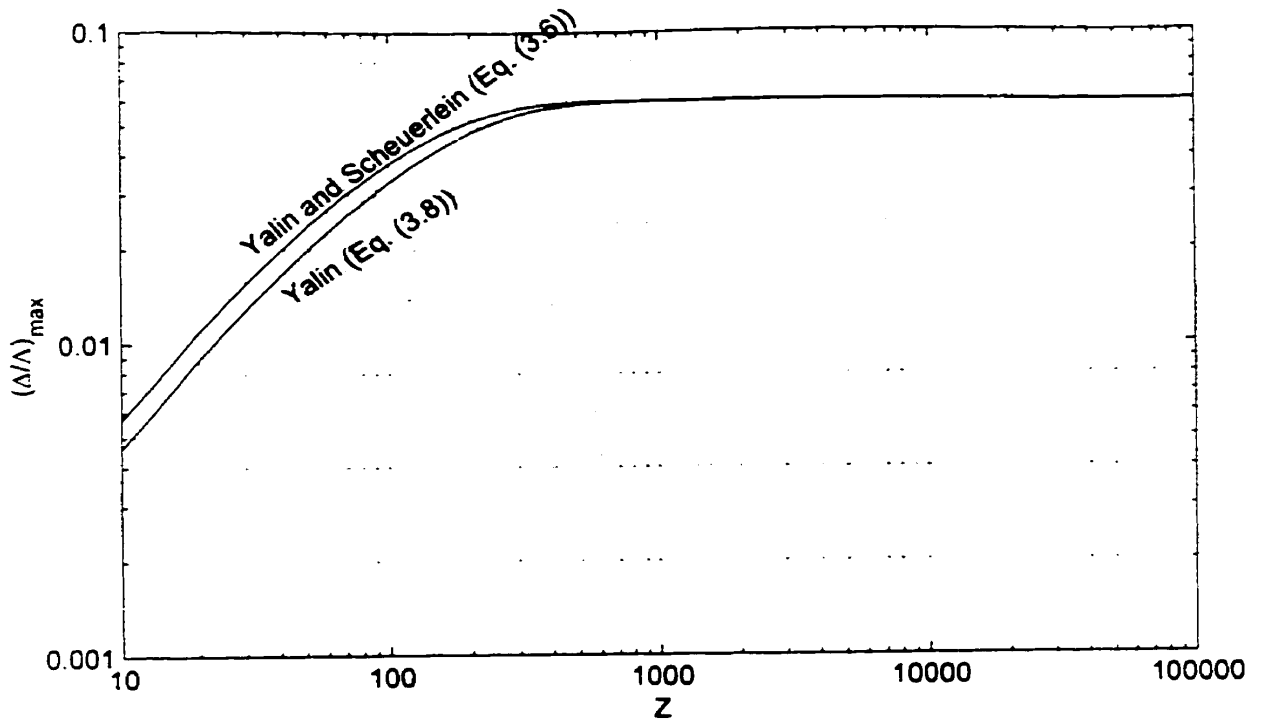


Figure 3.1: Plot of $(\Delta/\Lambda)_{\max}$ versus Z

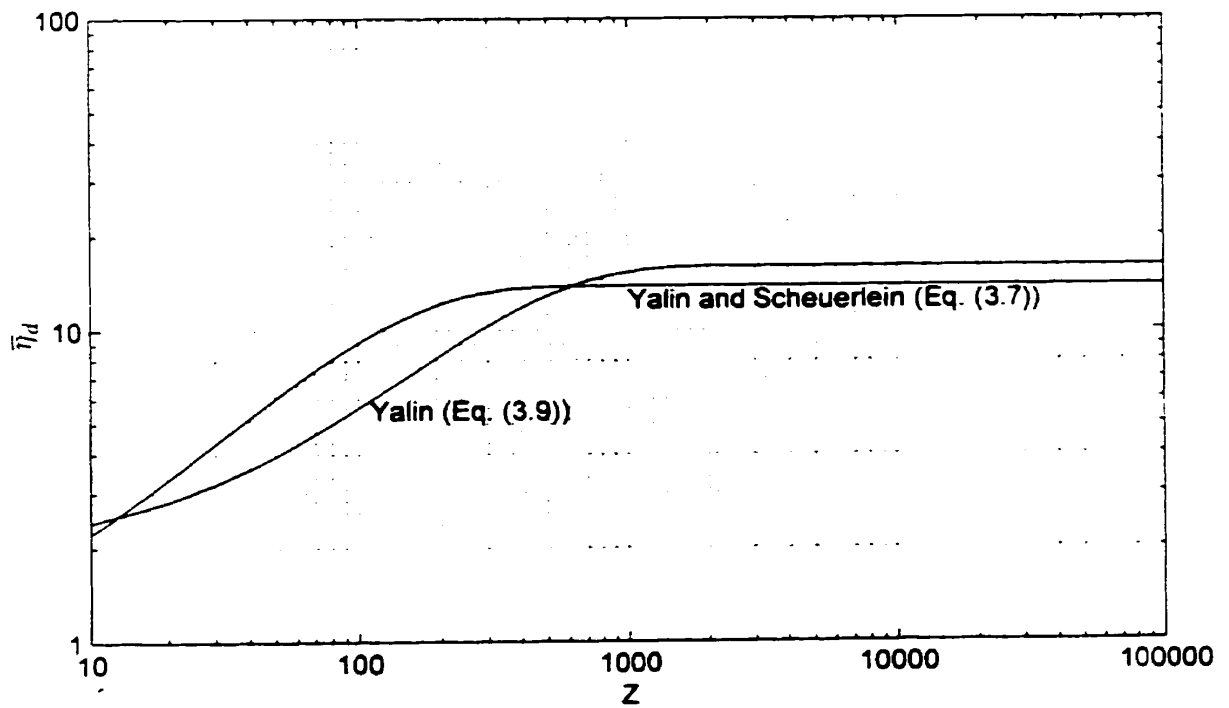


Figure 3.2: Plot of $\bar{\eta}_d$ versus Z

The Δ/Λ -curves are plotted versus η in Figs. 3.3 and 3.4.: Fig. 3.3 corresponds to the expressions of Yalin and Scheuerlein (1988), whereas Fig. 3.4 corresponds to the expression of Yalin (1992). The discrepancy between the Δ/Λ -curves in Figs. 3.3 and 3.4 is larger for smaller values of Z . For large values of Z , the Δ/Λ -curves in Figs. 3.3 and 3.4 are nearly the same. If the data plotted in Fig. 2.3 is transferred to these figures, then it becomes clear that the expressions of Yalin and Scheuerlein (1988) yield a better agreement with the data than the later expressions by Yalin (1992), for small Z -values.

The Δ/Λ -curves of Yalin and Scheuerlein are plotted versus η in Fig. 3.5 in a normal-normal scale. Although this plot is equivalent to that in Fig. 3.3, it is presented in this thesis so that the form of steepness function becomes clearer: for example, this figure clearly shows how a steepness curve approaches asymptotically to the η -axis.

From Figs. 3.2 and 3.3 it should be clear that $\bar{\eta}_d$ (according to Yalin and Scheuerlein (1988)) is an increasing function of Z , which yields $\bar{\eta}_d = 16$ for all values of $Z > \approx 500$. From Figs. 3.1 and 3.3, one infers that $(\Delta/\Lambda)_{max}$ is also an increasing function of Z , which yields $(\Delta/\Lambda)_{max} \approx 0.06$, for all values of $Z > \approx 500$. Thus in Fig. 3.3, the $(\Delta/\Lambda; \eta)$ -curves for $Z \geq 500$ are all in coincidence.

From the content of this Section, it should be clear that, of all the equations suggested by Yalin and his co-workers, those presented in Yalin and Scheuerlein (1988) fit the data used by these authors the best. For this reason, in this thesis, only the expressions of Yalin and Scheuerlein will be used from here onwards.

3.2.2 Expression of van Rijn (1982)

As mentioned in the Literature Review, this author gives Δ/Λ as

$$\frac{\Delta}{\Lambda} = \Phi(T, Z) = 0.015 \left(\frac{1}{Z}\right)^{0.3} (1 - e^{-0.5T})(25 - T) \quad (3.10)$$

where T is the “transport stage parameter” as defined by Eq. (2.22), viz

$$T = \frac{(v'_*)^2 - (v_{*,\sigma})^2}{(v_{*,\sigma})^2} .$$

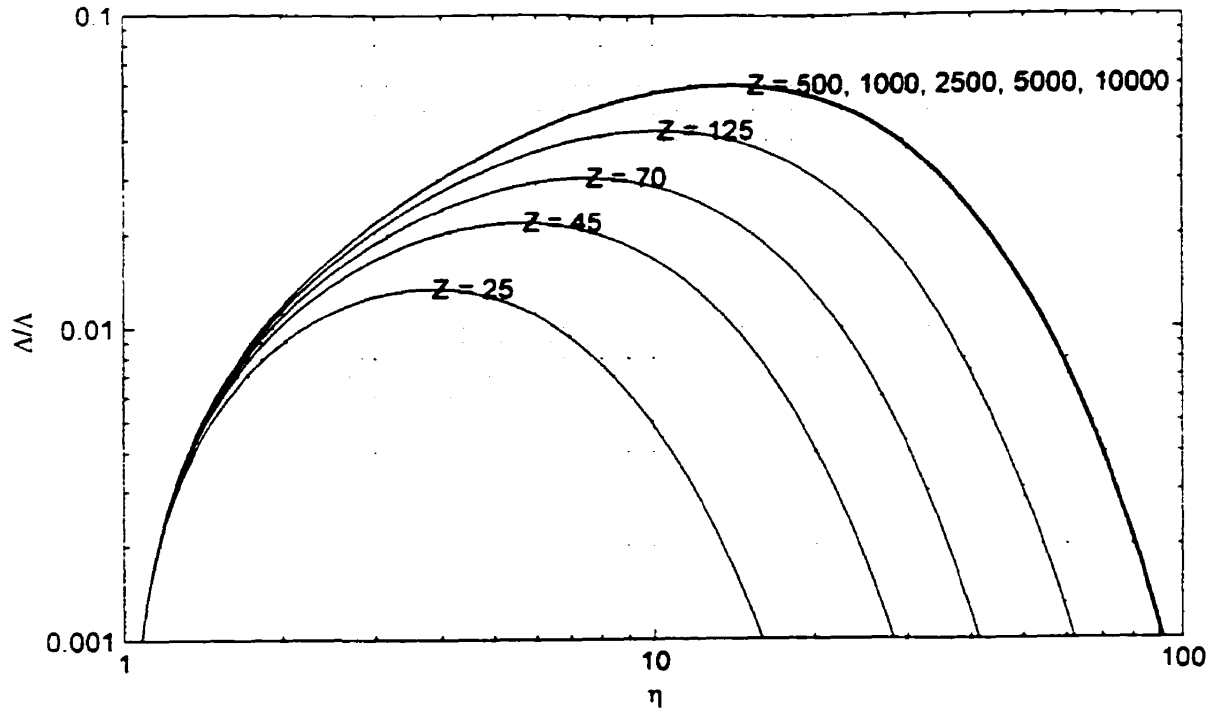


Figure 3.3: Family of Δ/Λ -curves by Yalin and Scheuerlein (1988)

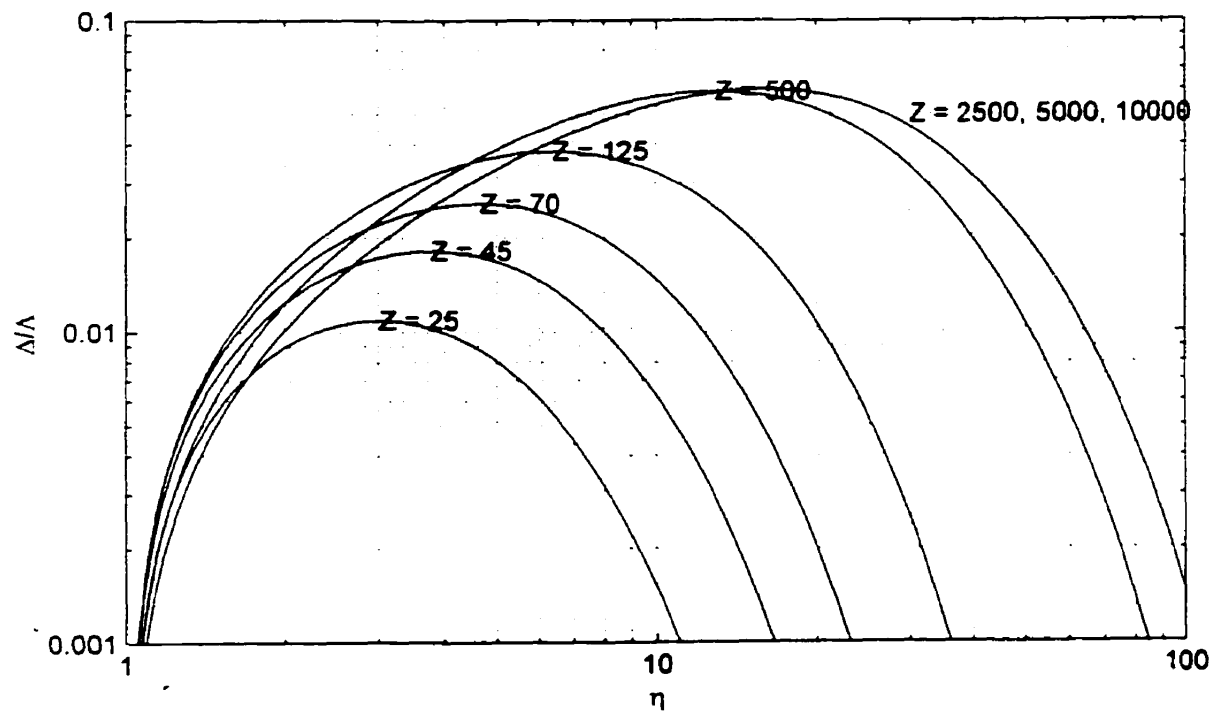
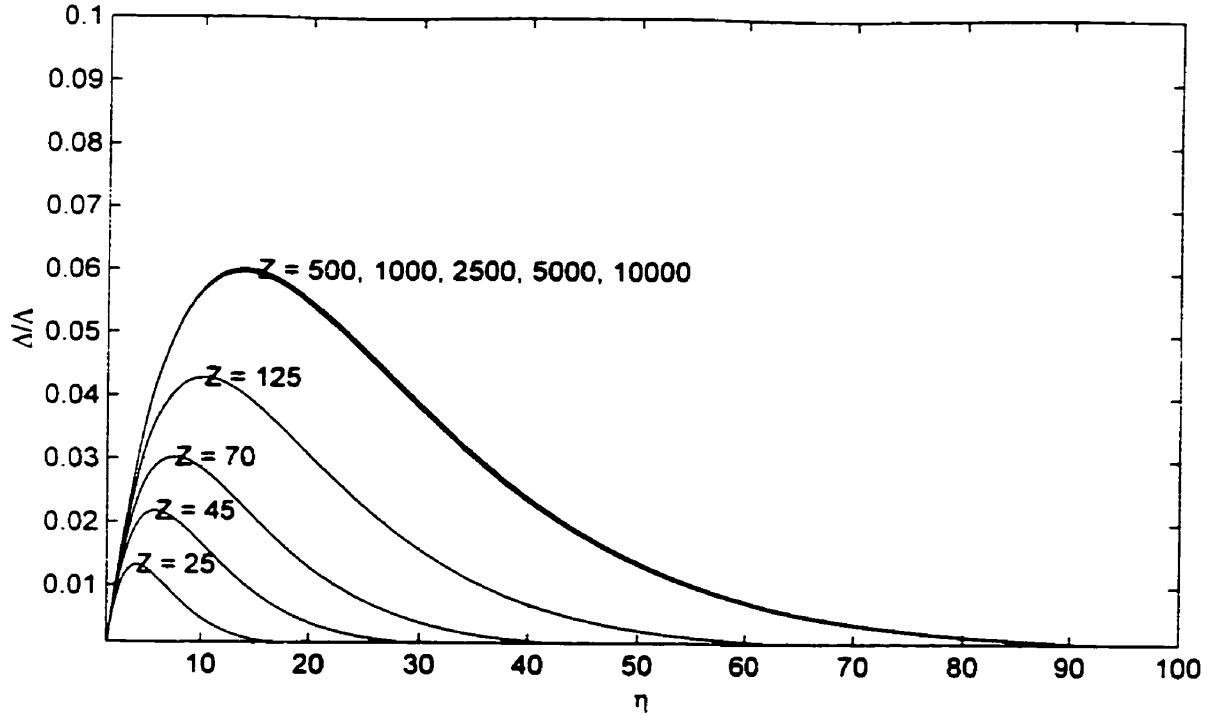


Figure 3.4: Family of Δ/Λ -curves by Yalin (1992)



**Figure 3.5: Family of Δ/Λ -curves by Yalin and Scheuerlein (1988)
(normal scale)**

Here v'_* is the grain shear velocity. van Rijn defined v'_* in terms of the dimensional grain Chézy coefficient C_0 , i.e., as

$$v'_* = v \frac{\sqrt{g}}{C_0}$$

(see Eq. (2.23)). The friction factor C_0 is given by van Rijn as

$$C_0 = 18 \log \left(12 \frac{R}{3D_{90}} \right) \quad (3.11)$$

where R is the hydraulic radius.

Eq. (3.11) can be written as

$$C_0 = 7.82 \ln \left(12 \frac{R}{3D_{90}} \right) \quad (3.12)$$

Since $C_0 = c_0 \sqrt{g}$, where c_0 is the (flat bed) dimensionless Chézy coefficient, Eq. (3.12) yields

$$c_0 = 2.5 \ln \left(12 \frac{R}{3D_{90}} \right) \quad (3.13)$$

If $3D_{90}$ is identified with the grain roughness k_s , then it becomes clear that Eq. (3.13) is, essentially, the well known logarithmic velocity expression of rough turbulent flows (see e.g. Schlichting (1968), Yalin (1992)):

$$c_0 = 2.5 \ln \left(11 \frac{R}{k_s} \right) . \quad (3.14)$$

Just like in Yalin and Karahan's model, Eq. (3.10) of van Rijn implies that the function Δ/Λ is represented by a family of Δ/Λ -curves for the $(\Delta/\Lambda; T)$ -plane, each individual curve corresponding to $Z = (const)_i$.

The family of Δ/Λ -curves for the $(\Delta/\Lambda; T)$ -plane is shown in Fig. 3.6. Δ/Λ first increases with T , then it reaches a maximum, and finally it decreases for large values of T , yielding the "flat bed at advanced stages". Let \bar{T}_d denotes the value of T corresponding to $(\Delta/\Lambda)_{max}$ of a Δ/Λ -curve, and \hat{T} the value of T for which the flat bed ($\Delta/\Lambda = 0$) at advanced stages is reached. As can be inferred from Fig. 3.6. $\bar{T}_d = const = 5$, whereas $\hat{T} = const = 26$ for any Z .

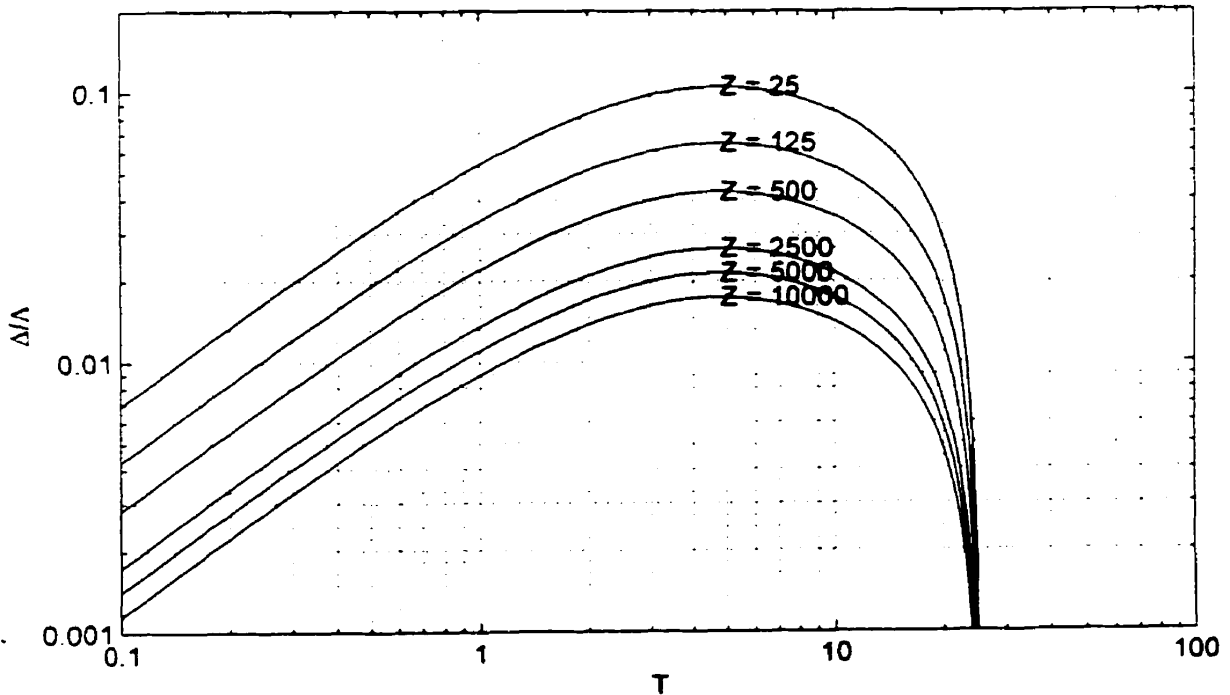


Figure 3.6: Family of Δ/Λ -curves by van Rijn (1982)

3.3 Comparison of expressions by Yalin and Scheuerlein and expression by van Rijn

It is difficult to compare the expressions by Yalin and Scheuerlein and that by van Rijn because of the fact that one involves η and another involves T . In this section, an attempt is made to relate the two parameters η and T and thus to compare the two expressions.

Substitution of $v_*' = v/c_0$ into the expression of T , viz Eq. (2.22), yields

$$T = \frac{v^2}{c_0^2} \cdot \frac{1}{v_{*cr}^2} - 1 \quad (3.15)$$

Since $v = cv_*$, the expression above can be written as

$$T = \frac{c^2}{c_0^2} \cdot \frac{v_*^2}{v_{*cr}^2} - 1 \quad (3.16)$$

i.e.

$$T = \left(\frac{c}{c_0}\right)^2 \eta - 1 \quad (3.17)$$

As mentioned previously, it is well known that the bed form steepness is an increasing function of Z . Thus if Z is very small, then the bed forms are also very small and $c/c_0 \rightarrow 1$, implying $T \approx \eta - 1$. In this case, the Δ/Λ -curves of Yalin and van Rijn corresponding to small Z , plotted in Figs. 3.3 and 3.6, respectively, can be compared. Take $Z = 25$. According to Yalin and Scheuerlein, $\bar{\eta}_d \approx 4$, $(\Delta/\Lambda)_{max} \approx 0.013$ and $\Delta/\Lambda = 0.001$ (implying flat bed at advanced stages) occurs for $\hat{\eta} \approx 18$. According to van Rijn, $\bar{T}_d = 5$, yielding $\bar{\eta}_d \approx 6$, $(\Delta/\Lambda)_{max} \approx 0.1$, and $\Delta/\Lambda = 0.001$ (implying flat bed at advanced stages) occurs for $\hat{T} \approx 25$, implying $\hat{\eta} \approx 26$. If the data of Yalin and Karahan (1979) for $Z = 25$ plotted in Fig. 2.3 is considered, then the values of $(\Delta/\Lambda)_{max} \approx 0.1$ and $\hat{\eta} \approx 26$ are shown to be unrealistic.

Consider now the case when Z is very large. According to van Rijn, the flat bed at advanced stages is reached for $T = \hat{T} = 25$. Since the steepness of bed forms at this stage is negligible, $c/c_0 \rightarrow 1$ and $T \approx \eta - 1$. Thus, $\hat{T} = 25$ of van Rijn implies $\hat{\eta} = 26$. However, the data indicates that the flat bed at advanced stages for large values of Z

is reached for values of $\hat{\eta} \gg 26$ (see e.g. Fig. 2.3, as well as Julien (1992) and Julien and Klaassen (1995)). The value of $\hat{\eta} \approx 100$ of Yalin and Scheuerlein (1988) appears to be much more realistic.

4.0 DEVELOPMENT OF PRESENT DUNE STEEPNESS EXPRESSION

4.1 General

As mentioned in Chapter 3, the dune steepness expressions of Yalin and co-workers (Yalin and Karahan (1979), Yalin and Scheuerlein (1988), Yalin (1992)) and van Rijn (1982) are based on the analysis of, essentially, laboratory data. Indeed, consider first the sources of the (extensive) data used by Yalin, which are given in the legend in Fig. 2.2. With the exception of the few data points from the Mississippi River, and even fewer data points from the Amazon River, the data was collected from laboratory experiments. The data sources used by van Rijn are given in the legend in Fig. 2.7. This author used considerably smaller amount of laboratory data than Yalin (only 84 data points); and a total of 22 data points from some rivers in the Netherlands (the IJssel, Waal and Rijn), the Rio Parana in Argentina, the Mississippi River, as well as data from some Japanese irrigation channels. The reason why these two authors have used such a small amount of river data is obviously that the geometric properties of dunes have seldom been reported in the literature. This is probably due to difficulties in measuring the dunes in the field (in contrast to measuring dunes in a laboratory flume). In fact, in the literature, almost no new measurements of dunes have been reported in recent years. The only exception are the field measurements of dunes reported by Julien (1992), and Julien and Klaassen(1995). The nature of the field data presented in these works is discussed below.

In summary, one can say that previous dune steepness expressions were developed on the basis of (essentially) laboratory data, with limited use of field data. This fact has been realized and emphatically pointed out by Julien (1992) and Julien and Klaassen (1995). In particular, these authors proceed to suggest that there is a disagreement between measured and predicted Δ/Λ for river data.

At this point, it should be mentioned that in terms of the dimensionless variables

X, Y and Z , the distinction between laboratory data and field (river) data is in the values of Z . In the case of laboratory experiments, typical values of $Z = h/D$ are in the range $Z < \approx 1500$; whereas in the case of river data, typical values of Z are $Z \gg 1500$.

The contents of the present chapter confirm that existing expressions for dune steepness are not valid for both laboratory (small Z) and field data (large Z). An attempt is made to determine an expression for dune steepness which is valid for the entire spectrum of Z -values.

From Chapter 3, it should be clear that, of the existing dune steepness expressions, the most sound from dimensional analysis standpoint, as well as agreement with the data (albeit laboratory data) is achieved by the expression of Yalin and Scheuerlein. The expression suggested by the author later in this chapter is a generalized form of the expression by these authors, and is meant to be applicable for the entire spectrum of possible Z -values.

4.2 Description of existing laboratory and field data

4.2.1 Data sources

The main data sources used in this thesis are Adams (1990) and the well known report by Brownlie (1981). In addition, data reported by Julien (1992) as well as data found in recent literature was used. The data is too extensive to be included in this thesis: the interested reader is referred to da Silva, Banerjee and Zhang (In press), where all of the data is presented.

The data compiled by Adams (1990) consists of 1,150 flume tests and 180 field tests, collected by forty three different authors (see References A). For each test, the following information is given: grain size D , submerged specific weight γ_s , flow depth h , slope S , flow velocity v , the bed form type, and the values of bed form length Λ and height Δ .

The data compiled by Brownlie comprises 5,263 laboratory tests and 1,764 field

tests, collected by 50 authors (see References B). For each test, the following information is given: grain size D , submerged specific weight γ_s , flow depth h , slope S , flow velocity v , and the bed form type. However, the bed form geometric characteristic (Λ and Δ) are not given. It should be noticed that most of the authors in Adams (1990) are included in Brownlie (1981). However, since this author does not define Λ and Δ , the data of Adams (1990) was preferred in all plots where Δ/Λ is used.

The data compiled by Julien comprises data from the rivers Jamuna, Missouri, Parana, and Zaire. These references to the original works where the data was first published is given in References C.

4.2.2 Data description

Different authors used different methods, criteria and accuracy in their data collection. This lack of standardization must inevitably result in scatter when data from different authors are combined. Thus, it seems appropriate to briefly present the main criteria used by different authors. It should be kept in mind, however, that “the extent of scatter is mainly because of the measurement of the elusive quantity Δ by various authors and thus methods, and *also* because of the stochastic nature of the bed deformation phenomenon itself” (Yalin and Karahan (1979)).

4.2.2.1 Grain size

In choosing the representative grain size D , various percentiles of the grain size distribution curve (from 35 to 90) were used by different authors.

4.2.2.2 Bed form type

As can be inferred from the previous section, usually authors classify the bed form observed as dunes, ripples, etc. However, it should be noted that there is a lack of homogeneity in different authors when classifying data points. For example, Vanoni and Brooks (1957) defined all their data as dunes with $4 < X < 7$, and average steepness of 0.15. These were very likely ripples. For the purpose of this thesis, data

were classified as ripples when $X < \approx 5.5$ and as dunes when $X > \approx 30$. In the case of some plots presented in this chapter, as well as in Chapter 5, data with $X > 20$ were plotted. This lower limit enables use of more data, from which more meaningful results can be obtained; its use is justified because for $20 < X < 30$ the ripples superimposed on dunes are negligible.

4.2.2.3 Development time

The present analysis assumes that bed forms are fully developed, i.e. they are in equilibrium with steady flow conditions. Yet, the full development of bed forms may not always have been achieved. Yalin and Karahan (1979) attribute much of the scatter in the sand wave data produced in the past to the fact that it does not correspond exclusively to fully developed sand waves. As explained by these authors, usually a certain waiting time, T_w , is specified and the flume measurements are conducted soon after this time has elapsed. Yet, being a quantity inversely proportional to the sediment transport rate, the development duration of sand waves (T) varies rather strongly with the relative flow intensity η . For example, if η is large, T can be 20 minutes; if η is small, T can be 20 hours (for the same granular material in the same flume). Thus, if T_w is "standardized", one can never be certain that $T_w \geq T$ is achieved for every run (every η), and thus that all measurements correspond to the fully developed sand waves. The development time is especially critical for field data, due to the fact that bed form formation always lags the flow conditions which create the bed forms. Most of the field data available to the author is that reported by Julien (1992) and Julien and Klaassen (1995). These data were collected during rather unsteady flow conditions. Thus, a criterion had to be adopted to determine whether the dunes were nearly fully developed. The criterion adopted is that dunes were considered developed if $5h \leq \Lambda \leq 8h$.

4.2.2.4 Grain roughness

In this thesis, following Yalin (1992), the granular roughness of bed surface is taken as $k_s \approx 2D$. However, it should be noted that different grain roughness expressions were adopted by these authors in their methods. For example, van Rijn used $k_s = 3D_{90}$, Yalin used $k_s = 2D_{50}$. Other expressions are suggested by authors such as Kampuis ($K_s = 2.5D_{90}$), Gladki ($k_s = 2.3D_{80}$), Hey ($k_s = 3.5D_{84}$) and Mahmood ($k_s = 5.1D_{84}$).

4.3 Determination of present dune steepness expression

4.3.1 General

As mentioned earlier in this chapter, the dune steepness expression developed herein rests on the earlier works by Yalin and Karahan (1979), Yalin and Scheuerlein (1988) and Yalin (1992). As can be inferred from Chapter 3, the expressions by these authors are of the type

$$\frac{\Delta}{\Lambda} = \left(\frac{\Delta}{\Lambda}\right)_{max} \zeta e^{1-\zeta}, \quad \text{where } \zeta = \frac{\eta - 1}{\bar{\eta}_d - 1} \quad (4.1)$$

The difference in the successive works by these authors is the different expressions which were adopted for $(\Delta/\Lambda)_{max}$ and $\bar{\eta}_d$.

In the following two sections, a series of plots are presented. The purpose of these plots is to investigate the nature of dune steepness as a function of η and Z . In Section 4.3.4, use is made of the insight gained from these plots in order to find an expression for the dune steepness.

4.3.2 Plots of dune steepness versus relative flow intensity

Consider the plots shown in Figs. 4.1, 4.2 and 4.3, where values of Δ/Λ are plotted versus (measured) η ; the data is classified according to Z . (For the present purpose,

disregard the broken lines in these figures - they will be discussed later on). For these plots, the bed forms were classified as dunes if $X > 20$. This lower limit for X , which leads to inclusion of more data in these plots, is justified in light of the uncertainty of dune existence region.

Fig. 4.1 corresponds to “small” values of Z ($Z \leq 1000$, say) typical of laboratory flows. The sources of the data in this plot are given in the legend in the figure; the data points in the ranges $20 \leq Z \leq 30$, $40 \leq Z \leq 50$ and $65 \leq Z \leq 75$ are due to Yalin and Karahan (1979). The value of Δ/Λ of each data point was computed as the ratio between the *measured* dune height Δ , and the *measured* dune length Λ .

Fig. 4.2 corresponds to “large” values of Z ($Z \geq 1000$, say) typical of field data. Owing to the scarcity in the available literature of field measurements of Λ and Δ in comparison to measurements of h and $c = v/v_*$, the values of Δ/Λ of the data points in Fig. 4.2 were calculated (from the measured h and c) with the aid of Engelund-Yalin equation, viz

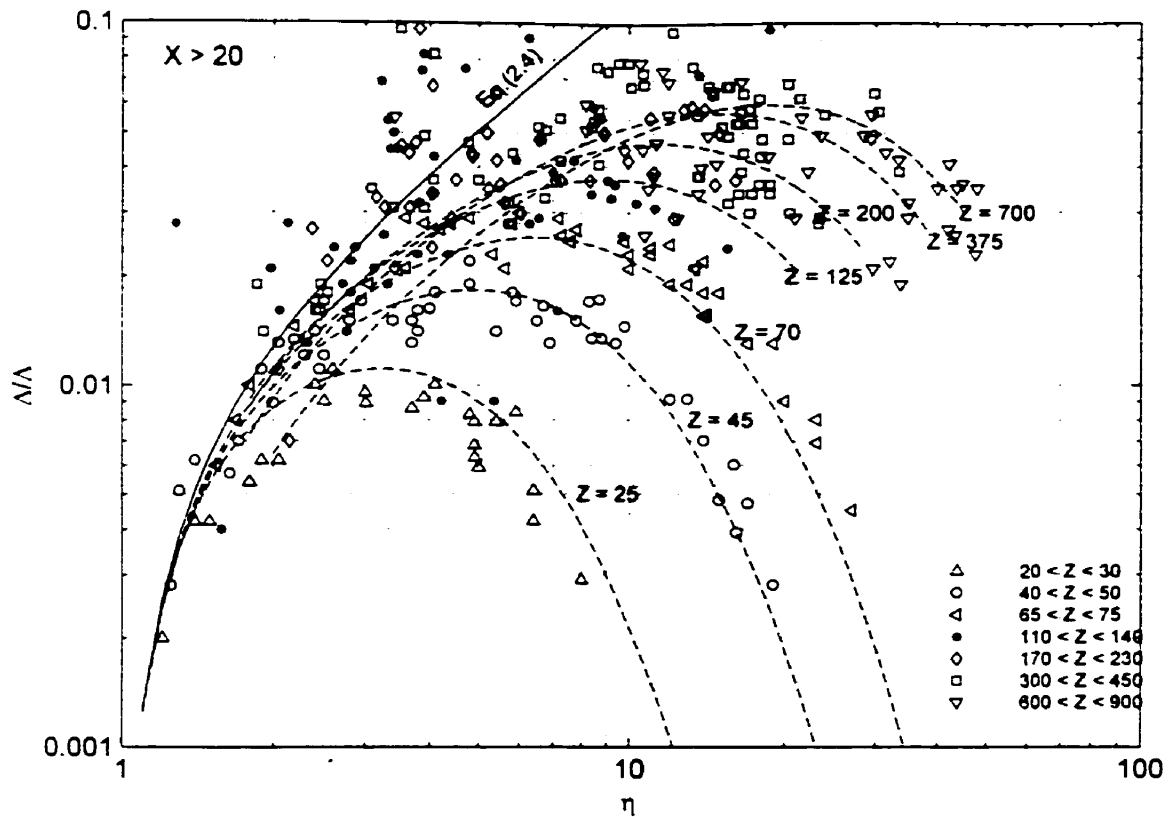
$$\frac{1}{c^2} = \frac{1}{c_0^2} + \frac{1}{2} \left(\frac{\Delta}{\Lambda} \right)^2 \frac{\Lambda}{h}, \quad (4.2)$$

where Λ/h and c_0 were calculated with the aid of the following expressions

$$\frac{\Lambda}{h} = 2\pi \quad \text{and} \quad c_0 = \frac{1}{\kappa} \ln \left(11 \frac{h}{k_s} \right) \quad (4.3)$$

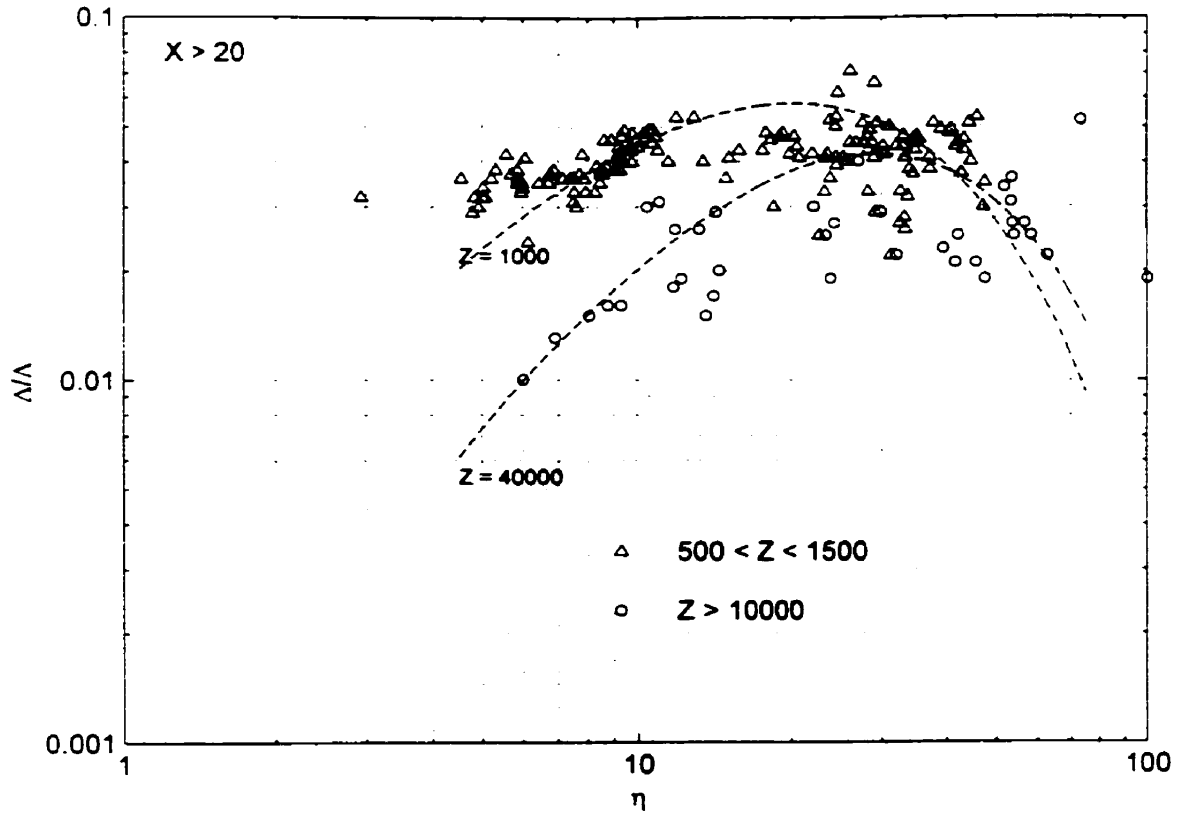
(with $k_s = 2D$). The sources of the data used in this figure are given in its legend.

Fig. 4.3, just like Fig. 4.2, corresponds to “large” values of Z ($Z \geq 1000$, say). However, in contrast with the data plotted in Fig. 4.2, for the data plotted in Fig. 4.3. measured values of Δ and Λ (in addition to h and c) were reported by their authors. Thus, the value of Δ/Λ of each data point was computed as the ratio between the *measured* dune height Δ , and the *measured* dune length Λ . It should be pointed out that *all* river dune data available to the author for which Δ and Λ were measured are plotted in this figure. As mentioned several times earlier in this thesis, the data is scarce.



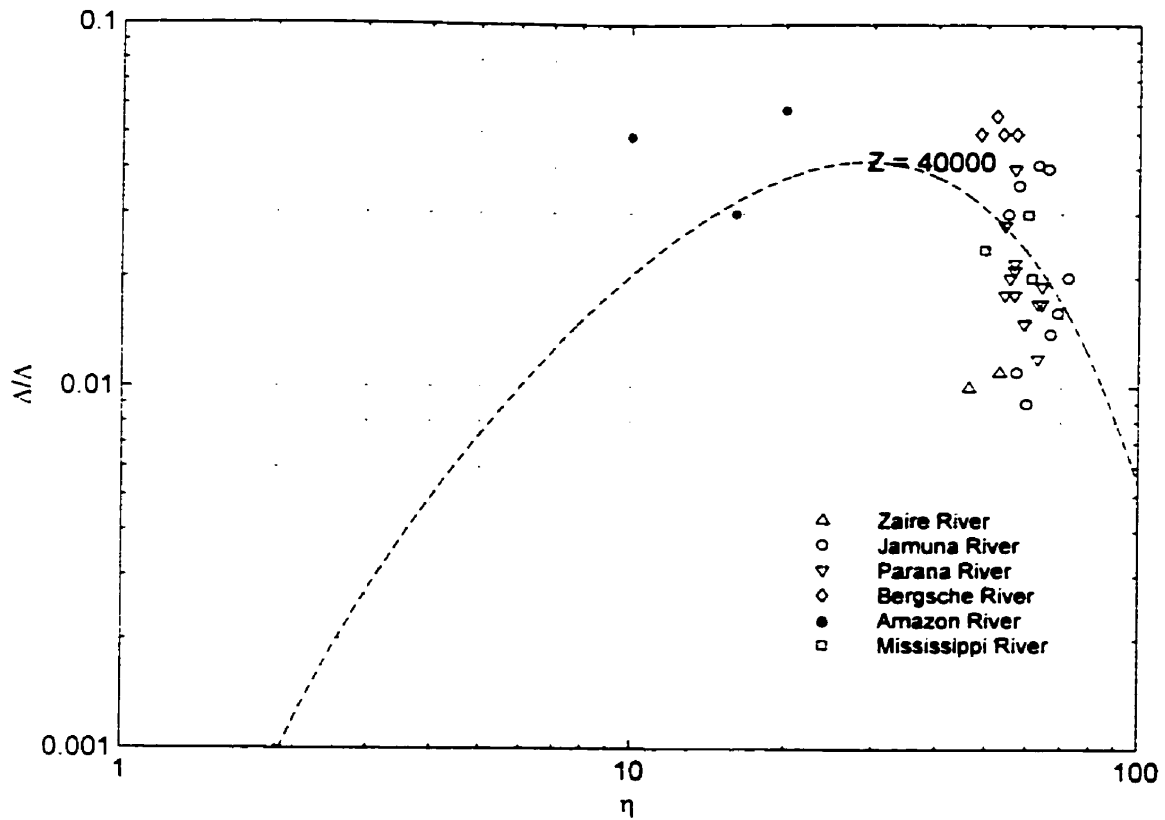
	Data sources
Laboratory data	Ashida and Tanaka (1967); Bishop (1977); Casey (1935); Fredsoe (1979, 1981); Grigg (1970); Hubbell and Sayre (1964); Hung and Shen (1979); Jain and Kennedy (1971); Matsunashi (1967); Nordin and Algert (1967); Nordin (1964); Raichlan and Kennedy (1965); Shen and Cheong (1977); Simons, Richardson and Albertson (1961); Simons, Richardson and Hausbild (1963); Singh (1960); Stein (1965); Vischer (1983); Wang and White (1993); Williams, (1970); Znamenskaya (1963).
Field data	Martinec : Luznice River (1967); Shinohara and Tsubaki: HII River (1959).

Figure 4.1: Plot of measured Δ/Λ versus η ($Z < 1000$)



	Data sources
Laboratory data	Government of East Pakistan (1966, 68, 69); Nordin (1976); Onishi, Jane and Kennedy (1972); Pratt (1970); Simons, Richardson and Guy (1956-61, 1966); Stein (1965).
Field data	da Cunha: Portugal River data (1969); Nedeco: South American River and canal data (1973); Nordin and Beverage: Rio Grand River data (1965); Seitz: Snake and Clearwater River data (1976); Shinohara and Tsubaki: HII River data (1959); Toffaleti: Mississippi River data (1968).

Figure 4.2: Plot of estimated Δ/Λ versus η ($Z > 1000$)



Data sources	
Field data	Adriaanse: Bergsche River (1986); Julien: Jamuna River and Parana River (1992); Lane and Eden: Mississippi River (1940); Nordin: Amazon River; Peters (1978) and Termes (1986): Zaire River.

Figure 4.3: Plot of measured Δ/Λ versus η ($Z > 1000$)

4.3.3 Plots of c/c_0 versus η

Consider Eq. (4.2). This equation can be written as

$$\frac{c}{c_0} = \frac{1}{\sqrt{1 + \frac{1}{2} \left(\frac{\Delta}{\Lambda}\right)^2 \frac{\Lambda}{h} c_0^2}} \quad (4.4)$$

For the case of dunes only, $\Delta/\Lambda = \phi(\eta, Z)$ and $\Lambda/h = \phi(\eta, Z)$ (see Chapter 3). Therefore $c/c_0 = \phi(\eta, Z)$. In the plane $(c/c_0; \eta)$, Eq. (4.4) implies a family of curves, each curve corresponding to a different value of Z .

Fig. 4.4 a to k are the plots of both laboratory and field data in the plane $(c/c_0; \eta)$. (For the present purpose, disregard solid lines in these figures). The data sources are given in Table 4.1. Only data with $X > 30$ were plotted. The value of Z indicated in each of these figures is the average Z -value of the data points in each plot.

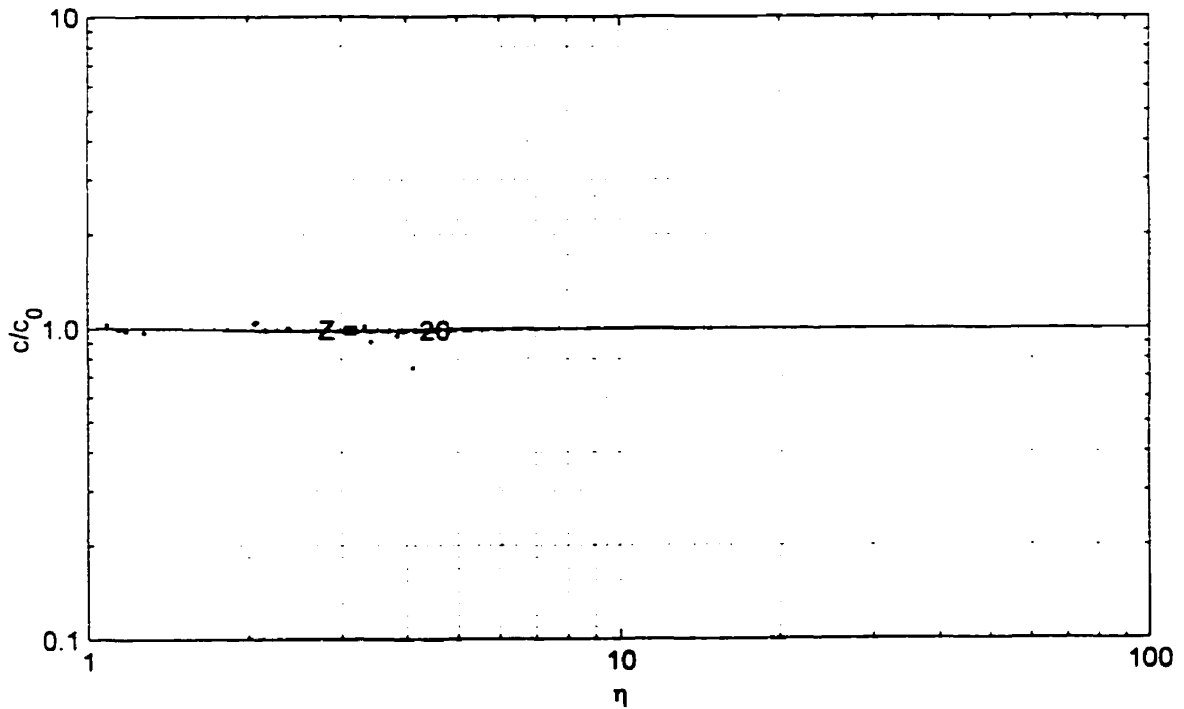


Figure 4.4 a: Plot of c/c_0 versus η ($0 < Z \leq 25$)

It should be mentioned that Eq. (4.4) is valid only for two-dimensional flow ($B/h > \approx 10$, say). However, a considerable amount of the available laboratory data

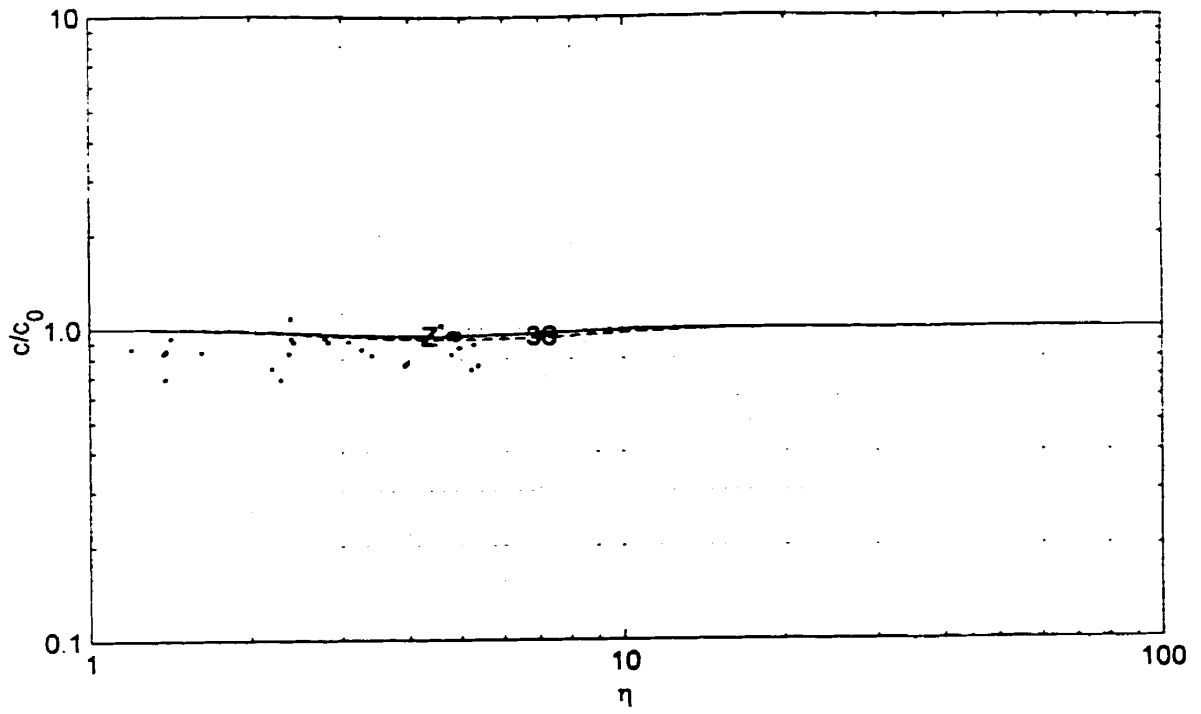


Figure 4.4 b: Plot of c/c_0 versus η ($25 < Z \leq 50$)

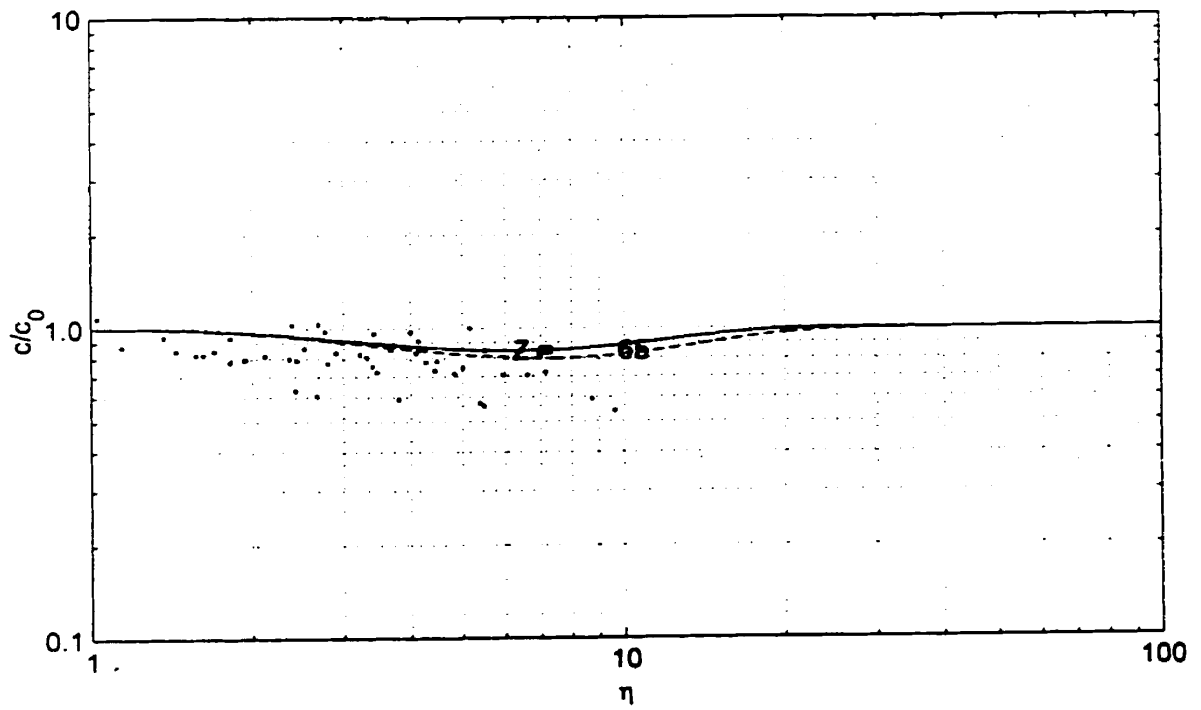


Figure 4.4 c: Plot of c/c_0 versus η ($50 < Z \leq 75$)

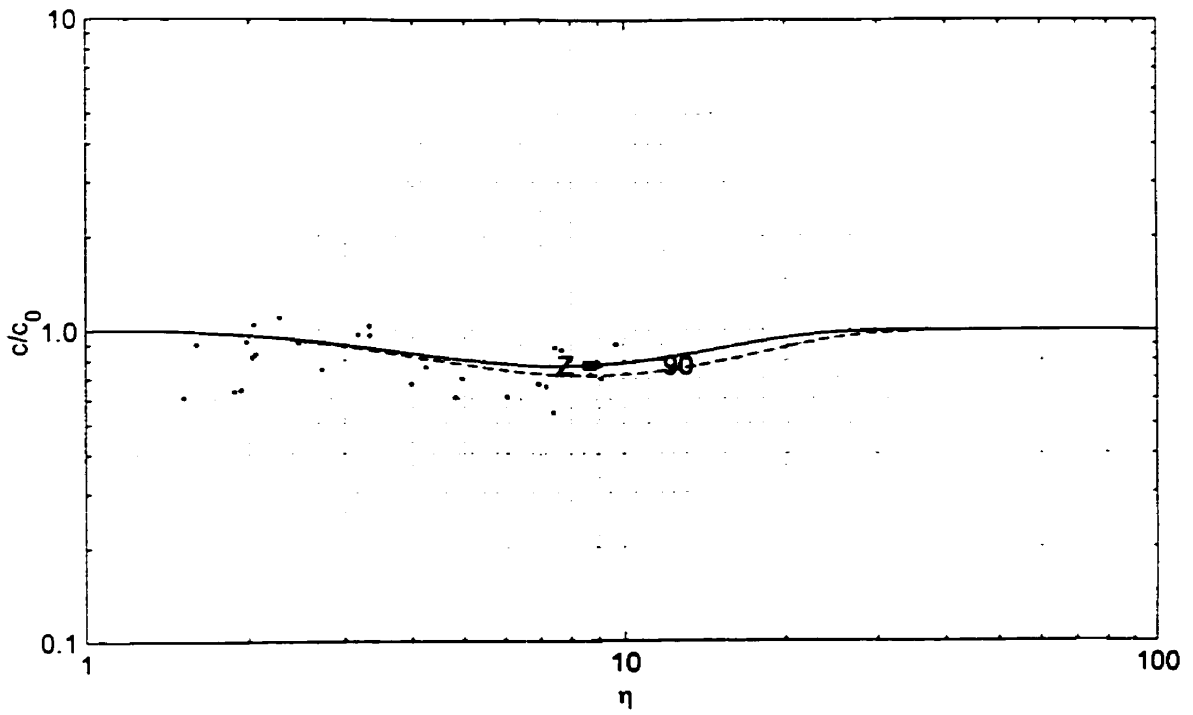


Figure 4.4 d: Plot of c/c_0 versus η ($75 < Z \leq 100$)

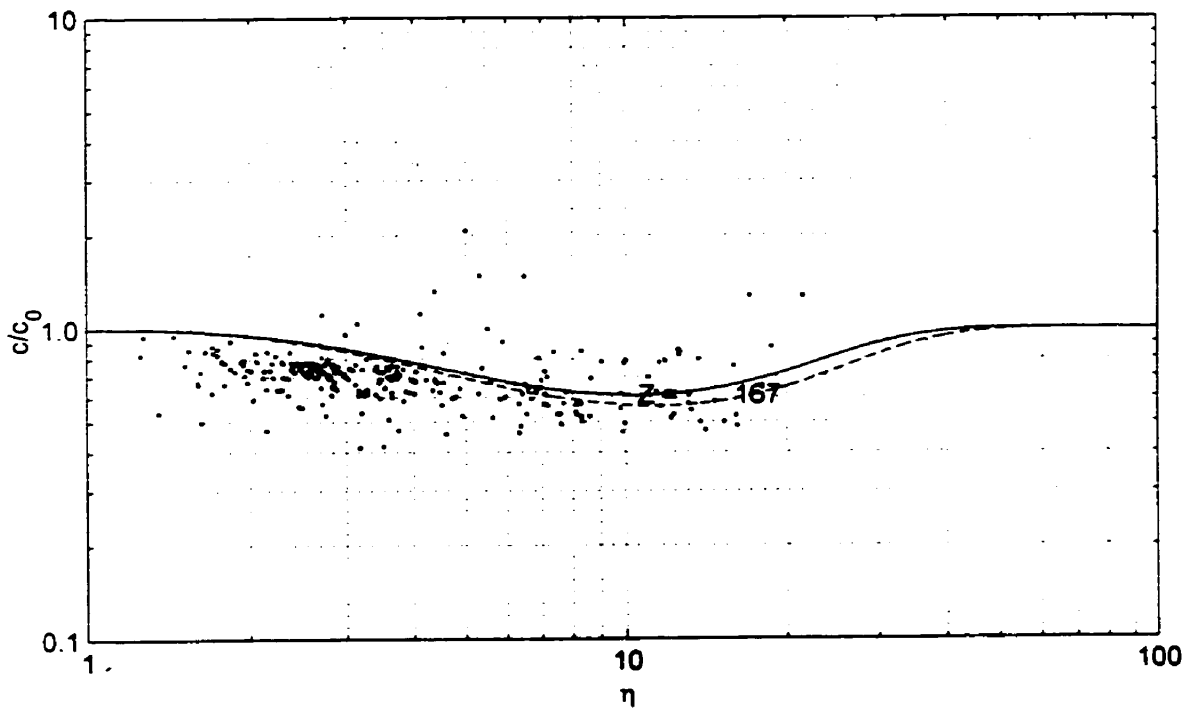


Figure 4.4 e: Plot of c/c_0 versus η ($100 < Z \leq 250$)

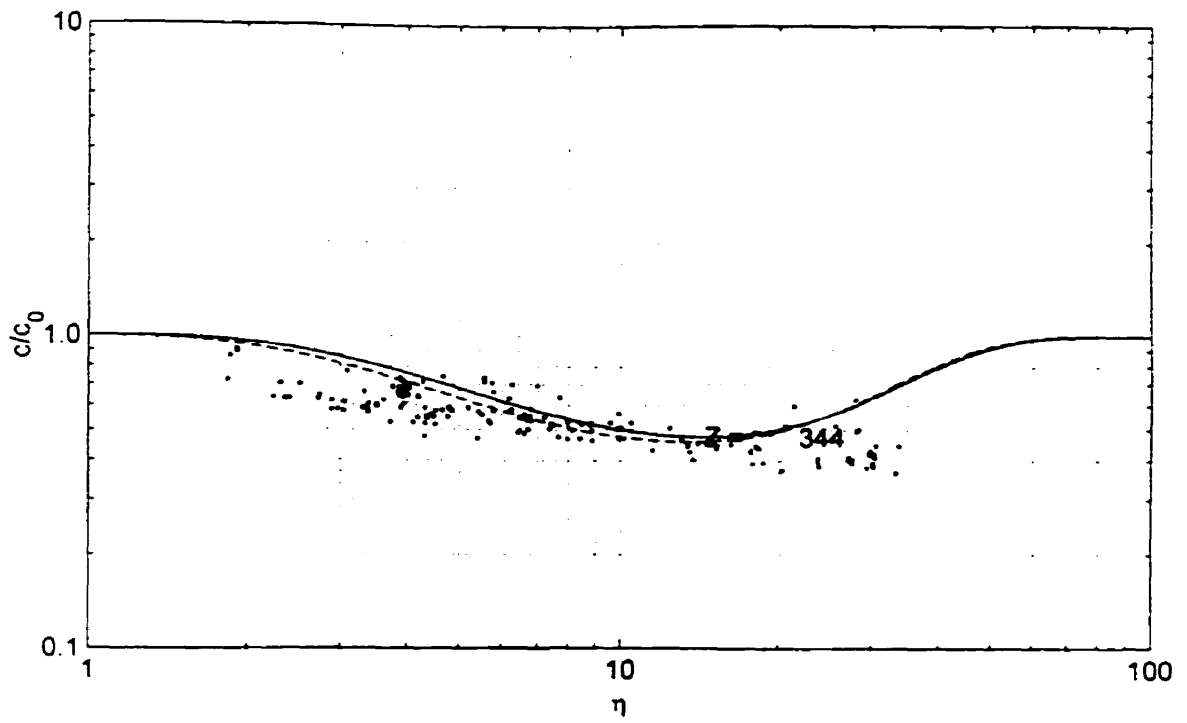


Figure 4.4 f: Plot of c/c_0 versus η ($250 < Z \leq 500$)

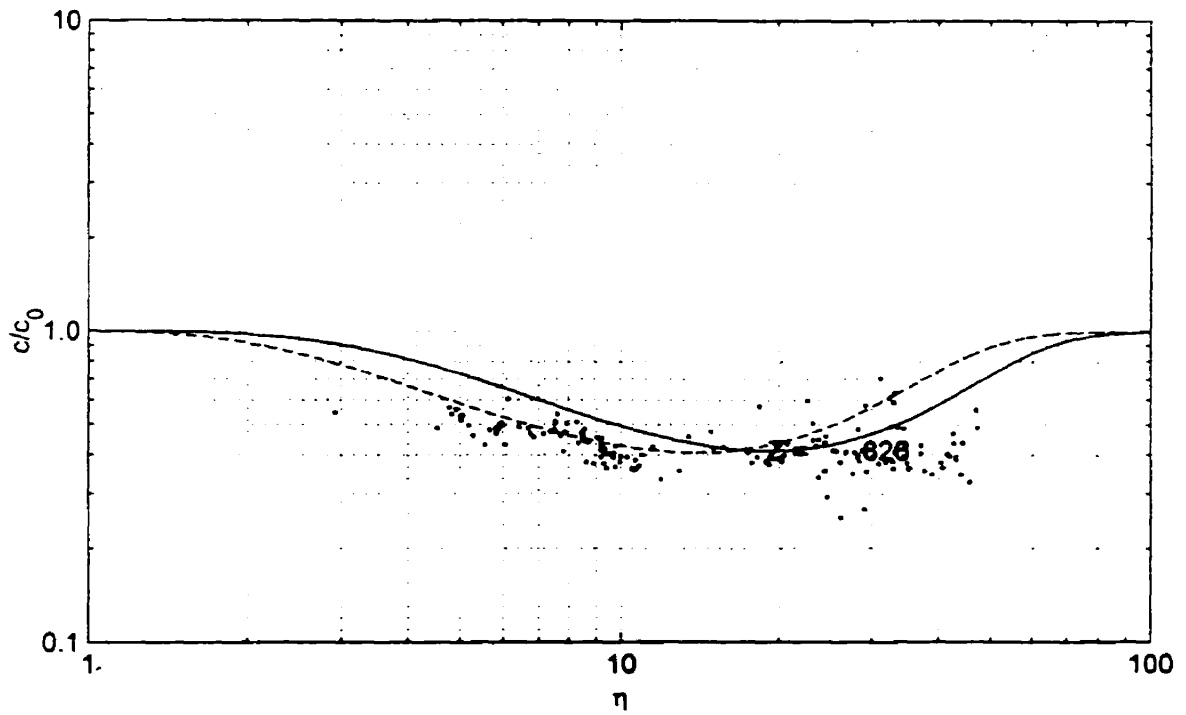


Figure 4.4 g: Plot of c/c_0 versus η ($500 < Z \leq 1500$)

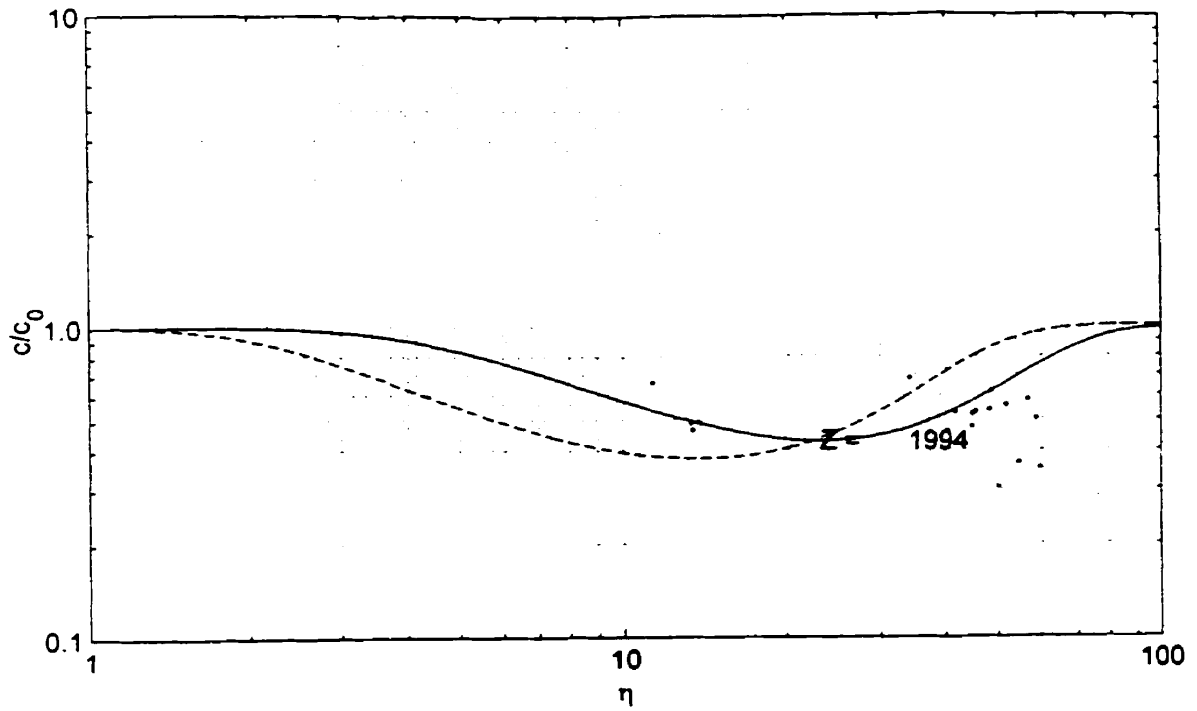


Figure 4.4 h: Plot of c/c_0 versus η ($1500 < Z \leq 2500$)

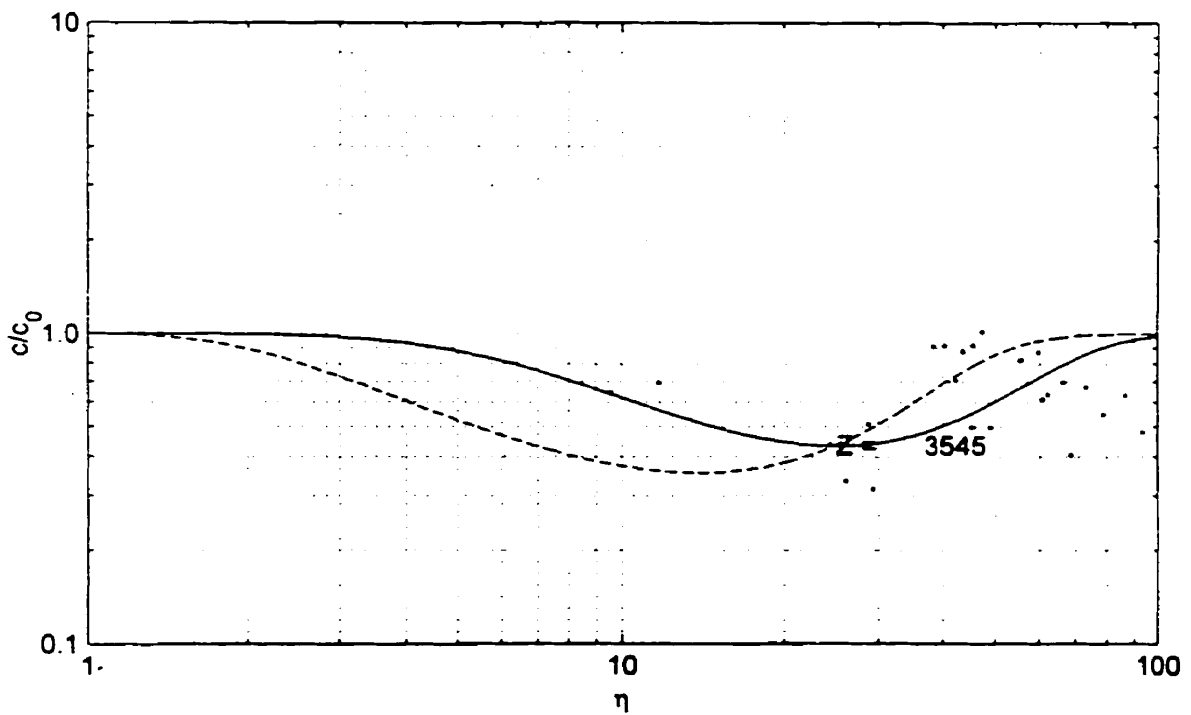


Figure 4.4 i: Plot of c/c_0 versus η ($2500 < Z \leq 5000$)

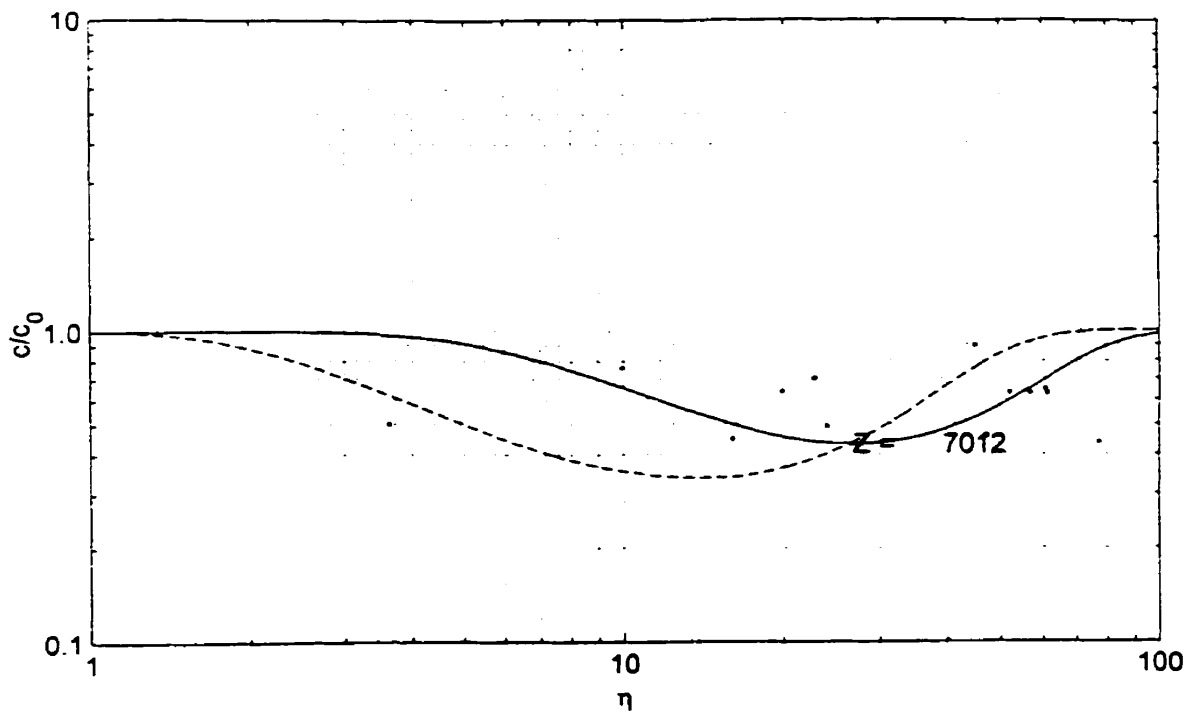


Figure 4.4 j: Plot of c/c_0 versus η ($5000 < Z \leq 10000$)

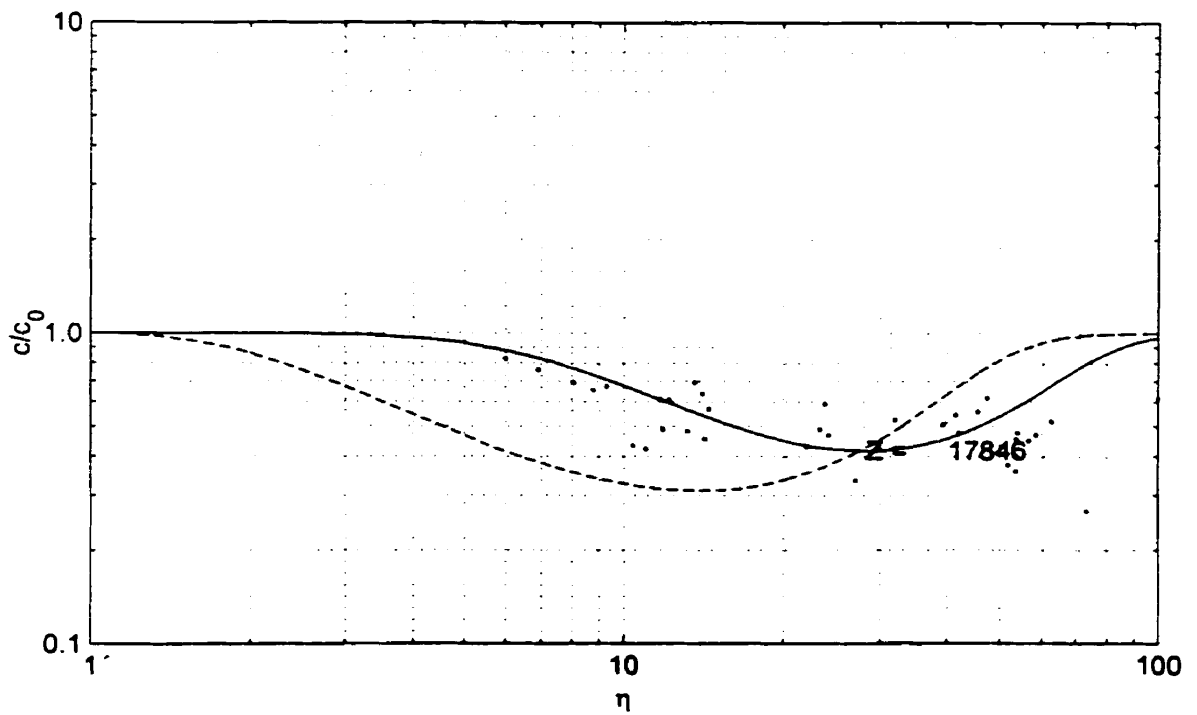


Figure 4.4 k: Plot of c/c_0 versus η ($Z \geq 10000$)

	Data sources
Laboratory data	Ashida and Tanaka (1967); Bishop (1977); Chyn (1935); Gibbs and Neill (1972); Gilbert (1914); Government of East Pakistan (1966, 68, 69); Grigg (1970); Hubbell and Sayre (1964); Jorissen (1938); Nordin (1976); Pratt (1970); Simons, Richardson and Guy (1956-61, 1966); Singh (1960); Stein (1965); U. S. Waterways Experiment Station (1935, 1936); Williams (1970); Znamsenskaya (1963).
Field data	da Cunha: Portugal River data (1969); Einstein: Mountain Creek data (1944); Nedeco: South American River and canal data (1973); Nordin and Beverage: Rio Grand River data (1965); Seitz: Snake and Clearwater River data (1976); Shinohara and Tsubaki: HII River data (1959); Toffaleti: Mississippi River data (1968).

Table 4.1: Data sources of Fig. 4.4

has $B/h < 10$. This implies that the wall effect may not be negligible. In order to plot these data, the data has been adjusted so that the wall effect is “removed” from it. The method of adjustment of the data is explained in Appendix A.

From Eq. (4.2), it follows that when $\Delta/\Lambda \rightarrow 0$, then $c/c_0 \rightarrow 1$; and when Δ/Λ is large, then $c/c_0 < 1$. The larger is Δ/Λ (that is, the more prominent the dunes), the larger is the deviation from $c/c_0 = 1$. Therefore, in Figs. 4.4 a to k, the further away from the horizontal line implying $c/c_0 = 1$ the data points, the larger is Δ/Λ . The value of η corresponding to the maximum value of Δ/Λ , i.e. $(\Delta/\Lambda)_{max}$, of each curve, which is termed $\bar{\eta}_d$, for each value of Z can be inferred from these plots: $\bar{\eta}_d$ is the abscissa corresponding to the largest deviation of the data point cloud from the horizontal cline c/c_0 .

4.3.4 Dune steepness expression

4.3.4.1 General

The point patterns in Figs. 4.1 and 4.2 seem to indicate that, as Z increases, the value of $\bar{\eta}_d$ (i.e., the value of η corresponding to $(\Delta/\Lambda)_{max}$ for each curve associated with a specified Z) increases far beyond the limit $\bar{\eta}_d = 16$ suggested by Yalin and his co-workers. Moreover, from Fig. 4.2, it appears that, for large values of Z , the value of $(\Delta/\Lambda)_{max}$ is lower than the value $(\Delta/\Lambda)_{max} = 0.06$ suggested by Yalin and his co-workers. Moreover, the data in Fig. 4.2 does not support a model in which the Δ/Λ -curves corresponding to different Z -values would all merge into the solid line shown in Fig. 4.1 (as suggested by Yalin and his co-workers). The plot in Fig. 4.3, given the scarcity of the data, does not give any information on the nature of the dune steepness as a function of η and Z .

Taking all of the above into account, it seems appropriate to adopt for dune steepness the following generalized form of Eq. (4.5):

$$\frac{\Delta}{\Lambda} = \left(\frac{\Delta}{\Lambda}\right)_{max} \left(\zeta e^{1-\zeta}\right)^m, \quad \text{with } \zeta = \frac{\eta - 1}{\bar{\eta}_d - 1} \quad (4.5)$$

where $(\Delta/\Lambda)_{max}$ and $\bar{\eta}_d$, which are functions of Z , are to be determined, and the

introduction of the power m implies that the (Δ/Λ) -curves do not all merge into one single line. In the next subsection, expressions for $\bar{\eta}_d$, $(\Delta/\Lambda)_{max}$ and m are introduced.

4.3.4.2 Determination of $\bar{\eta}_d$, $(\Delta/\Lambda)_{max}$ and m

From Figs. 4.1 and 4.2, as well as from Figs. 4.4 a to k, the values of $\bar{\eta}_d$ for Δ/Λ -curves corresponding to different Z -values were (very roughly) estimated as shown in Table 4.2 (where the first three values of $\bar{\eta}_d$ are the same as given by Yalin and Karahan (1979) in Fig. 2.3).

Z	$\bar{\eta}_d$	$(\Delta/\Lambda)_{max}$
25	3.03	0.01
45	4.85	0.018
70	6.78	0.028
125	9.00	0.038
200	11.00	0.04
350	15.00	0.058
1000	20.00	0.06
40000	30.00	0.04

Table 4.2: Values of $(\Delta/\Lambda)_{max}$ for Δ/Λ -curves

From Figs. 4.1 and 4.2, the values of $(\Delta/\Lambda)_{max}$ for Δ/Λ -curves corresponding to different Z -values were (very roughly) estimated as shown also in Table 4.2.

The values of $\bar{\eta}_d$ shown in Table 4.2 are plotted versus Z in Fig. 4.5. In order to find an equation passing through the points shown in Fig. 4.5, it was assumed they would fit a curve of the type:

$$\bar{\eta}_d = \frac{30Z^n + \alpha}{Z^n + \beta} \quad (4.6)$$

Using the following three points:

$$Z = 7; \quad \bar{\eta}_d = 1$$

$$Z = 100; \quad \bar{\eta}_d = 8$$

$$Z = 1000; \quad \bar{\eta}_d = 20$$

n , α and β were determined as 0.72, -46 and 70, respectively. The resulting expression for $\bar{\eta}_d$ is thus:

$$\bar{\eta}_d = \frac{30Z^{0.72} - 46}{Z^{0.72} + 70} \quad (4.7)$$

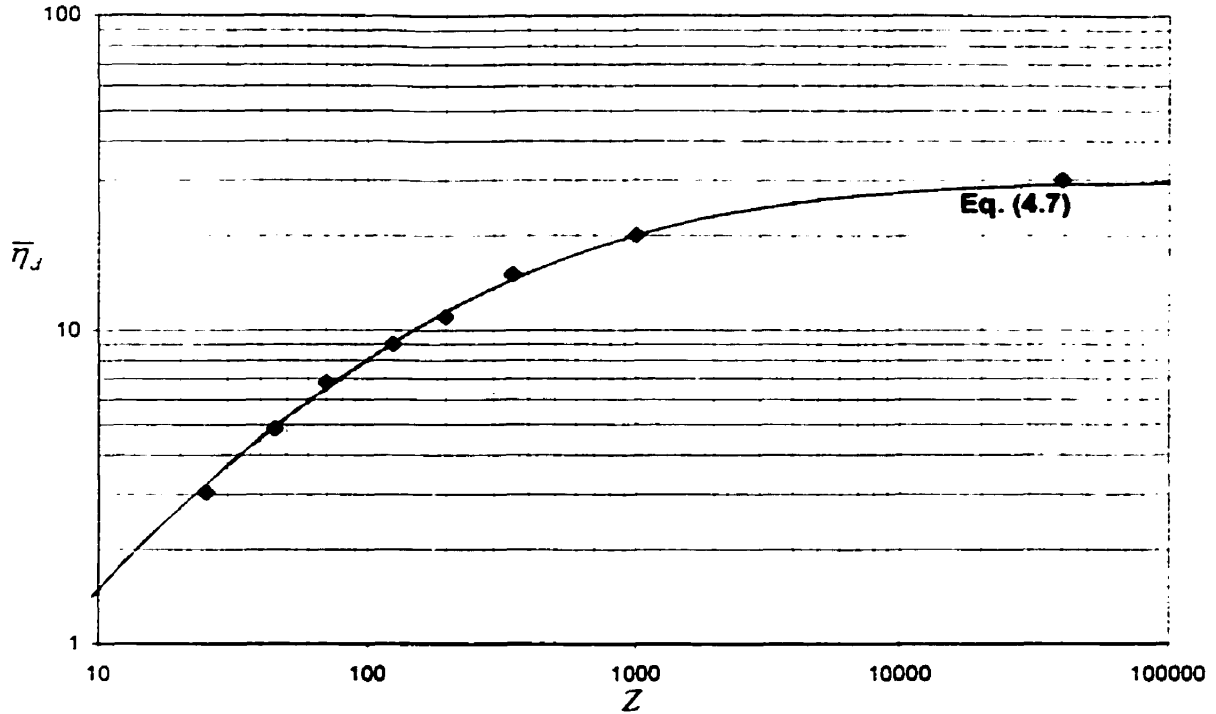


Figure 4.5: Values of $\bar{\eta}_d$ versus Z and the fit curve

The values of $(\Delta/\Lambda)_{max}$ shown in Table 4.2 are plotted versus Z in Fig. 4.6. The solid line in this figure represents the following equation, which was found to pass satisfactorily through the points:

$$\left(\frac{\Delta}{\Lambda}\right)_{max} = 0.04 \left(1 - e^{-0.0119Z}\right) \left[\frac{0.5}{1 + 3(\log Z - 2.8)^2} + 1 \right] \quad (4.8)$$

It should be noticed that Eq. (4.8) represents the sum of two parts:

$$\left(\frac{\Delta}{\Lambda}\right)_{max} = \left(\frac{\Delta}{\Lambda}\right)' + \left(\frac{\Delta}{\Lambda}\right)''$$

Here

$$\left(\frac{\Delta}{\Lambda}\right)' = 0.04 \left(1 - e^{-0.0119Z}\right) \quad (4.9)$$

is shown in Fig. 4.6. The term $(\Delta/\Lambda)''$ is the difference between the two curves in Fig. 4.6.

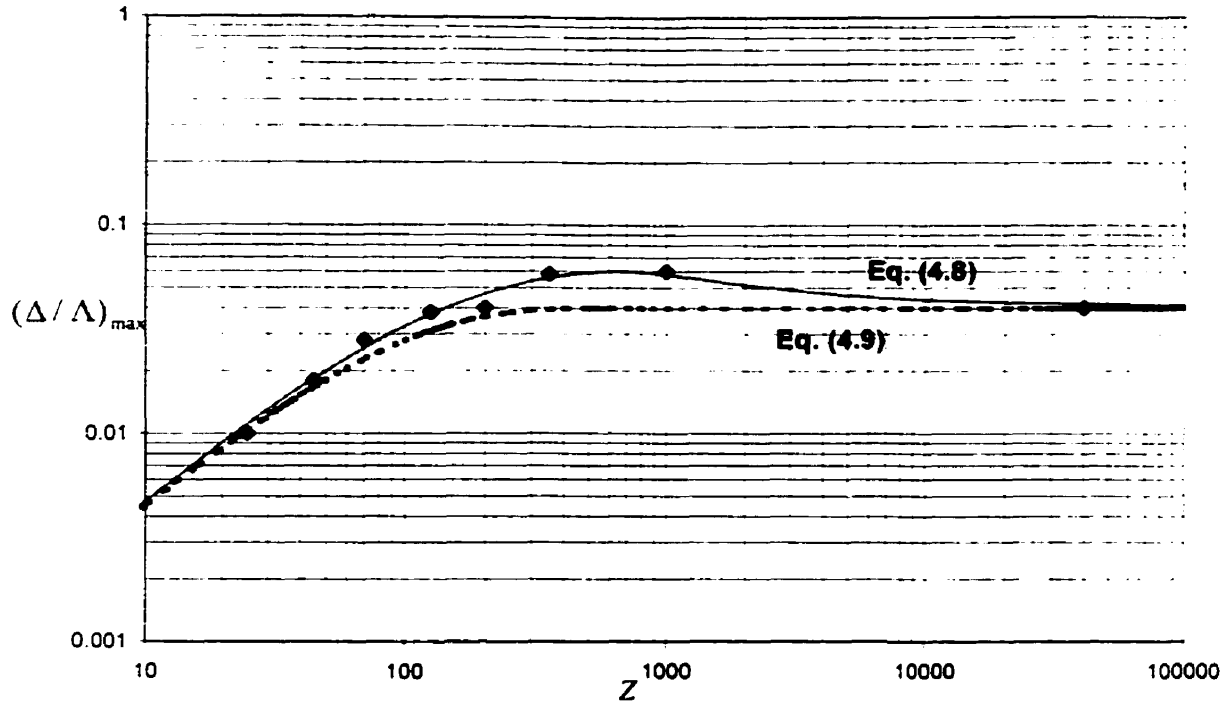


Figure 4.6: Values of $(\Delta/\Lambda)_{max}$ versus Z and the fit curve

In order to determine the power m , the following procedure was followed. For $Z = 100, 350, 3600$ and 20000 , the curves c/c_0 were calculated with the aid of Eq. (4.4), where the new expression for Δ/Λ was used, and a value for m was selected as to yield the best agreement between the curves in Figs. 4.4 d, 4.4 f, 4.4 i and 4.4 k. The values of m in Table 4.3 are plotted in Fig. 4.7. It was found that the equation

$$m = \begin{cases} 1 & \text{if } Z \leq 100 \\ -0.0682(\log Z - 3.3)^3 + 0.346(\log Z - 3.3) + 1.3 & \text{if } 100 < Z < 40000 \\ 1.6 & \text{if } Z \geq 40000 \end{cases} \quad (4.10)$$

passes satisfactorily through the points in Fig. 4.7.

Z	m
90	1.0
350	1.1
3600	1.4
20000	1.6

Table 4.3: Values of m corresponding to different Z -numbers

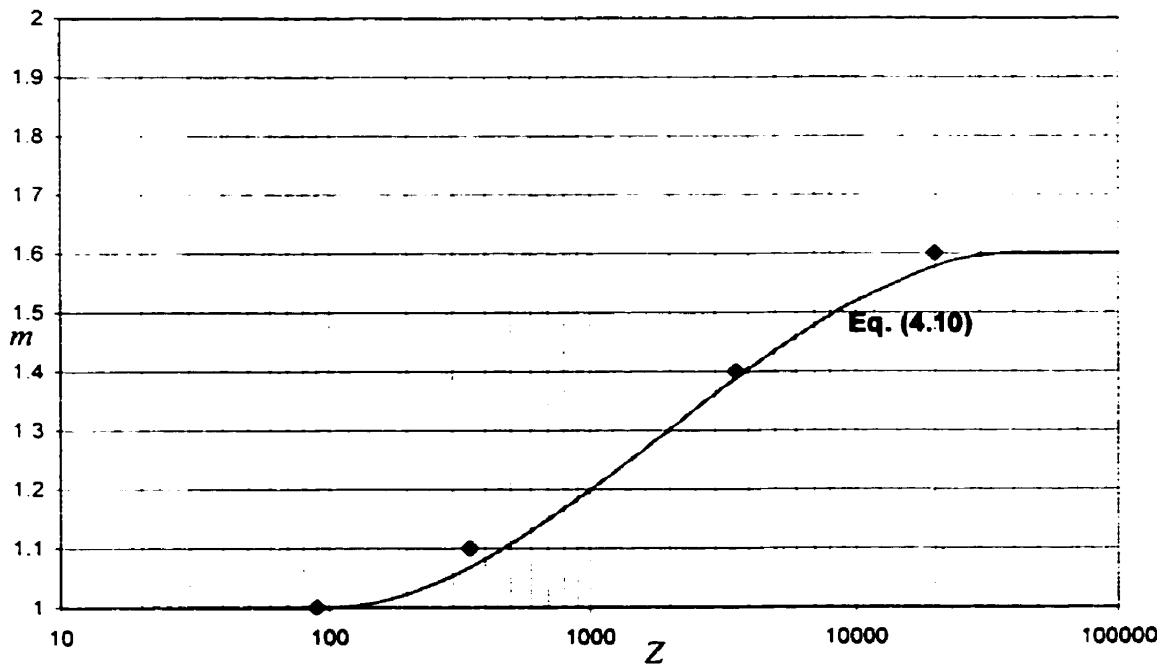


Figure 4.7: Values of m versus Z and the fit curve

4.3.4.3 Final dune steepness expression

Considering the previous sections, the present dune steepness expression can be written as:

$$\frac{\Delta}{\Lambda} = 0.04 \left(1 - e^{-0.0119Z}\right) \left[\frac{0.5}{1 + 3(\log Z - 2.8)^2} + 1 \right] (\zeta e^{1-\zeta})^m \quad (4.11)$$

where

$$\zeta = \frac{\eta - 1}{\bar{\eta}_d - 1} \quad \text{with} \quad \bar{\eta}_d = \frac{30Z^{0.72} - 46}{Z^{0.72} + 70}$$

and

$$m = \begin{cases} 1 & \text{if } Z \leq 100 \\ -0.0682(\log Z - 3.3)^3 + 0.346(\log Z - 3.3) + 1.3 & \text{if } 100 < Z < 40000 \\ 1.6 & \text{if } Z \geq 40000 \end{cases}$$

The broken lines in Figs. 4.1 to 4.3 are the plots of the expressions above. They seem to fit the data adequately.

The solid lines in Figs. 4.4 a to k are the plots of Eq. (4.4), where the above expressions for Δ/Λ are used. These lines were computed for the Z -values which corresponds to the mean average of Z -values of the data in each plot. These lines seem also to reflect adequately the behavior of the data for the entire spectrum of Z -values. The broken lines in these same figures were determined from Eq. (4.4) using Yalin and Scheuerlein (1988) equation for Δ/Λ . The lines deviate more from the data than the solid lines, especially for large values of Z .

The present Δ/Λ -curves are plotted in Fig. 4.8. This figure should be compared with Fig. 3.3, in order for the differences between the expressions of Yalin and Scheuerlein (1988) and the present dune steepness expression to be easily visualized.

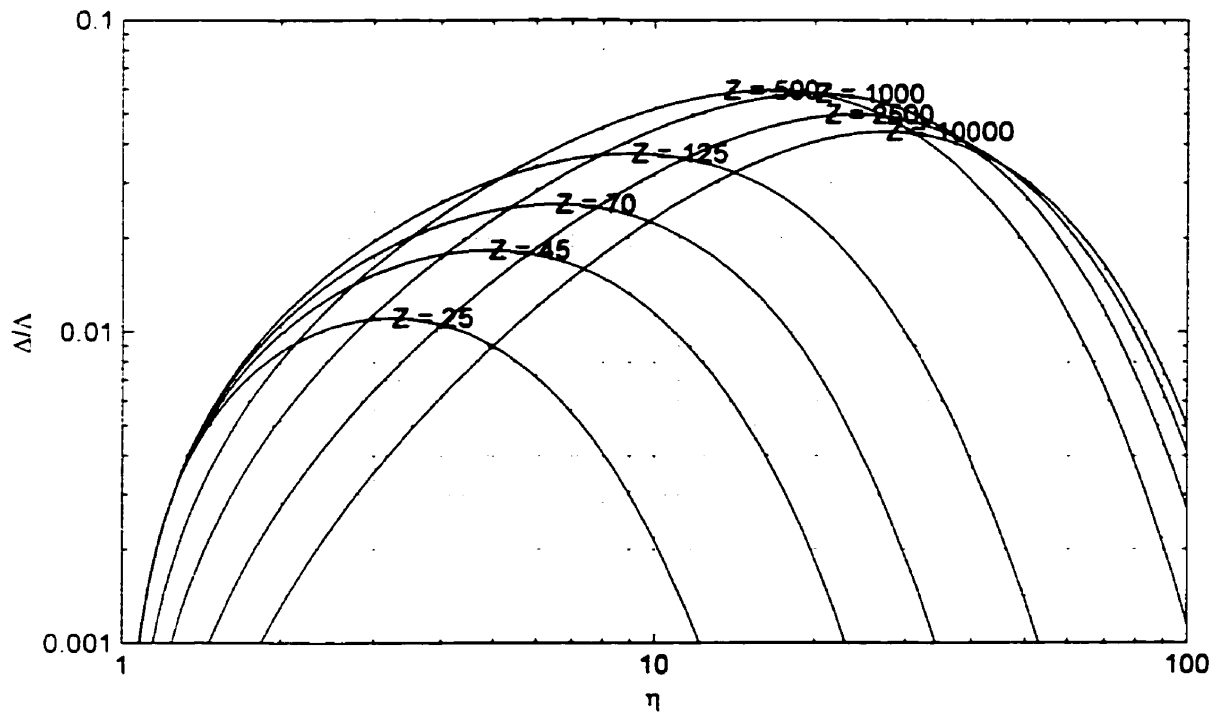


Figure 4.8: Family of present Δ/Λ -curves

5.0 APPLICATION TO PREDICTION OF FRICTION FACTOR AND FLOW DEPTH

The main purpose of this chapter is to verify the dune steepness expression introduced in this thesis by the author.

As should be clear from previous chapters, the experimental data available in the literature is very extensive. Almost all authors report the flow related characteristics (Q, v, B, h, S, c). Yet, much fewer report the bed form geometric characteristics (either because they were not measured, or because the authors were not focusing in their studies on bed forms). This is especially true for field (river) data. In fact, all available data containing the bed form geometric characteristics were used by the author to develop the dune steepness expression introduced in this thesis. Thus, a different means to verify the expression had to be found. Since most of the authors define the value of the friction factor in their experiments, the present dune steepness expression is herein verified via its effect on the friction factor (by comparing measured values of friction factor with computed (via dune steepness) values of friction factor).

The present chapter serves also the purpose of comparing the most prominent methods of determination of friction factor (where “most prominent” implies the most frequently mentioned in the literature and most frequently used in practice).

5.1 Discussion of prominent methods

Although several methods for determination of friction factor have been introduced in Chapter 2, some of these have but a historical interest. The most prominent methods now in use are those by Engelund-Yalin (1966), (1964), van Rijn (1982), and White, Paris and Bettess (1979). The following is a brief discussion of these methods.

According to Engelund-Yalin, the friction factor of flow over a bed covered by one type of bed forms is given by

$$\frac{1}{c^2} = \frac{1}{c_0^2} + \frac{1}{2} \left(\frac{\Delta}{\Lambda} \right)^2 \frac{\Lambda}{h} . \quad (5.1)$$

Here c_0 (which is due to the granular roughness k_s ($\approx 2D$)) is given, for rough turbulent flows under consideration, by the well known logarithmic form

$$c_0 = \frac{1}{\kappa} \ln \left(11 \frac{h}{k_s} \right) = \frac{1}{\kappa} \ln \left(11 \frac{Z}{2} \right) \quad (5.2)$$

As can be inferred from Chapter 2, of all the expressions produced to date, Eq. (5.1) is the only one which has been derived from first principles. However, the accuracy of the friction factor computed from Eq. (5.1) depends on how accurate the expressions of Δ/Λ and Λ/h are. For a bed covered by dunes, Yalin and Scheuerlein (1988) suggested that Eqs. (2.12) and (2.19) for Δ/Λ and Λ/h , viz,

$$\frac{\Delta}{\Lambda} = 0.0127(\eta - 1)e^{-0.0778(\eta-1)/(1-e^{-0.01Z})} \quad (5.3)$$

$$\frac{\Lambda}{h} \approx 6 \quad (5.4)$$

be used.

As can be inferred from the content of this thesis, in general,

$$\Delta/\Lambda = \psi(\xi, \eta, Z) \quad \text{and} \quad \Lambda/h = \psi(\xi, \eta, Z) . \quad (5.5)$$

Since (in general)

$$c_0 = \psi(\xi, Z) , \quad (5.6)$$

Eq. (5.1) can be written as

$$c = \psi(\xi, \eta, Z) . \quad (5.7)$$

It is, of course, possible to bypass the geometric properties of bed forms, Δ/Λ and Λ/h , and determine c as a function, per se, of ξ, η, Z . This approach has been adopted by White et al. (1979). Indeed, consider Eq. (2.31), viz

$$F_{gr} = \left\{ \sqrt{32} \log(10Z) \right\}^{n-1} \sqrt{Y} c^{1-n} \quad (5.8)$$

and Eq. (2.35) which can be written as

$$F_{gr} = (F_{fg} - A)\phi(\xi) + A , \quad (5.9)$$

where

$$F_{fg} = \sqrt{Y} . \quad (5.10)$$

From Eq. (5.8), we obtain

$$c^{1-n} = \frac{F_{gr}}{\{\sqrt{32} \log(10Z)\}^{n-1}} \sqrt{Y} . \quad (5.11)$$

Substituting Eq. (5.9) into Eq. (5.11), one obtains

$$c = \left(\frac{(\sqrt{Y} - A)\Phi(\xi) + A}{\{\sqrt{32} \log(10Z)\}^{n-1} \sqrt{Y}} \right)^{\frac{1}{1-n}} , \quad (5.12)$$

which is valid for $\xi > 1$ (i.e., $n < 1$). Since A and n are both functions of ξ (see Eqs. (2.36) and (2.37)), then

$$c = \phi(\xi, Y, Z) \quad (5.13)$$

This expression, which can be equivalently written as (see Section 1.4)

$$c = \phi(\xi, \eta, Z) , \quad (5.14)$$

is indeed in agreement with Eq. (5.7). In other words, from the standpoint of dimensional analysis, White et al.'s method is sound. However, in contrast to Yalin's method for the determination of friction factor, Eq. (5.12) has the following disadvantages:

1- the expression is not derived from first principles, and thus does not throw any light on the physical mechanisms involved.

2- since the bed form geometric properties do not appear in the expression, the method does not give any information on bed forms. Yet in practice, the prediction of both dune steepness and length is sometimes relevant.

The method by van Rijn rests on Eq. (2.49), viz

$$c = \frac{v}{v_*} = \frac{1}{\kappa} \ln \left(11 \frac{R}{K_s} \right) \quad (5.15)$$

where K_s is the equivalent roughness. According to this author,

$$K_s = 3D + \psi(\Delta/\Lambda, \Delta) . \quad (5.16)$$

Eqs. (2.24) and (2.25) (of van Rijn) are to be used for Δ/Λ and Δ .

The form (5.15) adopted by van Rijn is objectionable. Indeed, it is well known that the logarithmic form is valid only if h/k_s is “large” ($h/k_s > 8$, say), i.e. when the roughness elements are considerably smaller than the flow depth h (or the hydraulic radius R). Yet, dune height can be as large as one third of the flow depth ($\Delta/h \leq 1/3$). Thus, the form (5.15) should not be valid for the cases in which $h/K_s < 8$, say.

5.2 Application of present dune steepness to friction factor prediction

Since Eq. (5.1) is the only equation which has been derived from first principles, the author suggests this equation be used for the determination of friction factor. Yet, the dune steepness equation of Yalin and Scheuerlein (1988) (Eq. (5.3)) should be replaced by

$$\frac{\Delta}{\Lambda} = 0.04 (1 - e^{-0.0119Z}) \left(\frac{0.5}{1 + 3(\log Z - 2.8)^2} + 1 \right) (\zeta e^{1-\zeta})^m . \quad (5.17)$$

Here, ζ and m are given as

$$\zeta = \frac{\eta - 1}{\bar{\eta}_d - 1}$$

$$m = \begin{cases} 1 & \text{if } Z \leq 100 \\ -0.0682(\log Z - 3.3)^3 + 0.346(\log Z - 3.3) + 1.3 & \text{if } 100 < Z < 40000 \\ 1.6 & \text{if } Z \geq 40000 \end{cases}$$

5.3 Computational procedures

5.3.1 Determination of friction factor

In what follows, computational procedures are suggested for the determination of friction factor and flow depth using each of the methods discussed above.

i) Computational steps to determine c from Yalin-Engelund method, using Yalin and Scheuerlein equations for the bed form geometric characteristics:

Given the values of D, h and S ,

1. Determine $v_* = \sqrt{gSh}$ and Y_{cr} .
2. Determine ξ, Y, η, Z .
3. Determine c_0 from Eq. (5.2), where κ (Von Karman constant) = 0.4.
4. Determine Δ/Λ from Eq. (5.3).
5. Determine Λ/h from Eq. (5.4).
6. Use $c_0, \Delta/\Lambda$ and Λ/h to determine c from Eq. (5.1).

ii) Computational steps to determine c from Engelund-Yalin method, using the author's dune steepness equation:

Given the values of D, h and S , follow the steps above, but use Eq. (5.17) in Step 4.

iii) Computational steps to determine c from van Rijn's method:

Given the values of Q, B, D and h ,

1. Determine Y_{cr} and Z .
2. Determine $v_{*cr} = (\gamma_s D Y_{cr} / \rho)^{1/2}$.
3. Determine $c_0 = \frac{1}{\kappa} \ln \left(11 \frac{R}{3D} \right)$.
4. Determine $v = Q / (Bh)$.
5. Determine $v'_* = v / c_0$.
6. Determine $T = (v'^2_* - v^2_{*cr}) / v^2_{*cr}$.
7. Determine

$$\frac{\Delta}{\Lambda} = 0.015 \left(\frac{1}{Z} \right)^{0.3} (1 - e^{-0.5T})(25 - T) \quad (5.18)$$

$$\Lambda = 7.3h \quad (5.19)$$

8. Determine $K_s = 3D + \Delta 1.1(1 - e^{-25\frac{\Delta}{\lambda}})$.

9. Determine c from $c = 2.5 \ln(11\frac{R}{K_s})$.

iv) Computational steps to determine c from White et al.'s method:

Given the values of D, h and S ,

1. Determine $v_* = \sqrt{gSh}$.

2. Determine ξ, Y and Z .

3. Determine the parameters n and A from Eqs.(2.36) and (2.37).

4. Compute $F_{fg} = \sqrt{Y}$ (Eq. (2.32)).

5. Compute F_{gr} from Eq. (2.34).

6. Calculate c from Eq. (2.30), viz

$$F_{gr} = \left\{ \sqrt{32} \log(10Z) \right\}^{n-1} \sqrt{Y} c^{1-n}$$

This form of computation is the same as suggested by the authors. Alternatively, Eq. (5.12) can be used.

5.3.2 Determination of flow depth

The determination of flow depth requires an iterative procedure, as described below:

Given Q, B, D and S ,

1. Assume h_i .

2. Determine c from one of the methods given in Section 5.2.1.

3. Determine $h_{i+1} = Q/(Bcv_*)$.

4. If $h_{i+1} - h_i > 1/100$, let $h_i = 0.5h_i + 0.5h_{i+1}$, then go back to Step 2; If $h_{i+1} - h_i < 1/100$, then $h_{computed} = h_{i+1}$.

5.4 Comparison with experiment

In this section, it is intended to compare measured values with computed values of c . For this purpose, a large number of experimental data is used. The data sources are given in Table 5.1.

	Data sources
Laboratory data	Ashida and Tanaka (1967); Bogardi and Yen (1936); Bishop (1977); Casey (1935); Chyn (1935); East Pakistan Water and Power Development Authority (1967); Gibbs and Neill (1972); Gilbert (1914); Government of East Pakistan (1966. 68. 69); Grigg (1970); Ho, Pang-Yung (1939); Hubbell and Sayre (1964); Jorissen (1938); Kennedy (1961); MacDougall (1933); Mavis. Liu and Soucek (1937); Meyer-Peter and Muller (1948); Nordin (1976); Pratt (1970); Sato, Kikkawa and Ashida (1958); Simons and Richardson (1966); Simons, Richardson and Guy (1956-61); Singh (1960); Stein (1965); U. S. Waterways Experiment Station (1936); Williams, (1970); Willis (1979).
Field data	Mahmood et al.: Acop Canal data (1979); da Cunha: Portugal River data (1969); Einstein: Mountain Creek data (1944); Knott: Trinity River data (1974); Nedeco: South American river and canal data (1973); Nordin and Beverage: Rio Grand River data (1965); Samide: N. Saskatchewan River and Elbow River data (1971); Seitz: Snake and Clearwater River data (1976); Shinohara and Tsubaki: HII River data (1959); Toffaleti: Mississippi River data (1968); Toffaleti: Rio Grand near Bernalillo (1968).

Table 5.1: Data sources of Figs. 5.1 - 5.8

The measured values of c were determined from the data by

$$c = \frac{v}{v_*} = \frac{Q/(Bh)}{\sqrt{gSh}} \quad (5.20)$$

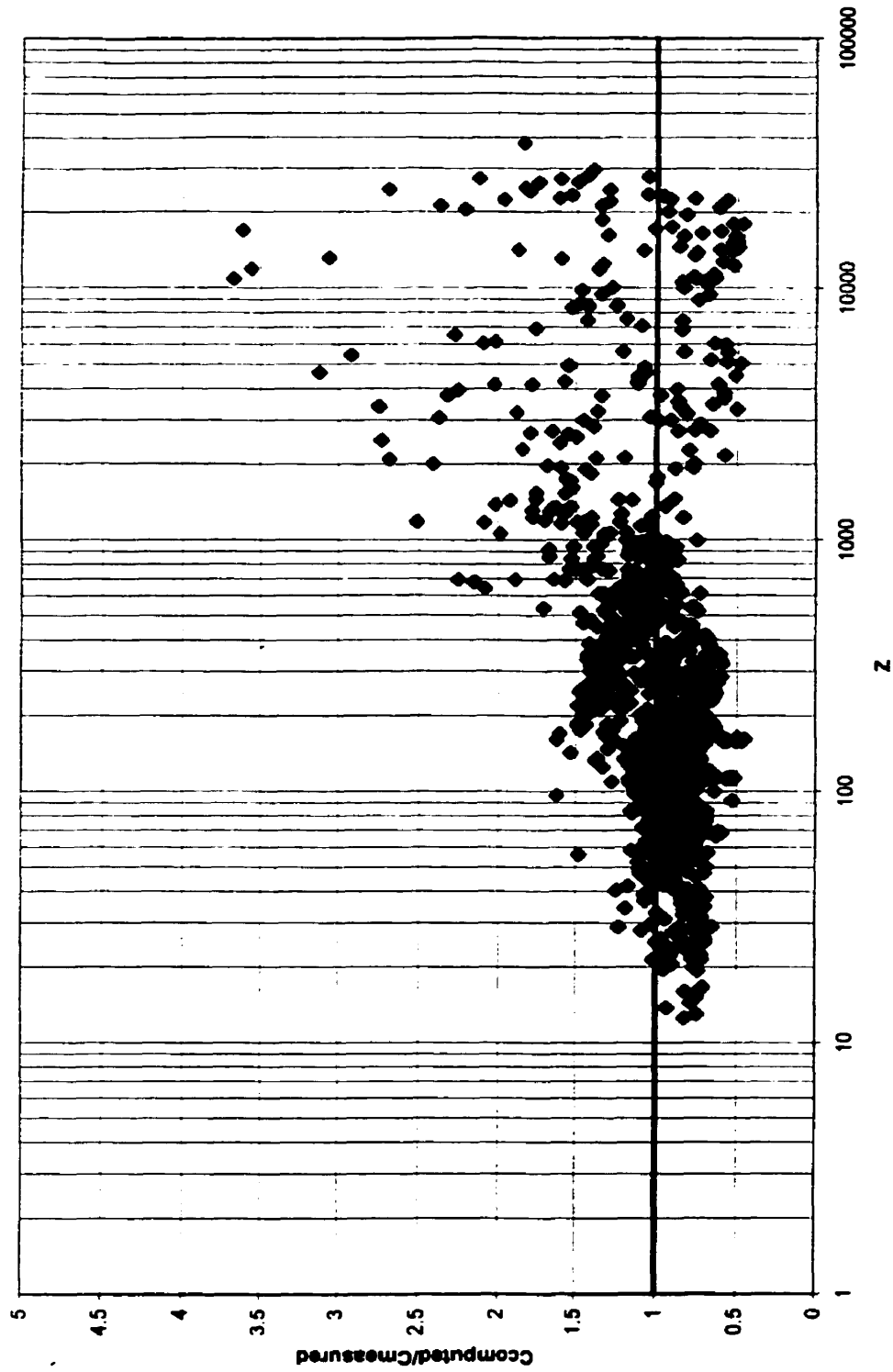
where the reported (measured) values of Q , B , h and S were used. The computed values of c for the corresponding experimental points were determined from the methods discussed in Section 5.3.1, viz:

1. Engelund-Yalin (Eq.(5.1)), where Δ/Λ and Λ/h are given by Yalin and Scheuerlein (1988).
2. Engelund-Yalin (Eq. (5.1)), where Δ/Λ is given by the present steepness equation (Eq. (5.17)) and Λ/h is given by Eq. (5.4).
3. van Rijn (Eq. (5.15)), where Δ/Λ and Δ are given by van Rijn's equations (2.24) and (2.25).
4. White et al. (Eq. (5.12)).

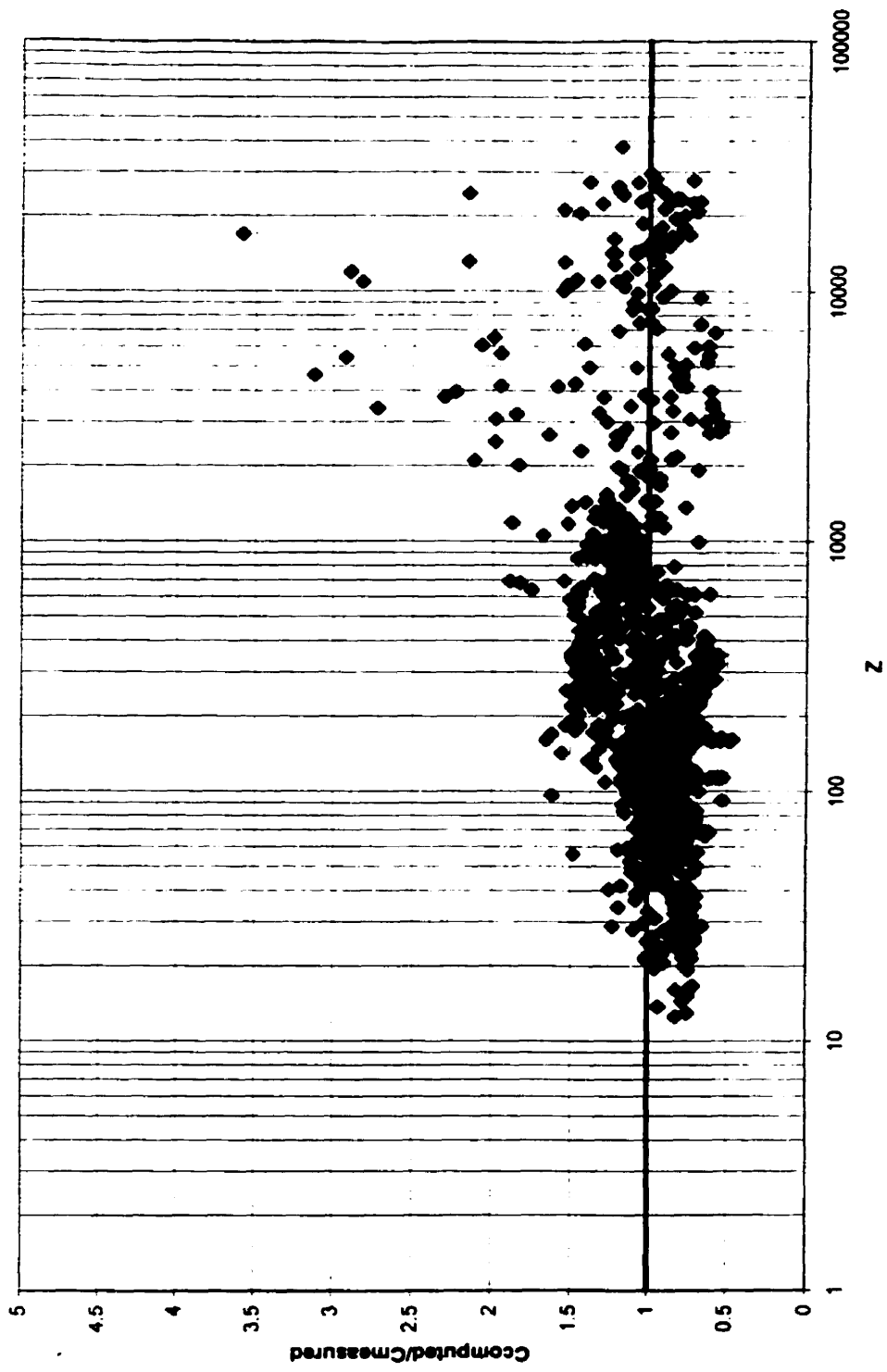
This thesis is founded on the idea that current methods of determination of dune steepness and thus friction factor, which appear to be valid for laboratory data (characterized by small Z -values) are not quite adequate for river data (characterized by large Z -values). For this reason, in what follows the ratio of computed c to measured c is plotted versus Z in Figs. 5.1 to 5.4.

In Fig. 5.1, $c_{computed}$ was obtained from Engelund-Yalin form (Eq. (5.1)), with Δ/Λ and Λ/h given by Yalin and Scheuerlein (1988). In Fig. 5.2, $c_{computed}$ was obtained from the same form, with Δ/Λ given by the dune steepness equation introduced in this thesis (Eq. (5.17)). As expected, for small values of Z ($Z < 1000$, say) these two plots do not differ from each other. However, for large values of Z , the data in Fig. 5.2 scatters considerably less around the line implying $c_{computed}/c_{measured} = 1$ than the data in Fig. 5.1. Clearly, the dune steepness expression introduced by the author leads to an improved prediction of the friction factor.

It is intended now to compare the present method of determination of friction factor with the methods of van Rijn and White et al. In Fig. 5.3, $c_{computed}$ was



**Figure 5.1: Plot of $C_{\text{computed}}/C_{\text{measured}}$ versus Z
(Yalin and Scheuerlein)**



**Figure 5.2: Plot of $C_{\text{computed}}/C_{\text{measured}}$ versus Z
(new steepness expression)**

obtained from van Rijn form (Eq. (2.49)), with Δ/Λ and Δ given by Eqs. (2.24) and (2.25), also due to van Rijn. The plot in Fig. 5.3 is much more scattered than the plot in Fig. 5.2, for both laboratory and river data. As Z increases, the larger is the amount of scatter. In agreement with the discussion carried out in Section 5.1, the plot shows also that for small values of h/K_s , the method of van Rijn is not satisfactory.

In Fig. 5.4, $c_{computed}$ was obtained from White et al.'s method (Eq. (2.31)). From comparison of this figure with Figs. 5.1 - 5.3, one concludes that White et al.'s method leads to the smallest amount of scatter around the line $c_{computed}/c_{measured} = 1$. Therefore, it is the method which yields the most reliable prediction of friction factor. This is not surprising, for this method is based on direct fit of the friction factor to a large number of experimental data.

Figs. 5.5 to 5.8 are the counterparts of Figs. 5.1 to 5.4, where values of computed flow depth h are compared with values of measured h . The plots in Figs. 5.5 to 5.8 give further support to the above comments related to the friction factor plots.

In summary, from the comparison of computed and measured values of both friction factor c and flow depth h , we can see that the method of White et al. achieves the best agreement between the experimental data and computed results. van Rijn's results are generally not quite satisfactory for large Z -values. In addition, the computed values of those points with "small" values of h/K_s , (< 7) show substantial deviation from their measured counterparts as can be seen in Fig. 5.3 and Fig. 5.7. Yalin's method, which is based on first principle, improves considerably on van Rijn's and, by using the author's steepness expressions in Yalin's friction factor equation, further improvement has been achieved.

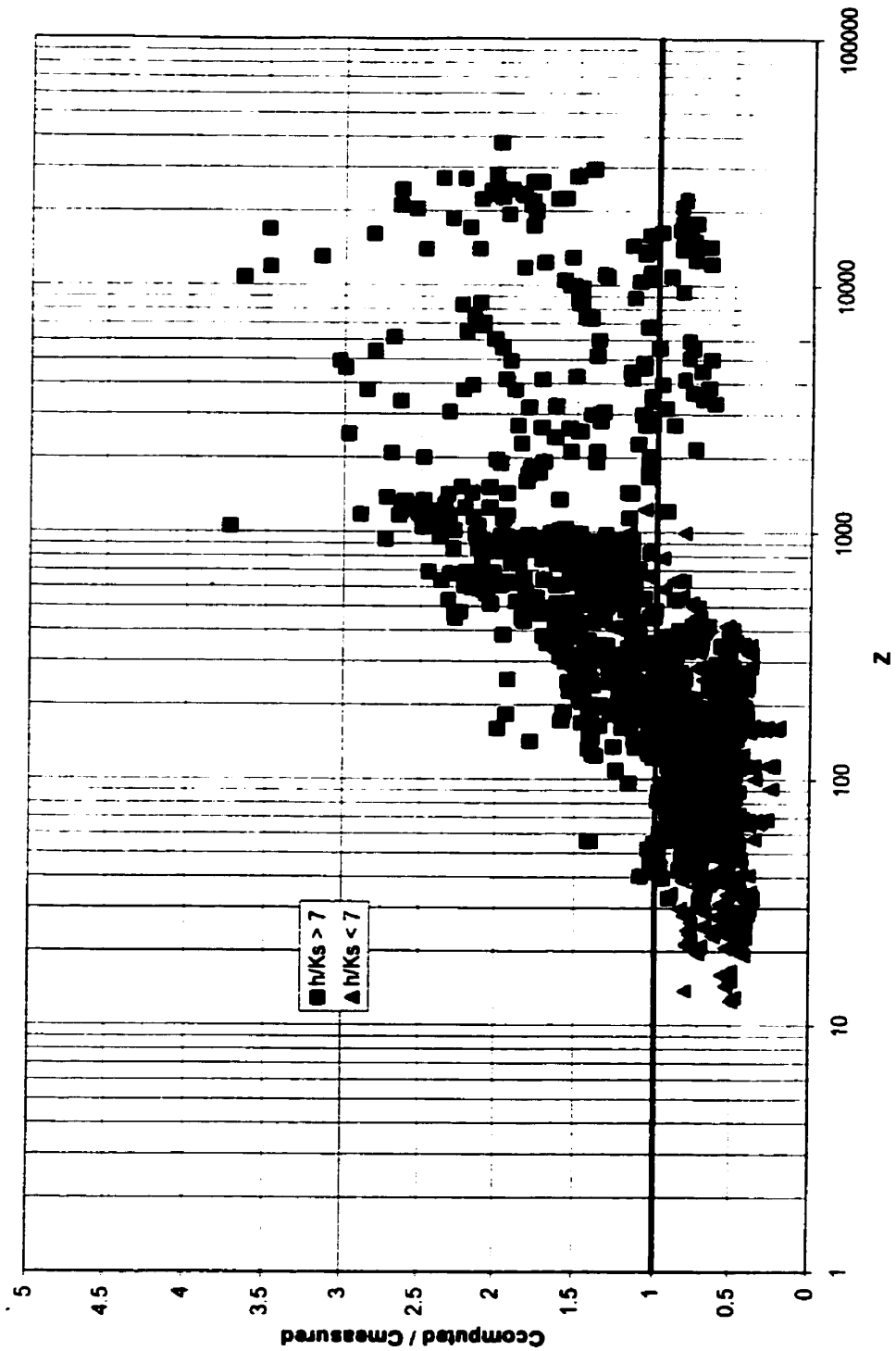


Figure 5.3: Plot of $C_{computed}/C_{measured}$ versus Z (van Rijn)

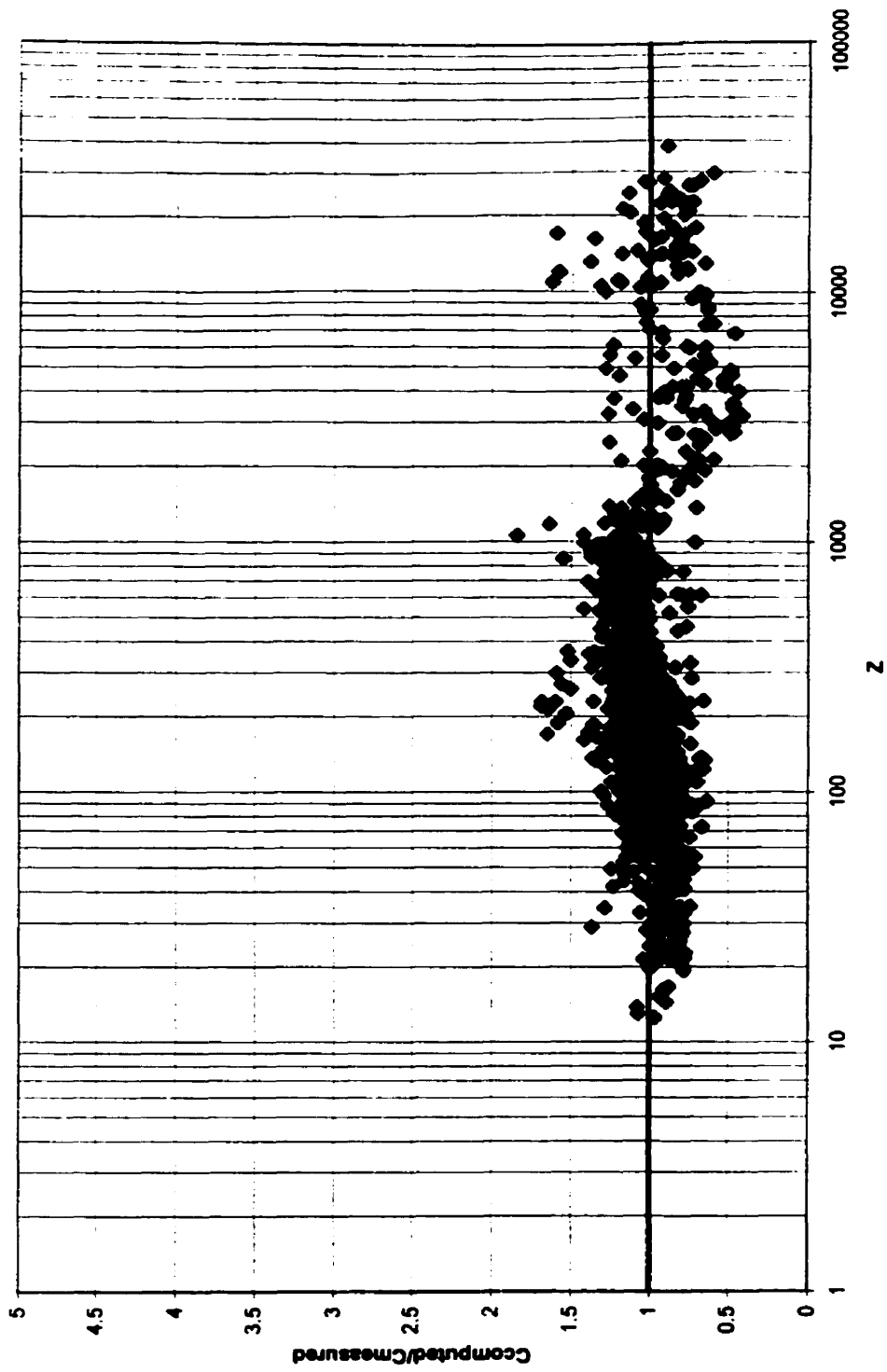


Figure 5.4: Plot of $C_{\text{computed}}/C_{\text{measured}}$ versus Z (White et al.)

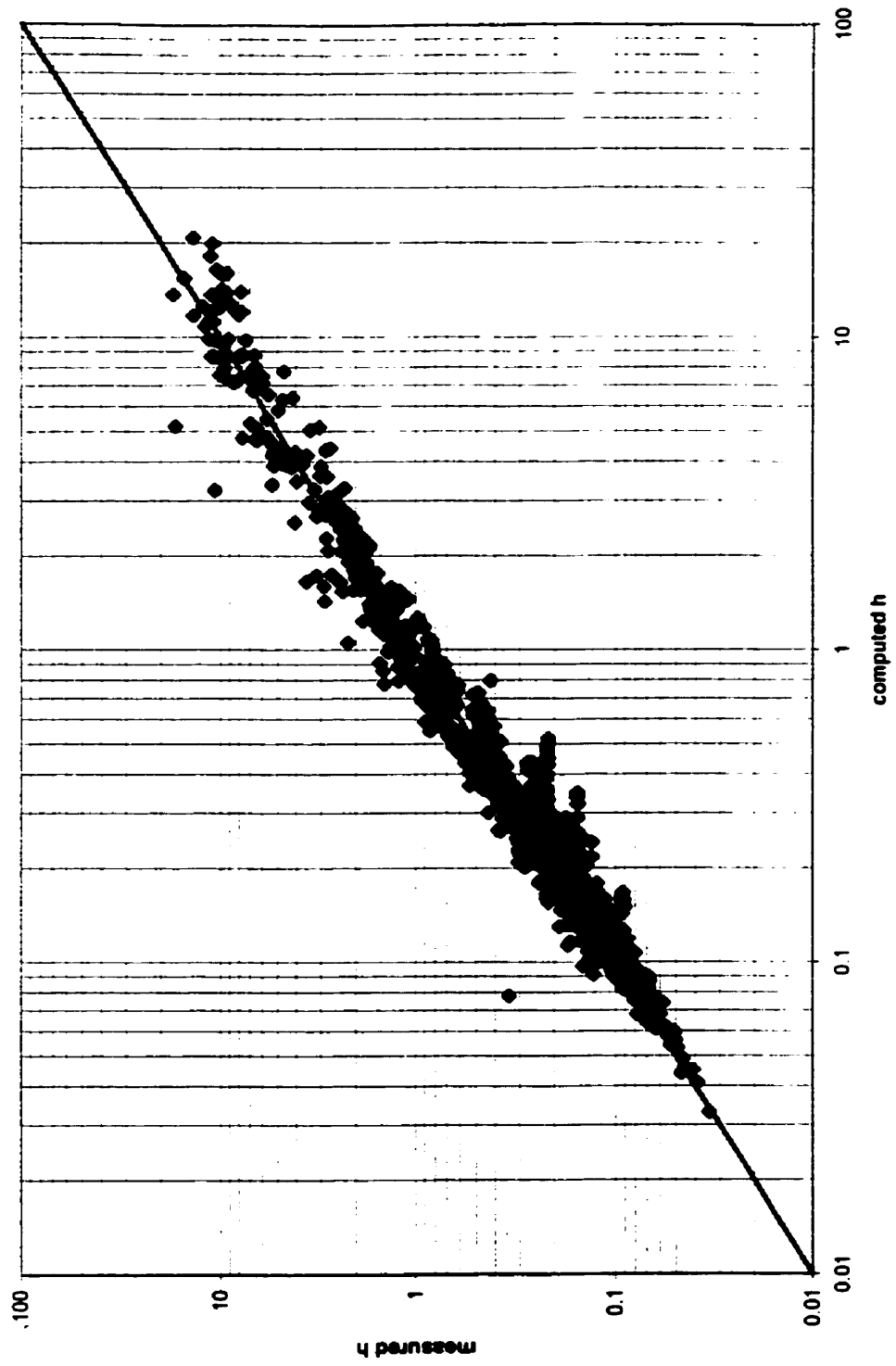


Figure 5.5: Plot of h_{measured} versus h_{computed} (Yalin and Scheuerlein)

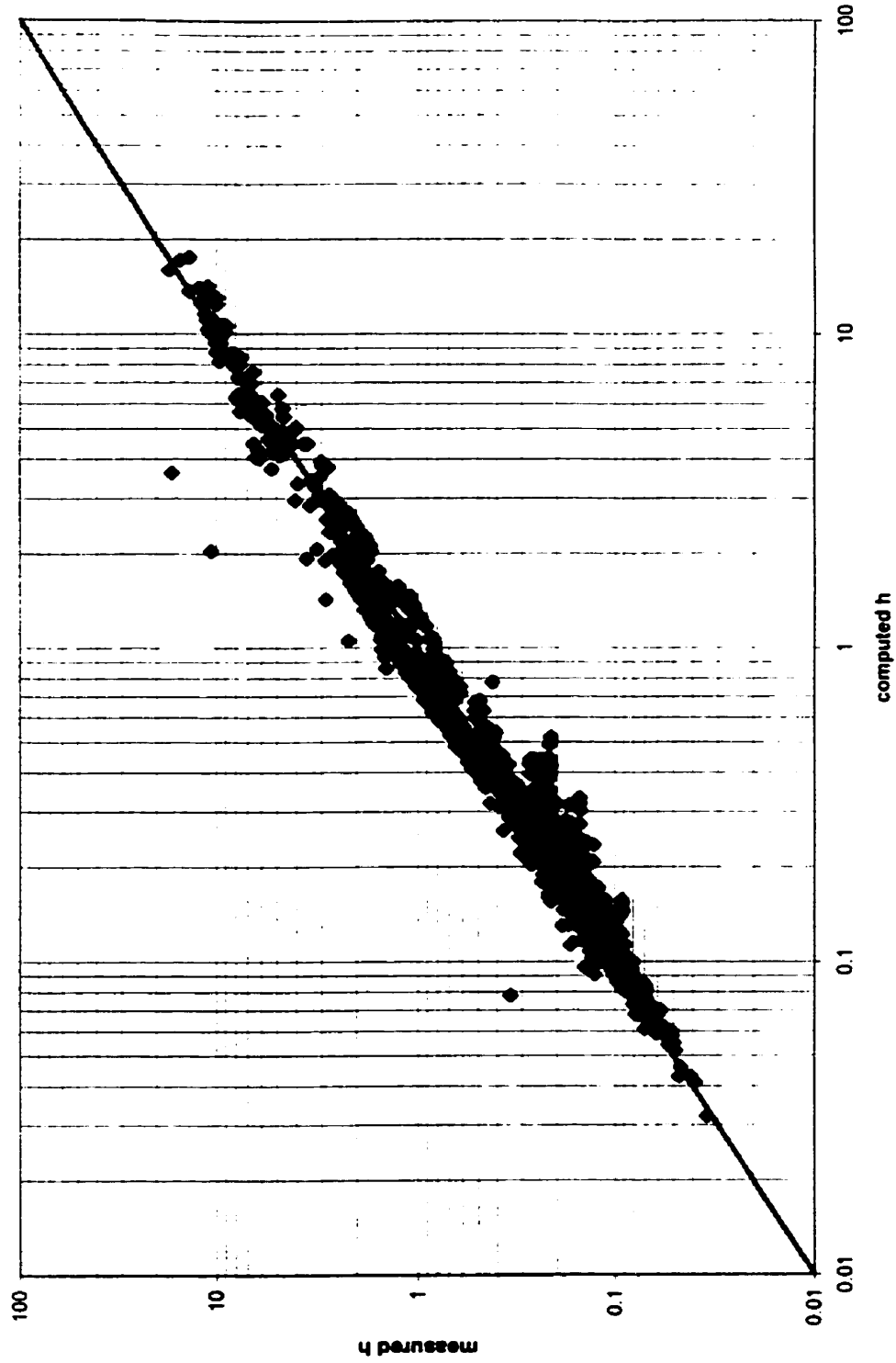


Figure 5.6: Plot of h_{measured} versus h_{computed} (new steepness expression)

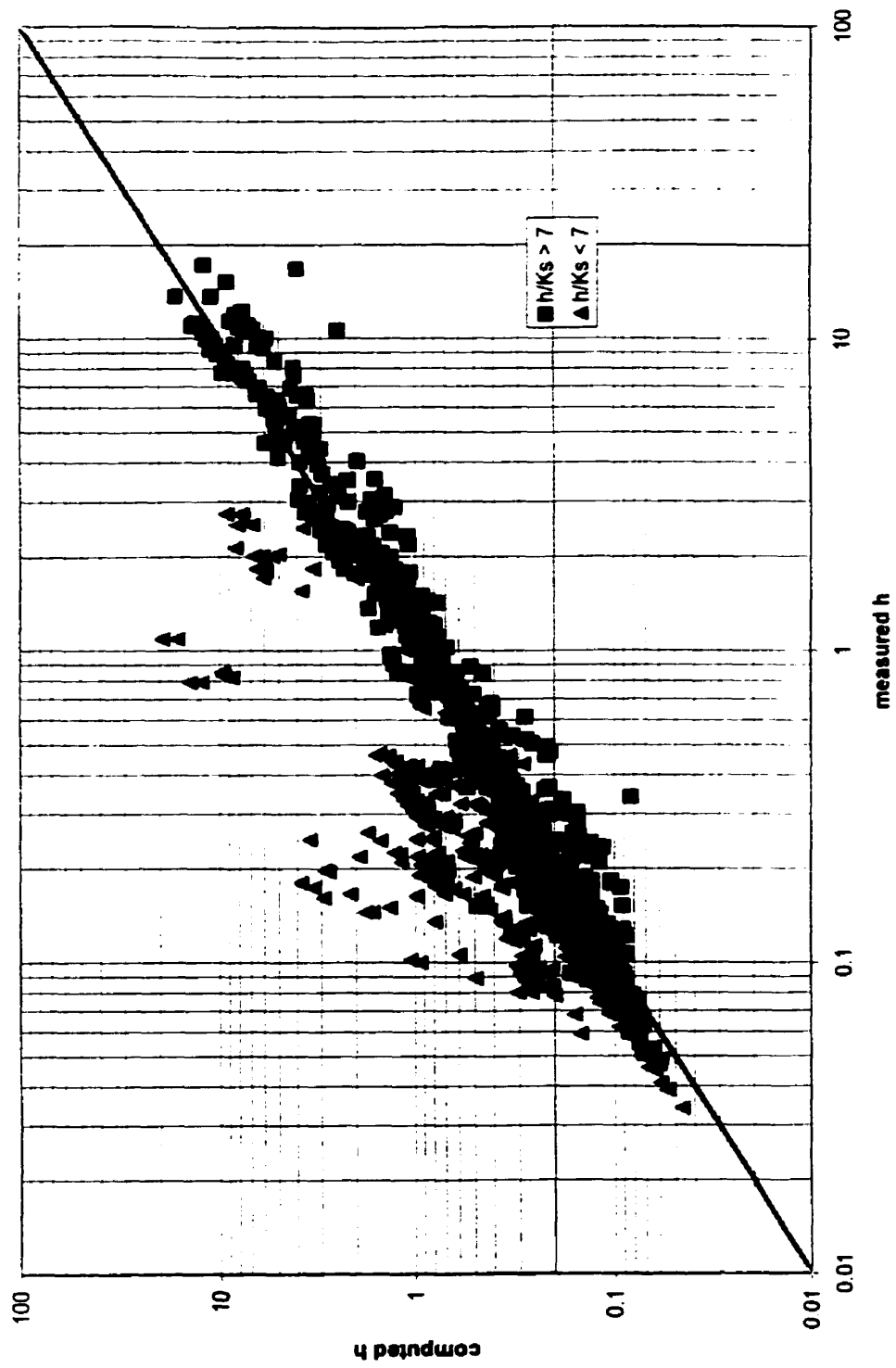


Figure 5.7: Plot of $h_{measured}$ versus $h_{computed}$ (van Rijn)

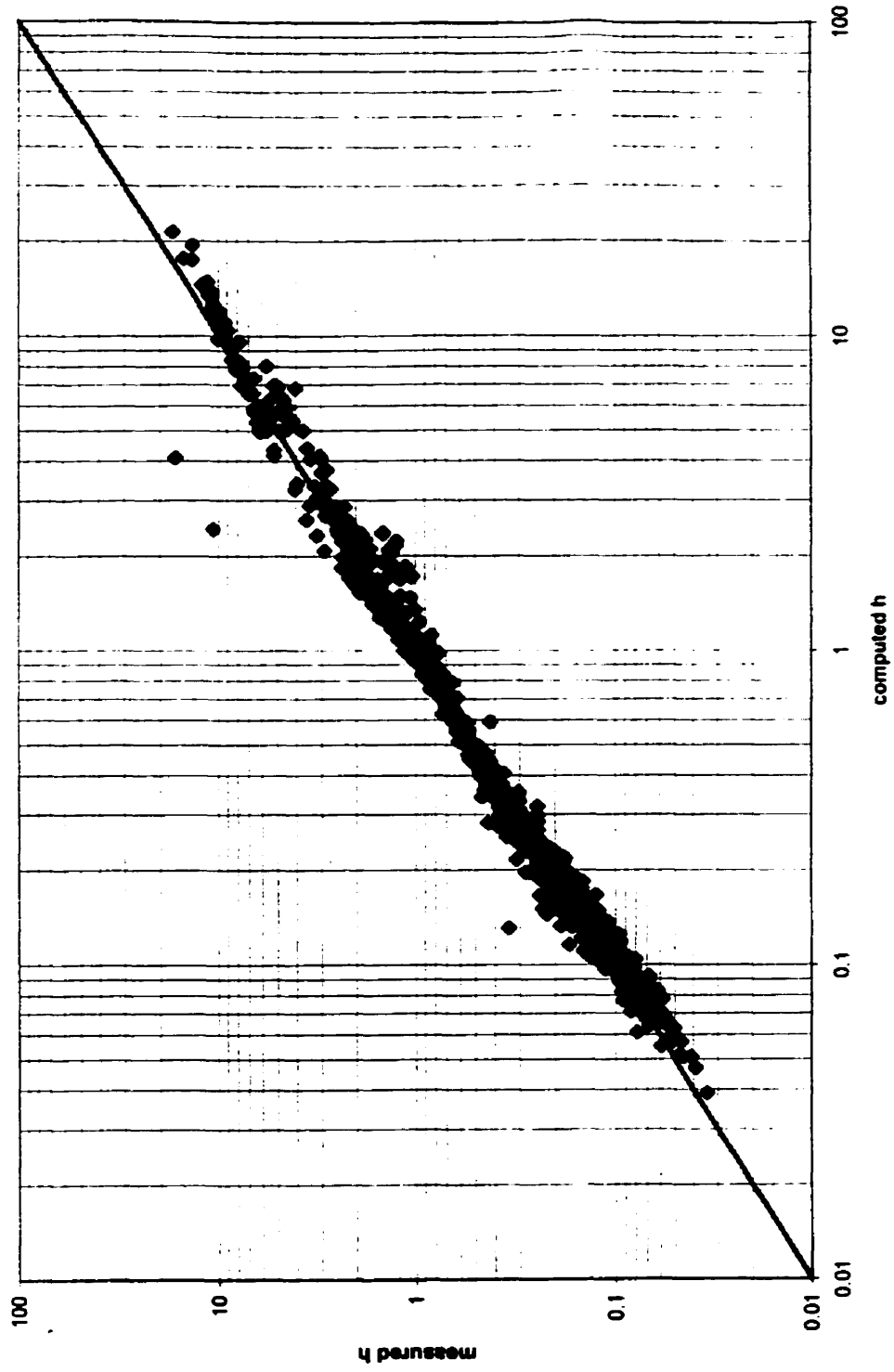


Figure 5.8: Plot of h_{measured} versus h_{computed} (White et al.)

6.0 CONCLUSIONS AND SUGGESTIONS FOR FUTURE RESEARCH

i) An in-depth analysis of the most popular expressions for dune steepness (namely of the expressions by Yalin and Scheuerlein (1988) and van Rijn (1982)) was carried out. It was revealed that the expressions by these authors were developed on the basis, mainly of laboratory data. Values of measured as well as estimated dune steepness were plotted in the $(\Delta/\Lambda; \eta)$ -plane (see Fig. 4.1 to 4.3). On the basis of these plots, it has been found that existing expressions for dune steepness seem to reflect adequately the behavior of dunes for small values of Z (typical of laboratory data). However, for large values of Z (typical of river data), existing expressions for dune steepness:

- overpredict the maximum steepness;
- underpredict the value of η for which this maximum steepness occurs;
- underpredict the values of η for which the “flat bed at advanced stages” is reached.

ii) A new dune steepness expression (Eq. (4.11)) is introduced in this thesis. This expression can be regarded as an extension (generalization) of previous expressions by Yalin and his co-workers for large values of Z . The expression was developed on the basis of all the available laboratory and river data. It is shown that this expression represents a better model for the expression of dune steepness.

iii) The author's dune steepness expression was used to determine the friction factor. It was shown that if this new expression is used, instead of the expressions suggested by Yalin and his co-workers, the prediction of friction factor is substantially improved.

iv) An in-depth analysis of the most popular methods of determination of friction factor and flow depth is carried out. The methods of Engelund-Yalin (1966), (1964), van Rijn (1982), and White et al. (1979) were analyzed. From the comparison of computed and measured values of both friction factor and flow depth, one infers that the method of van Rijn yields results which are generally not satisfactory for large

values of Z (typical of field data). In addition, the computed values of those points with "small" values of h/k_s (< 7) show substantial deviation from their measured counterparts (see Figs. 5.3 and 5.7). Engelund-Yalin's method, which is based on first principles, improves considerably on van Rijn's, especially for large Z -values. It was further noticed that the use of the author's steepness expression in Engelund-Yalin friction factor equation yields further improvement. Overall the method of White et al. yields the best agreement between measured and computed values of the friction factor. This is not surprising, for White et al.'s method was entirely derived from data analysis. However, the method does not provide insight into the physical mechanisms involved, and cannot be used if there is an interest in defining not only friction factor, but also the bed form geometric characteristics.

With regard to suggestions for future research, the two following points should be considered:

- 1- The development of expressions to characterize the bed form geometric characteristics, as well as the verification of such expressions requires accurate and extensive data. Although a large amount of laboratory data has been reported in the literature, river data is rather scarce. It is, however, possible that data has been collected around the world (by governmental agencies, consulting firms, etc.), for one purpose or another, but remains unpublished. An effort should be undertaken to gather unpublished data, and to promote active data collection.

- 2- The present thesis, just like most works carried out to date, rests on the fact that the flow is steady and bed forms are fully developed. However, in practice, there are many occasions where the conditions deviate substantially from these assumptions. Thus, bed form geometry under unsteady flow conditions should be investigated in the future.

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Appendix A
Wall Friction

The total cross section is divided into two regions as shown in Fig. A.1. The Regions 1 are under the wall effect and Region 2 is not. The area, wetted parameter, and hydraulic radius for Region 1 are as follows:

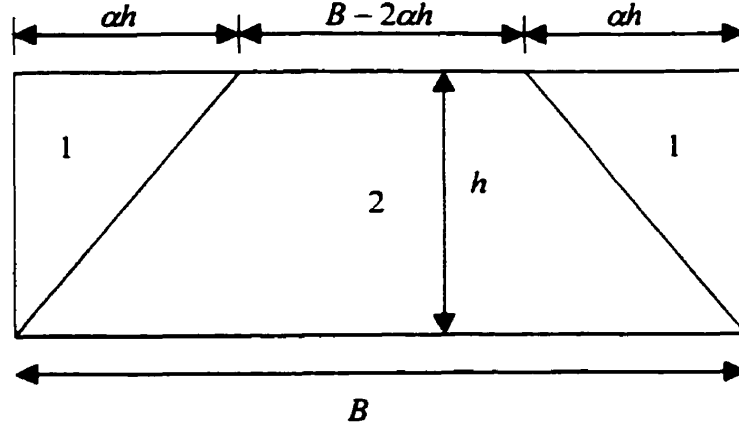


Figure A.1: Regions under wall friction effect

$$A_1 = \alpha \frac{h^2}{2} \quad (\text{A.1})$$

$$P_1 = h \quad (\text{A.2})$$

$$R_1 = \alpha \frac{h}{2} \quad (\text{A.3})$$

For Region 2,

$$A_2 = (B - \alpha h)h \quad (\text{A.4})$$

$$P_2 = B \quad (\text{A.5})$$

$$R_2 = \frac{(B - \alpha h)h}{B} \quad (\text{A.6})$$

According to Chézy equation, the flow rate Q is given by

$$Q = Ac\sqrt{gSR} \quad (\text{A.7})$$

Thus, the flow rate Q_1 and Q_2 for regions 1 and 2 are

$$Q_1 = \frac{1}{2}\alpha h^2 c_1 \sqrt{gS\alpha \frac{h}{2}} \quad (\text{A.8})$$

and

$$Q_2 = (B - \alpha h)hc_2\sqrt{gS\frac{(B - \alpha h)h}{B}}, \quad (\text{A.9})$$

respectively. The total flow rate $Q = 2Q_1 + Q_2$ is given by

$$Q = \alpha h^2 c_1 \sqrt{gS\alpha\frac{h}{2}} + (B - \alpha h)hc_2\sqrt{gS\frac{(B - \alpha h)h}{B}} \quad (\text{A.10})$$

Since $v = Q/(Bh)$, where Q is given by Eq. (A.10), the average flow velocity v is obtained as:

$$v = \sqrt{gSh} \left[\frac{\alpha^{3/2} h}{\sqrt{2} B} c_1 + \left(1 - \alpha \frac{h^{3/2}}{B} \right) c_2 \right] \quad (\text{A.11})$$

Assuming that $c_2 = c_0$, and $c_1 = \alpha_* c_0$ ($c_0 = \frac{1}{\kappa} \ln(11h/k_s)$), then

$$\frac{v}{\sqrt{gSh}} = c = \left[\frac{\alpha^{3/2}}{\sqrt{2}} \alpha_* \frac{h}{B} + \left(1 - \alpha \frac{h^{3/2}}{B} \right) \right] c_0 \quad (\text{A.12})$$

where c is the total friction factor for the entire cross-section. Eq. (A.12) can be written as

$$\frac{1}{c^2} = \frac{1}{\left[\frac{\alpha^{3/2}}{\sqrt{2}} \alpha_* \frac{h}{B} + \left(1 - \alpha \frac{h^{3/2}}{B} \right) \right]^2} \cdot \frac{1}{c_0^2}. \quad (\text{A.13})$$

On the other hand (see da Silva (1995)), the effect of the wall can be incorporated into c in the following manner:

$$\frac{1}{c^2} = \frac{1}{c_*^2} + \frac{1}{c_w^2} \quad (\text{A.14})$$

Combining Eqs. (A.13) and (A.14), the following expression is determined for $1/c_w^2$:

$$\frac{1}{c_w^2} = \frac{1}{c^2} - \frac{1}{c_*^2} = \left[\frac{\alpha^{3/2}}{\sqrt{2}} \alpha_* \frac{h}{B} + \left(1 - \alpha \frac{h^{3/2}}{B} \right) \right]^{-2} \cdot \frac{1}{c_0^2} - \frac{1}{c_*^2} \quad (\text{A.15})$$

In order to remove the effect of bank friction from the data, it is thus sufficient to determine c from the data with the aid of the following equation:

$$\frac{1}{c^2} = \frac{1}{c_*^2} - \frac{1}{c_w^2}, \quad (\text{A.16})$$

where $c_* = Q/Bh\sqrt{gSh}$, and c_w is given by Eq. (A.15). The following values were adopted for α and α_* :

$$\alpha = 2.5 ; \quad \alpha_* = 1.0 .$$

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