Performance Analysis of Direct Sequence Code Division Multiple Access Systems

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A dissertation submitted to the
Department of Electrical and Computer Engineering
in conformity with the requirements for
the degree of

Doctor of Philosophy

Queen's University
Kingston, Ontario, Canada

April, 2000

0-612-52848-0

To my parents. Mümin and Şadiye Sunay
and to Yaşar Aktuğ. in memoriam.

ABSTRACT

At a time where there is an ever growing move towards wireless communications, it is important to provide the literature with a rigorous, accurate performance analysis tool for DS CDMA systems since most third generation systems are based on this multiple access scheme. In this light, this thesis is concerned with providing an analysis scheme to calculate the error probabilities of DS CDMA systems in additive white Gaussian noise (AWGN) and multipath fading channels with diversity combining when correlator receivers are used to coherently receive the signal. Unlike the popular approximations used for performance analysis, the scheme developed here is valid for systems at a wide variety of operating points.

Developing an accurate analysis scheme for DS CDMA is important for two reasons. First, it provides a better understanding of such systems and second, such an analysis scheme could be used to find just how accurate the popular approximations that are currently in use to estimate the system performance are and whether they have any limitations.

The error probability expressions derived throughout the thesis are based on a Fourier series approximation of the error function, $Q(x)$. Once the error probability expressions are derived, we evaluate the performance of biphase and quadriphase spread, coherent DS CDMA systems in AWGN and multipath fading channels. Synchronous, quasi-synchronous and asynchronous systems are considered. For multipath fading channels, we investigate the effects of diversity combining on the system performance. We consider selection diversity, equal gain combining and maximal ratio

combining. We investigate the effects of nonzero synchronization errors on the system capacity and conclude that the such errors cause significant capacity reductions in the system. We observe that in all cases considered, the synchronization errors effect the system performance by potentially reducing the energy of the desired signal component of the received signal and by introducing self interference.

We also calculate error probabilities for various DS CDMA systems using the Standard Gaussian Approximation (SGA), Improved Gaussian Approximation (IGA) and the Fourier series based expression we derived in this thesis. The scenarios are selected such that conditions for the validity of the Central Limit Theorem are questionable. We investigate a scarcely populated system, a system with dominant interferers, a multi-service system with uneven number of subscribers to the offered services and system in a frequency non-selective Rayleigh fading channel. We observe that the SGA in all of these cases yields optimistic results, especially when the number of users active in the system is small. The IGA on the other hand, gives accurate results for a scarcely populated system, a system with a dominant interferer and a multi-service system. However, in the dominant interferer and multi-service cases, if the strength of the dominant interferer is significantly large, or if the signaling rates between the different services are large, the IGA does not work at all. A modification can be made to overcome this problem at the expense of a reduced accuracy. For systems in fading channels neither of the Gaussian approximations seems to converge to the true error probability.

ACKNOWLEDGMENTS

No single person affects a student's graduate experience more than his advisor. I am very fortunate to have had my advisor Dr. Peter J. McLane. His generosity with his time, knowledge, experience and financial support has been invaluable. I believe he perfectly maintained the delicate balance of guidance and freedom that, I hope, will turn me into a successful researcher in years to come. The kindness he and his family showed me during my time at Queen's will always be remembered and appreciated.

This thesis is dedicated to my parents. Even though they were thousands of miles away during the course of my program, as always, I continually felt their boundless support, love, understanding and unfaltering approval. I am forever indebted to them. Many thanks to my brother and sister as well for their love and support.

This thesis is also dedicated to the memory of my high school mathematics teacher, Yaşar Aktuğ. While I regret that he was never able to see me complete my Ph.D., I am comforted by the knowledge of the pleasure it would have given him. His instruction in high school was primal in awaking a desire for learning within me that I hope will never expire.

Writing the thesis while juggling with a full time job in another country has not been easy. I wish to thank Dr. McLane for his understanding and trust to allow me to leave to pursue a full time job before finishing the thesis. I should thank to my parents, Dr. McLane and my dear friend Alemayehu Seyoum for the constant pushing that was necessary to get to this stage.

I wish to thank the faculty of the Department of Electrical and Computer Engineering at Queen's University for their excellent instruction and dedication. Special thanks to Dr. Norman Beaulieu for his development of the Fourier Series method for computing error rates which is used throughout the thesis.

I would also like to take this opportunity to thank the friends I have had, past and present. Over the years they have helped become the person I am today. It has been said many times that a person is only as good as those who surround him - and I am indeed fortunate.

Finally, I gratefully acknowledge the financial support of the Canadian Institute for Telecommunications Research (CITR) and Queen's University.

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Notations

$a_k(t)$	k'th user's data signal
$a_{k,j}$	j'th symbol of k'th user's data signal
$b_k(t)$	k'th user's spreading sequence
$b_k^i(t)$	k'th user's in phase spreading sequence
$b_k^i(t)$	k'th user's quadrature spreading sequence
$b_{k,j}$	j'th chip of k'th user's spreading sequence
$b_{k,j}^i$	j'th chip of k'th user's in phase spreading sequence
$b_{k,j}^q$	j'th chip of k'th user's quadrature spreading sequence
$\beta_{k,l}$	Path gain for the l'th path of the k'th user
$C_j(\lambda)$	Aperiodic Discrete Autocorrelation Function
D	Desired signal plus self interference term at the output of the receiver correlator
D_1	Desired signal term at the output of the receiver correlator
D_2	Self interference term due to timing errors at the output of the receiver correlator
$\delta(t)$	Unit impulse function
ϕ_k	k'th user's received signal phase offset
$\hat{\phi}_k$	Estimation of the k'th user's received signal phase offset
$\Phi_m(w)$	Characteristic function of the random variable m evaluated at w

G	Processing gain
$h_k(t)$	k'th user's observed channel
$h_{k,l}(t)$	l'th path of k'th user's observed channel
$H_0(z)$	Zeroth-order Struve function
$H_1(z)$	First-order Struve function
$J_0(z)$	Zeroth-order Bessel function of the first kind
$J_1(z)$	First-order Bessel function of the first kind
K	Number of users
L_p	Loss in the signal power in dB as it is transmitted through the mobile channel
λ	Constant specifying the nature of the DS CDMA system
M	Multiple access interference term at the output of the receiver correlator
$M(a, b, z)$	Confluent Hypergeometric Function
N	AWGN term at the output of the receiver correlator
$n(t)$	Additive White Gaussian Noise
P_k	k'th user's transmitted signal power
$p_T(t)$	Unit amplitude rectangular pulse of duration T
θ_k	k'th user's initial phase offset
$r(t)$	Received signal at the input of the correlator receiver
$s_k(t)$	k'th user's transmitted signal
T	Symbol period
T_c	Chip Period
τ_k	k'th user's received signal time offset

$\hat{\tau}_k$ Estimation of the k'th user's received signal time offset

$\mathcal{T}_k$ k'th user's initial message starting time

w_c Common signaling frequency

$\Psi(t)$ Unit amplitude chip waveform

Z_1 Input to the decision device at the correlator receiver tuned in to capture the first user's signal

Abbreviations

2G	Second Generation
2.5G	Advanced Second Generation
3G	Third Generation
AM	Amplitude Modulation
AMPS	Advanced Mobile Phone System
AWGN	Additive White Gaussian Noise
BPSK	Binary Phase Shift Keying
CDMA	Code Division Multiple Access
cdma2000	The generic name for the third generation system based on TIA/EIA-2000 family of standards
CLT	Central Limit Theorem
CSMA	Carrier Sense Multiple Access
DAMPS	Digital AMPS
dB	Decibel
DS CDMA	Direct Sequence CDMA
DQPSK	Differential Quadrature Phase Shift Keying
EDGE	Enhanced Data Rates for Global Evolution
EIA	Electronics Industry Association
ETACS	Extended European Total Access Cellular System
FDMA	Frequency Division Multiple Access
FH CDMA	Frequency Hopped CDMA
FM	Frequency Modulation
GMSK	Gaussian Minimum Shift Keying

GPRS	General Packet Radio Service
GSM	Groupe Spécial Mobile or Global System for Mobile
HSCSD	High Speed Circuit Switched Data
IGA	Improved Gaussian Approximation
iid	Independent, identically distributed
IMT-2000	International Mobile Telecommunications at 2000 MHz
IMTS	Improved Mobile Telephone System
ISDN	Integrated Switched Digital Network
ITU	International Telecommunications Union
NMT	Nordic Mobile Telephone System
PCS	Personal Cellular System
PDC	Pacific Digital Cellular
PDMA	Polarization Division Multiple Access
P(E)	Probability of Error
PMTS	Public Mobile Telephone Service
PN	Pseudonoise
POTS	Plain Old Telephone System
PSTN	Public Switched Telephone Network
QPSK	Quadrature Phase Shift Keying
RF	Radio Frequency
SDMA	Space Division Multiple Access
SGA	Standard Gaussian Approximation
SIR	Signal to Interference Ratio
SNR	Signal to Noise Ratio

TDMA	Time Division Multiple Access
TIA	Telecommunications Industry Association
USDC	U.S. Digital Cellular
WARC	World Administrative Radio Commission
W-CDMA	Wideband CDMA

Chapter 1

INTRODUCTION

Currently, wireless communications is enjoying a rapid development. According to some, this development is so revolutionary [23] that the implications of it will probably be one of the most important economic forces shaping the society in the first half of the next century [45]. Needless to say, wireless communications is receiving public attention as it never has before. The idea "Third Generation Wireless Communications and Related Services" has captured the attention of the media, and with it, the imagination of the public. Historians have already declared the 1990s as the "Telecommunications Decade." Hardly a week goes by without seeing a short paragraph or even an article on the subject appear in a popular magazine or newspaper (see for example, [45, 257, 65, 66, 67]). Many of the wireless communication terms that were only familiar to the researchers of the field until recently have been endorsed by the media. It seems that everyone has an idea what "Personal Communications" or "Multimedia Communications" might mean. People are used to seeing "Spread Spectrum" based cordless telephones on display at various shops. This brings an enormous demand on the newest state-of-the-art communications gadgets. Marketing surveys state that most of the existing wired telephone system[1] users want wireless communications [38].

In 1992, the World Administrative Radio Commission (WARC) of the Interna-

[1] Commonly referred as POTS: plain old telephone system

tional Telecommunications Union (ITU) formulated a plan to implement a global frequency band in the 2000 MHz range that would be common for all countries for universal wireless communications systems. This visionary concept stemmed from the desire to have flexible, seamless global service provision. This effort has been named as International Mobile Telecommunications 2000 (IMT-2000) by the ITU in 1995 [24]. Combining the notions of microcells, macrocells and satellites, the IMT-2000 system is envisioned to provide a wireless network that would not only enable a user in the system to be able to make or receive calls no matter where they are, but also provide enhanced multimedia capabilities that require high speed packet data access as well as circuit switched data. A large portion of the current research in wireless communications is focused on determining which modulation, multiple-access, voice coding and networking schemes would be suitable for such a wireless system. Unfortunately, this question is not easy to answer. The conclusions that have appeared in the literature have been mostly speculative and they vary significantly [3, 22, 38, 68, 69, 95, 147, 207, 242]. The preliminary results of the on-going standards efforts have been just as varied [24]. ITU, encouraging global service provision, stresses the importance of converging these essentially competing technologies. It seems that the final decision, if one is ever made, will rely on politics as much as the technology itself.

Among the multiple access schemes that are being considered for the IMT-2000, code division multiple access (CDMA) constitutes the main scope of this thesis. CDMA was first proposed during World War II as a means to avoid jamming and eavesdropping of combat messages [48, 171, 194]. Subsequent discoveries have suggested that CDMA would be suitable for use in commercial wireless communications as well [69]. In fact, some argue that CDMA will be the ultimate tool to achieve high quality, multiple-service, global commercial communications [207, 247, 250]. Other researchers believe that CDMA, though mathematically intriguing, is an over-hyped idea that is bound to be disappointing [3, 38].

Will CDMA really emerge as the only scheme for future wireless communications or will it be one of the many schemes in use? If used, what kind of a performance should one expect from a CDMA system? How easy is it to implement a CDMA based personal communications system that supports a multitude of services and if implemented, will it be sufficient for the anticipated demand for such systems? Until analyses of CDMA based systems along with other competing technologies are performed rigorously, these questions cannot be answered justly. Following on this argument, we present a rigorous, accurate method to calculate the performance of various single cell CDMA systems in additive white Gaussian noise (AWGN) and multipath fading channels in this thesis. Multi-cell operation is not considered, but it is felt that the method developed herein could be applied to some aspects of such systems.

1.1 The Wireless Evolution: A Historical Retrospective

Since much of the current hype on the CDMA technology stems from its possible use in third generation commercial wireless communications, it is appropriate to present here a brief historical retrospective of the wireless communication evolution.

Being able to communicate with people on the move, an idea that was entertained in many science fiction novels, first came to life in 1897 when Marconi demonstrated that the radio was capable of providing continuous contact with ships sailing the English channel. Since then, the telecommunications and electronics fields saw considerable developments which in turn made new and improved mobile services plausible. The first of such services was established in 1934 when police forces of a number of U.S. cities started using amplitude modulation (AM) to provide wireless communications between the police cars and the police stations [185]. AM was very sensitive to vehicle ignition noise and therefore, the performance of this system was very poor. The invention of frequency modulation (FM) by Edvin Armstrong in 1935 and consequently the need for reliable mobile networks during World War II accelerated the research on all types of communications including mobile. It was during this time when the first spread spectrum communications system was deployed [198]. Using the technologies that emerged during the war era, notably the FM, the first Public Mobile Telephone Service (PMTS) was established in select U.S. cities in 1946. This system provided service in an area of radius of approximately 50 km. Mobility was achieved mainly by the use of very high transmission powers in the system. In 1950, a spectrally more efficient mobile system, the Improved Mobile Telephone Service (IMTS) replaced PMTS [185]. Neither of these techniques caught the attention of the general public because they both had very limited coverage areas, they were not compatible with the existing wired Public Switched Telephone Network (PSTN) and they were very expensive.

The ability to provide wireless communication to an entire population was conceived only after the development of the cellular concept in Bell Laboratories in 1968 [139]. The aim in developing the cellular concept was not only to provide wireless coverage to a very large geographical area but also to accommodate a large number of subscribers without diminishing the service quality. This required an efficient use of the limited available spectrum. The envisioned cellular system was made up of transmitters, called base stations. The coverage of each base station was limited to a small geographical area called a cell so that the same radio channels could be reused by another base station some distance away [71]. Base stations were inter-linked and a switching technique called a handoff was proposed to enable a call to proceed uninterrupted when the mobile moved between different cells. Unfortunately, the solid-state technology of 1960s did not enable the realization of the cellular concept at that time. With the development of highly reliable, miniature, solid-state radio frequency hardware in 1970s, the door to implementing cellular systems finally opened. The first of such systems was established in Japan in 1979 by the NTT in the 400 MHz band using a channel bandwidth of 25 kHz. North America and Europe soon followed the trend with the deployment of Advanced Mobile Phone System (AMPS) in North America in 1983 [267] and the Nordic Mobile Telephone System (NMT) and Extended European Total Access Cellular System (ETACS) in Europe in 1981 and 1985, respectively. AMPS uses the 824-894 MHz frequency band and utilizes a channel bandwidth of 30 kHz, whereas ETACS uses the 900 MHz frequency band with a channel bandwidth of 25 kHz. All first generation, analog, cellular systems, like NTT, AMPS, NMT and ETACS use FM as their modulation schemes.

Cellular networks are based on the notion of users simultaneously sharing a finite amount of spectrum to achieve spectral efficiency. This should be accomplished in such a way that the interference from all the users that share the same spectrum is kept at a minimum. The network management schemes that provide such conditions are called multiple access schemes. Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) are the three major access techniques used to share the available bandwidth in wireless communication systems. These concepts will be described in the following section.

The first generation cellular networks, NTT, AMPS, NMT and ETACS which all use FDMA for multiple access prompted a rapid growth and demand in cellular systems all around the world [267]. This growth has transformed mobile communications from a special service to a select few to a form of communications that is part of our everyday lives. The growth in the number of mobile subscribers has placed a strain on the cellular systems in many cellular systems in metropolitan areas, making it increasingly difficult to operate and maintain the quality of such systems. As the number of subscribers continued to increase, cells had to become smaller. Due to the frequency-reuse concept, adjacent cells could not use the same frequency to minimize the possible interference, so a limited number of frequencies had to be reused at closer distances, creating an increasing source of interference.

These problems along with the fragmentation problems of the incompatible analog, first generation cellular networks deployed all across the world prompted the move from analog to digital in wireless communications. Digital radio promised to be void of many of the limiting qualities of its analog counterpart. In [22], it is stated that the telecommunication giant, AT&T originally envisioned a move from analog to digital would help

- use the limited radio frequency spectrum more efficiently to ultimately increase the number of subscribers that can be accommodated,
- improve the voice quality in heavy traffic conditions beyond what is possible with AMPS,
- provide a multitude of services besides the basic voice service,
- simplify the task of operating and maintaining cellular systems and,
- provide a smooth transition from AMPS to digital (i.e., provide dual mode services).

In 1991, the first ever second generation, digital, cellular network, the Global System for Mobile or formerly known as Groupe Spécial Mobile (GSM), was deployed in Europe in the 900 MHz band using a channel bandwidth of 200 kHz. GSM uses TDMA for multiple access and GMSK as its modulation scheme. GSM was deployed to serve as the continental European cellular service and following the guidelines of ISDN promised a wide range of services. GSM proved to be highly successful. In 1993, the use of GSM crossed the boundaries of Europe when it was implemented in some South American and Asian countries as well as Australia. It is predicted that there will be around 20-50 Million GSM subscribers globally before the year 2000 [185].

Shortly after the deployment of GSM, the first U.S. Digital Cellular (USDC) system[2], which is now standardized as TIA/EIA-54 (an improved version of TIA/EIA-54 has also been standardized as TIA/EIA-136) was established in major U.S. cities in the 824-894 MHz band[3]. Unlike the GSM, which operates in its own frequency band, the North American TIA/EIA-54 system has to share its frequency band with the AMPS system. TIA/EIA-54/136 uses TDMA as its multiple access scheme and $\pi/4$-DQPSK as its modulation scheme. The system was designed to share the same specifics as AMPS so that the cellular carriers could offer a dual mode of operation. The smooth transition from analog to digital in the same frequency band was an important factor in the development of the TIA/EIA-54/136 standards. When deployed, carriers in large cities where capacity shortages were already present in the analog AMPS services, the switch from AMPS to USDC was instantaneous. Smaller cities, on the other hand, waited until more users were equipped with the digital phones to make the switch. In 1993, Japan deployed its first digital cellular network, the Pacific Digital Cellular (PDC) based on similar specifics as TIA/EIA-54.

In 1993, a second digital cellular system was proposed in the U.S. with the promise of a capacity increase over TIA/EIA-54 [188, 189] by making use of the CDMA multiple access. Unlike the other second generation, digital, cellular systems, the CDMA-based system is a wideband system in which every user has access to the entire frequency allocation of 1.25 MHz in the 824-894 MHz band. This system was later standardized as TIA/EIA-95-A [226]. Like the TIA/EIA-54/136 standards, TIA/EIA-95-A is a dual mode standard where both analog AMPS and digital CDMA operations are supported. In 1994 the first TIA/EIA-95-A dual mode telephones were in stores [183]. The TIA/EIA-95-A standard generated much hype in the field and somewhat slowed down the penetration of TIA/EIA-54/136 across the country with

[2] Also referred to as the Digital AMPS (DAMPS)

[3] The 824-894 MHz band is commonly referred to as the "Cellular Band."

the anticipation of a possible take over of the market by TIA/EIA-95-A.

On January 9, 1996, the Hutchinson Telephone Company Limited launched the world's first commercial CDMA digital cellular phone system in Hong Kong by using Motorola's cellular CDMA cell sites and network equipment and Qualcomm's CDMA mobile phones [184, 262]. Initial consumer surveys indicated preference of CDMA over the GSM mainly because of CDMA's soft handoff capability and longer battery life. The deployed CDMA system transmits an RF power level that is only 1/25th to 1/100th of that of the analog cellular phone. In what is considered to be the world's most difficult radio transmission environments, CDMA users could not feel when their calls were being handed over to another base station. As of March 1996, the Hutchinson CDMA system had 40,000 subscribers [26].

In 1995, the US government auctioned the 1.8-2.0 GHz band for cellular use[4]. In light of the recent field test results and the fast penetration of CDMA into the Hong Kong market, the TIA/EIA-95 based CDMA has emerged as the leading digital wireless technology in the PCS auctions[5]. The major winner in the PCS auctions for the PCS band, Sprint Technologies Venture, has announced that it would only use CDMA. Other important PCS suppliers such as PCS PrimeCo and Ameritech followed trend. From a total consumer market of 506.90 million POPs, CDMA operators captured 256.679 million whereas TIA/EIA-136 operators and PCS-1900 operators have captured 118.313 million and 131.963 million, respectively [27]. Similarly, 11 of the 14 largest cellular operators have chosen CDMA for their upcoming digital cellular operations at 800 MHz [27].

With the explosive growth in the demand for wireless communications, enhancements on the current second generation systems[6] as well as a move to a third genera-

[4]The 1.8-2.0 GHz frequency band is commonly referred to as the "PCS Band."

[5]The TIA/EIA-95-A based CDMA system for the PCS band has been standardized as ANSI J-STD-008 [8].

[6]The enhanced second generation (2G) systems are usually referred to as 2.5G systems.

tion (3G) global wireless communications system in the form of IMT-2000 are being planned. ITU has set forth the requirement that the 3G systems all provide variable bit rate, asymmetric transmission, packet data and multimedia services while offering at least 144 kbps for high mobility users and 2 Mbps for low mobility users with local coverage [86]. The evolution paths for the three most popular second generation systems towards 3G are summarized in Table 1.1. It is clear from the table that the 3G

2G Technology	Evolution	Maximum Data Rate	Expected Deployment
TIA/EIA-95-A	Current	9.6 kbps or 14.4 kbps	Available now
	TIA/EIA-95-B	64 kbps	2000
	3G cdma2000 Phase I (1X)	144 kbps	2002
	3G cdma2000 Phase II (3X)	384 kbps outdoors 2 Mbps indoors	2003
TIA/EIA-136	Current	9.6 kbps	Available now
	EDGE	384 kbps	2003
	3G W-CDMA	384 kbps outdoors 2 Mbps indoors	No deployment plans yet
GSM	Current	9.6 kbps or 14.4 kbps	Available now
	HSCSD	57.6 kbps	2000
	GPRS	115.2 kbps	2001
	EDGE	384 kbps	2002
	3G W-CDMA	384 kbps outdoors 2 Mbps indoors	2003

Table 1.1: Summary of the various evolution paths from 2G to 3G

standards efforts have resulted in various regional wireless system proposals. Different regions have developed system proposals based on the current second generation systems that are popular in their own geographies. It should be noted here that all of the three most popular 2G systems, including those that currently use TDMA, are evolving towards wideband CDMA structures. As of April 2000, however, the wideband CDMA systems of GSM evolution and TIA/EIA-95 evolution do not seem to be compatible with one another. With the regional third generation standards emerging in the Americas, Europe and Asia Pacific, ITU is concentrating on finding technical and perhaps more importantly political means to provide global roaming. Provision of global roaming will undoubtedly rely on how willing the different regions are in modifying parts of their 3G standard proposals towards this goal. Thus, there are no obvious winners on the road to 3G yet. The impact of just the notion of 3G has been overwhelming. It is clear that the telecommunications revolution is far from being over; it is merely starting.

1.2 Multiple Access Schemes

The ultimate goal of a wireless communications service provider is to be able to simultaneously accommodate as many users as possible and still manage to maintain the quality of service for even the most disadvantaged user. This has to be accomplished using the limited allocated spectrum. Multiple access schemes allow many mobile users to share the same bandwidth simultaneously without causing unacceptable interference to one another. Therefore, an effective multiple access scheme is vital in the success of a wireless communication system.

Frequency Division Multiple Access (FDMA), Time Division Multiple Access (TDMA) and Code Division Multiple Access (CDMA) are the three major access techniques used to share the available bandwidth in a wireless communications sys-

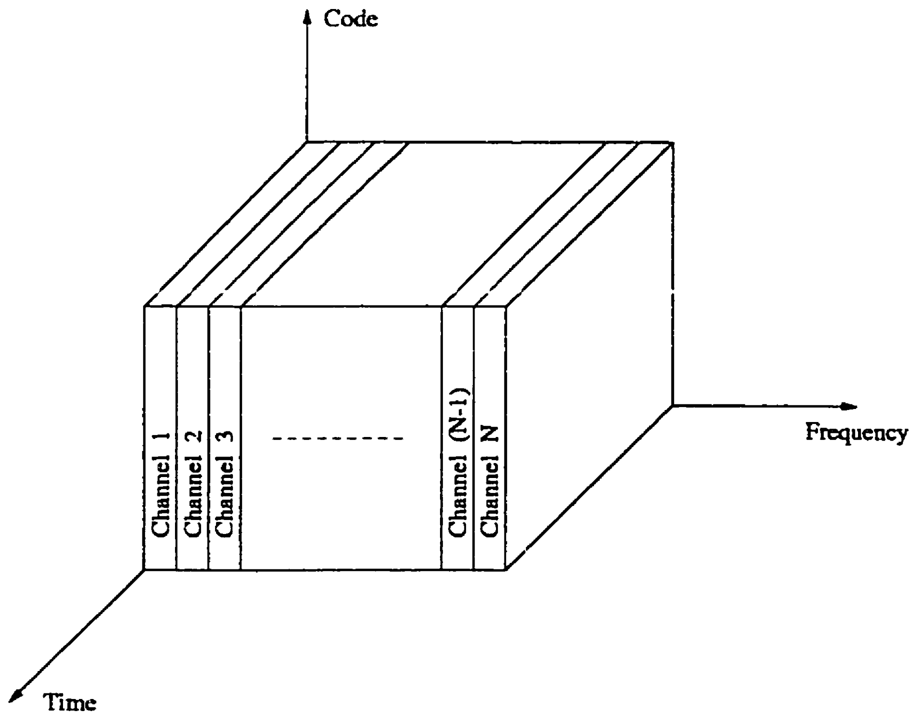

Figure 1.1: Basic FDMA Operation

tem. Other multiple access schemes have also been proposed in the literature (e.g. Carrier Sense Multiple Access (CSMA) [17], Spread ALOHA [3], Polarization Division Multiple Access (PDMA) and Space Division Multiple Access (SDMA) [121]) but have enjoyed limited acceptance.

1.2.1 Frequency Division Multiple Access (FDMA)

FDMA is based on the notion of dividing up the available bandwidth into channels and assigning different channels to different users thus providing frequency orthogonality between users' signals. The basic FDMA operation is illustrated in Figure 1.1. In an FDMA system, only one user has access to a given channel at a given time. Even if the assigned channel is not actively used for a period of time during the course of the connection, no other user can use the channel until the connection is terminated or the call is handed off to another base station. This is one of the main drawbacks of the FDMA scheme as the spectrum cannot be efficiently utilized this way. Once service is granted, subscribers of the FDMA system can transmit and receive at will. Since user orthogonality is provided in the frequency domain, FDMA requires tight RF filtering to minimize the multiple access interference.

It should be recalled that all of the first generation cellular networks such as AMPS use FDMA. Here, a user's signal occupies a single channel (made up of two simplex channels frequency division duplexed with a 45 MHz split - one per direction of transmission) while the call is in progress. When a call is completed, or when a handoff occurs, the channel is vacated so that another mobile subscriber can use it. Multiple or simultaneous users are accommodated in AMPS by giving each user a unique channel. In AMPS, narrowband FM is used to modulate the carrier. These systems use large cells, and therefore relatively high transmission powers making the base station equipments bulky. FDMA is a narrowband system. For this reason, first generation networks are severely effected by the time variation and frequency selectivity of the channel, which is typical for a mobile radio environment. Also, analog FDMA does not provide the desired call security in an affordable way. These features persuaded second generation designers to opt for TDMA and CDMA instead. However, one should note that, in the second generation networks, FDMA is still used in a hybrid form with TDMA (e.g. GSM and TIA/EIA-54/136) or CDMA (e.g. TIA/EIA-95) to achieve frequency efficiency through frequency reuse. It is suggested that a digital FDMA would provide the same security that a TDMA system provides and a microcell based digital FDMA system would not require bulky base stations [207] thanks to the recent developments in solid-state technology. One proposed development for the digital FDMA is to arrange for a user's data to be simultaneously carried on a number of different frequency channels to combat frequency selective

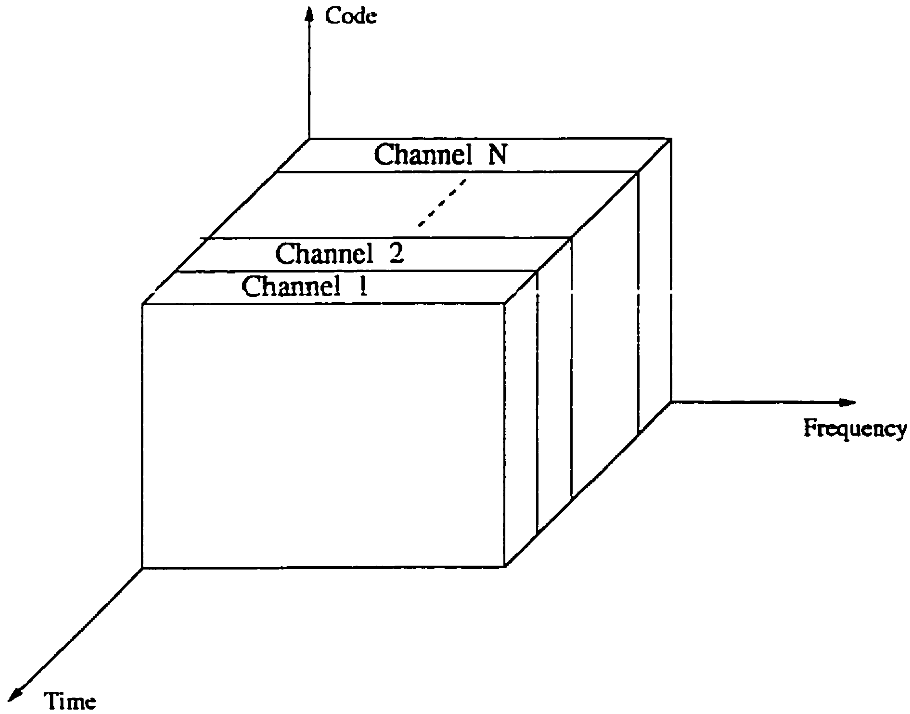

Figure 1.2: Basic TDMA Operation

fading. Another virtue of such a system would be that the digital FDMA would not require the signal processing complexity that TDMA and CDMA do since in FDMA the data rate is essentially the same as the multiplexed rate. Thus, [207] argues that it would not be surprising to see FDMA resurface as a viable multiple access scheme in the future.

1.2.2 Time Division Multiple Access (TDMA)

Unlike FDMA, which is based on the principle of providing frequency orthogonality amongst the users of the network to minimize the multiple access interference, TDMA manages multiple users by means of time orthogonality. In TDMA systems, the available system bandwidth which corresponds to an available time frame, is divided into time slots and in each time slot only one user is allowed to either transmit or receive. The basic TDMA operation is illustrated in Figure 1.2. Once service is granted to a specific user, that user has access to the same, periodically recurring time slot until its connection is terminated or the call is handed off to another base station. Thus, unlike FDMA systems where the communication is continuous once a link is set up, TDMA systems work in a discontinuous, buffer-and-burst method. The individual time slots not only carry the information but also a preamble that contains the destination address and synchronization information. The bursty nature of TDMA results in a more economical battery consumption than FDMA as the transmitter can be turned off while waiting for its assigned time slot. However, this comes at the expense of an increased synchronization overhead. Synchronization is very important in TDMA; signals arriving at the wrong time are pure interference in another user's time slot dramatically reducing the system performance. TDMA systems require the use of equalizers to cope with intersymbol interference due to multipath fading. The handoff process in TDMA systems is easy since the transmissions are discontinuous and therefore, during idle times, the subscriber unit can listen to other base stations.

Most of the second generation cellular communication technologies of today (GSM, TIA/EIA-54/136 and PDC) use TDMA to multiplex their users. GSM for example, utilizes two bands of 25 MHz, one for the forward link and the other for the reverse link. The 25 MHz for each direction of transmission is further divided into 200 kHz channels using FDMA. Each channel is time shared between as many as eight subscribers using TDMA. Radio transmissions are made at a channel rate of 270.833 kbps using binary GMSK modulation. Thus, the signaling bit duration is 3.692 μs, and the effective transmission rate per user is 33.854 kbps. With the TDMA associated overhead, user data is actually sent at a rate of 24.7 kbps [185].

Being a multiplexing scheme actively used for most of the second generation digital networks, TDMA is a serious contender for IMT-2000. TDMA stems from the time sharing methods used by minicomputers and mainframes to accommodate large number of tasks on centralized computers. With this in mind, it is easy to understand why both the telephone and the computer establishments, having a firm grasp of the TDMA technology through years of experience, would feel so at ease seeing TDMA as the sole multiplexer scheme for the third generation wireless networks [65].

The TIA/EIA-95-B standard proposing CDMA usage poses a significant challenge to the current TDMA technology. For this reason, TDMA is currently being improved by its proponents to make it more suitable for the needs of the future third generation systems. However, it has been stated that increasing user data rates, an anticipated trend with the introduction of services other than just voice, might make the TDMA burst rate unacceptably excessive [207].

1.2.3 Code Division Multiple Access (CDMA)

CDMA, born from the spread spectrum technology, is a multiple access scheme in which code orthogonality is used to provide simultaneous access to a number of users. In CDMA, a user's information signal is spread by means of a user specific code so that the transmitted signal occupies a bandwidth in excess of the minimum necessary to send the information [48, 171, 194]. The ratio of the transmitted signal bandwidth to the information signal bandwidth is referred to as the "processing gain" of the CDMA system. In CDMA, every user shares the same bandwidth and users are allowed to transmit or receive at will. The basic CDMA operation is illustrated in Figure 1.3.

The purpose of spreading the bandwidth is two-fold. First, by spreading the energy over a large bandwidth, the effective Signal-to-Noise Ratio (SNR) becomes extremely low. For example, if a signal originally had an SNR of 15 dB, spreading this signal with a processing gain of 64 would result in an SNR of $15 - 10\log_{10} 64 \simeq -3$dB due to the increased noise bandwidth. Due to this property, it is rather difficult to detect

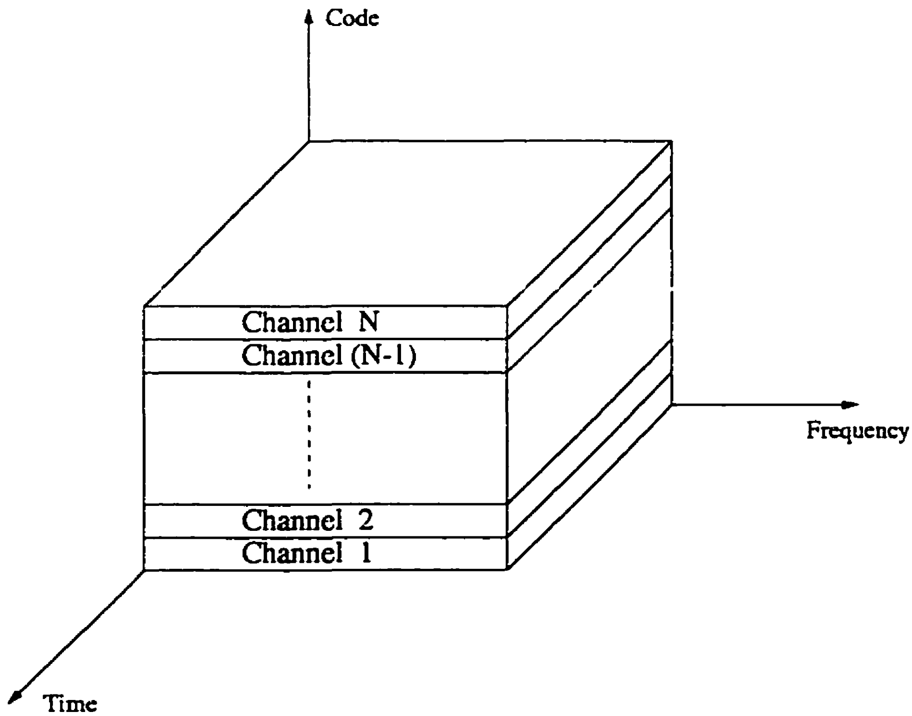

Figure 1.3: Basic CDMA Operation

the presence of a spread spectrum system.

The second benefit to spreading the signal in this fashion is that the signal becomes resistant to interference and intentional jamming. The common correlator receiver, upon synchronizing itself to the desired user's signal, uses that user's spreading code to coherently despread and reconstitute the original information signal. All other signals present in the system remain spread out, indistinguishable from the background noise. In other words, the receiver takes the SNR back to its original value while maintaining a high Signal-to-Interference Ratio (SIR). This reduces the average interference power relative to the average signal power in the bandwidth of interest so that the interferers do not noticeably degrade the system performance. In cases where it is not possible to obtain information on the received signal's phase, non-coherent receivers may also be used in CDMA systems [18, 93, 104]. As with other digital communications systems, use of non-coherent receivers will degrade the overall system performance by approximately 3 dB [176].

While CDMA is based on spread spectrum, there are considerable differences between the military use of spread spectrum and commercial CDMA. Spread spectrum was originally designed for a few users trying to avoid interception and jamming. Bandwidth efficiency was of secondary importance. CDMA, on the other hand, is the application of spread spectrum to the multiple access challenge. Here, the aim is to support as many users as possible, so bandwidth efficiency is of primary concern. The anti-jamming and low probability of intercept properties of spread spectrum can be used in several ways. First, by providing all users with ideally orthogonal spreading sequences, it is possible to support the same number of users as a TDMA or FDMA system in a given bandwidth assuming synchronous transmission. Second, by spreading the bandwidth of the signal beyond the coherence bandwidth of the mobile channel, we introduce frequency selective fading. This effect can be utilized as a form of diversity by employing a RAKE receiver [175]. Third, if different spreading codes are overlaid on top of the waveforms of each cell, intra-cell spreading codes as well as frequency bands can be reused between cells. All of these properties potentially enable CDMA to provide significant capacity advantages.

Of course, the pro-CDMA argument is not so simple. The big caveat is that unlike TDMA or FDMA, orthogonality is maintained only when ideal codes are used in a perfectly synchronous channel and the mobile channel is non-dispersive. Unfortunately, synchronism can not always be maintained perfectly in practice. It is also quite likely to encounter frequency selective fading channels in mobile environments. The imperfection affects the system performance in two ways. First, by introducing cross-correlation between spreading sequences, intra-cell interference is introduced which limits the system capacity. Second, the small cross-correlation introduces the need for strict power control. Even if there is a small amount of correlation between two spreading codes, if the received power of one user is significantly larger than the other user, the energy of the first will negatively impact the detection of the second. This problem of CDMA systems is commonly referred to as the "near-far problem" in the literature.

These problems are not inherent to the generic CDMA scheme but rather result from the fact that in practice, correlator receivers (or matched filters) are being used in CDMA systems to determine the most likely symbol. In an Additive White Gaussian Noise (AWGN) channel, this receiver structure is known to be optimal [176]. However, in a multiple access environment this is not the case. Since multiple access interference can sometimes be adequately modeled as a Gaussian process [178], one might assume that the correlator receiver is essentially optimal for multiple access interference. However, this is not true [239]. As can be observed in this thesis, there are several cases where the Gaussian Approximation for multiple access interference is grossly inaccurate. Additionally, the structured nature of multiple access interference allows for cancellation or rejection while AWGN has no inherent structure to exploit.

These facts lead Verdu in 1986 to improve the correlator receiver using non-Gaussian detection schemes [238]. This work showed that the optimal receiver for multiple access channels which exploits the structural properties of such interference provided significant gains over the conventional correlator receiver. Essentially, Verdu recognized the similarity between the intersymbol interference and multiple access interference. With this recognition it was shown that the optimal receiver in the presence of intersymbol interference, such as the Viterbi algorithm, could be adapted to provide optimal reception for the multiple access interference channel. Like many optimal solutions, the optimal receiver has a complexity that precludes its use in practical systems. For this reason, the conventional correlator receiver is still in active use today in commercial CDMA systems around the world. Therefore, the need for strict power control and the limit in capacity due to multiple access interference remain the two major concerns in today's CDMA systems.

Needless to state then is the following: key to the performance of CDMA is the selection of the user specific codes. To achieve perfect orthogonality between signals, the cross-correlation of every code pair should be equal to zero. However, this is not always possible. Sub-optimal codes are usually used in practice. Different codes are used for different systems. For example, if a system is synchronous, deterministic short codes that are orthogonal at zero time delay are the best choice. Walsh-Hadamard codes are examples of such codes [47]. In this case, whenever synchronism of the system is maintained so can the orthogonality of the codes (provided the channel is frequency non-selective) and thus the multiple access interference can be perfectly cancelled. If however, the system is asynchronous or if the synchronizers are not able to maintain the synchronism, deterministic codes are no longer desirable. This is because the cross-correlation values of such codes, though exactly equal to zero at zero time delay, can be quite large for certain non-zero values of time delay. For asynchronous systems, it is desired to construct a code group that has small cross-correlation values even when the time delay is non-zero. Pseudonoise (PN) sequences constructed using minimum length shift registers are excellent codes for asynchronous CDMA systems [171, 250].

The intra-cell interference due to the non-zero cross-correlations between the users' spreading sequences is naturally mitigated by the fact that the voice activity during a given call is usually around 37.5% [250] on a half-duplex link. Thus, the number of active users at a given moment in time is approximately 3/8'th of the number of actual number of users in the system. While TDMA and FDMA need sophisticated algorithms to take advantage of this inactivity, CDMA naturally takes advantage of it. By accepting some amount of interference, CDMA becomes a dynamic channel sharing scheme. This flexibility is a strong advantage for CDMA.

Perhaps the most unique characteristic of the CDMA scheme is its ability to provide a "soft capacity." With FDMA or TDMA where there is a fixed number of channels or slots for the users to share, the maximum number of users that can be serviced simultaneously is a fixed number and cannot be changed. Thus, these systems have a "hard capacity" ruled by the allocated bandwidth and signaling rate. With CDMA on the other hand, the number of calls that can be handled by a certain carrier is not a fixed number but rather depends on the interference characteristics of the environment. In fact, three primary system characteristics, coverage, quality and capacity can be balanced off of each other to arrive at the desired level of system performance. In other words, even higher capacity may be achieved through some degree of degradation in coverage and/or quality.

Irwin Jacobs, the CEO of Qualcomm, compares CDMA and TDMA to different strategies of communication at a cocktail party [65]. In the TDMA analogy, each person would restrict his or her talk to a specific time slot while everyone else remains silent. This system would work well as long as the party was managed by a dictator who controlled all conversations by complex rules and a rigid clock. In CDMA, on the other hand, everyone can talk at once but in different languages, technically referred to as codes. Each person listens for messages in his or her own language and ignores all other sounds as background noise. Although this system allows each person to speak freely, it requires constant control of the volume of the speakers. A speaker who begins yelling can drown out surrounding speakers and drastically reduce the number of conversations that can be sustained simultaneously. As mentioned before, this impairment of the CDMA technology is referred to as the near-far problem.

The TIA/EIA-95 standard uses two levels of controls to overcome the near-far problem. First is a mechanism that employs automatic gain control device on handsets to initially adjust the power sent by the handset to the level of power received by it from the base station. This is a rather rough adjustment and it does not come near to solving the problem, but it brings the solution within reach by using more complex and refined techniques. In the second power control step, the base station measures the handset's signal to noise ratio approximately 800 times every second. Depending on whether this ratio is above or below a constantly recomputed threshold, the base station sends a positive or negative pulse which corresponds to either raising or lowering the power of the handset by 1 dB. This solution, though accurate theoretically, was received with much skepticism amongst the TDMA supporters. Some claimed that the required power control for CDMA systems would end up being very costly and hence impractical. In fact as late as April 1995, some leading experts in telecommunications research claimed that CDMA would never be a viable technology for high capacity cellular applications [38].

Since then many field tests were conducted in different parts of the world which all proved otherwise [10, 39, 115, 116, 132, 141]. In fact, the two-tier power control algorithm of TIA/EIA-95 not only worked very well, but also helped the CDMA system provide a 3-5 fold increase in the system capacity in comparison to the current TDMA systems [26, 184, 262]. The soft capacity characteristic of CDMA also allows the service providers to shift this advantage towards providing a higher quality service and/or a wider coverage area by lowering the system capacity to the level of the second generation TDMA networks.

There are two possible ways of achieving CDMA: Direct Sequence CDMA (DS CDMA) and Frequency Hopped CDMA (FH CDMA). The TIA/EIA-95 standard is based on DS CDMA multiple access whereas much of the FH CDMA usage still lies mainly in military communications. Therefore, DS CDMA is enjoying significantly more attention than FH CDMA. In line with the current interest in DS CDMA, this thesis studies analysis techniques for the accurate calculation of the performance of DS CDMA systems that use coherent, correlator receivers. Both DS CDMA and FH CDMA are briefly described below. Since this thesis is on the analysis of coherent DS

CDMA systems, a more detailed, mathematical description of the DS CDMA scheme follows this chapter.

1.2.3.1 Direct Sequence CDMA (DS CDMA)

In DS CDMA, the spectrum spreading is accomplished by directly multiplying the narrowband message signal by a large bandwidth, user specific pseudonoise (PN) signal called the spreading sequence. In other words, the spreading sequence is made up of a series of symbols called chips whose duration is much smaller than the information bit duration. If the DS CDMA system processing gain is G, there exist G chips per information symbol. Multiplication of two unrelated signals produces a signal whose spectrum is the convolution of the spectra of the two component signals. In DS CDMA, since the spreading sequence has a much wider bandwidth than the message signal, the product signal has approximately the same bandwidth as the spreading sequence.

At the demodulator, the received signal is multiplied by exactly the same spreading sequence. If the locally generated spreading sequence is synchronized with the received spread signal, the result is the original message signal plus, possibly some higher frequency terms outside the bandwidth of interest. Hence, it is possible to easily filter out these high frequency terms to reproduce the original message signal essentially undistorted. Since the other users in the system have all different spreading sequences, multiplication with the locally generated spreading sequence of the desired user will ensure that the multiple access interference remains spread over the allocated system bandwidth causing very little disturbance in the bandwidth of interest.

1.2.3.2 Frequency Hopped CDMA (FH CDMA)

FH CDMA utilizes the large system bandwidth by periodically changing the carrier frequency of the narrowband message signal according to a user specific pseudonoise signal. The change in the frequencies is referred to as hopping. If the entire allocated bandwidth is, say, G times greater than the bandwidth of the message signal, the signal can be transmitted at any one of the G possible frequencies during a given time interval. Therefore, the instantaneous bandwidth of any one of the transmitted signals is equal to that of the corresponding narrowband message signals. The pseudorandom change of the carrier frequencies of a specific user randomizes the occupancy of the channels, thereby allowing multiple access.

In the FH CDMA system, if the rate of change of the carrier frequency is greater than the message signal rate, the system is referred to as a fast frequency hopped system. On the other hand, if the rate of change of the carrier frequency is less than or equal to the message signal rate, the system is referred to as a slow frequency hopped system.

At the FH CDMA receiver, the received signal is stripped off its time varying carrier frequency using a locally generated synchronized replica of the pseudonoise sequence. Similar to the DS CDMA case, this operation keeps the signals of the other users spread out making the FH CDMA system interference resistant.

1.3 Motivation

This thesis is concerned with providing a rigorous, accurate analysis scheme to calculate the performance of coherent DS CDMA systems in additive white Gaussian noise (AWGN) and multipath fading channels with diversity combining. Unlike the popular approximations used for performance analysis, the scheme developed here is valid for systems at a wide variety of operating points.

At a time where there is an ever-growing move towards wireless communications, it is important to provide a rigorous, accurate analysis scheme for DS CDMA systems which seem to be one of the main contenders in the on-going battle for wireless. It should be noted here that our aim is not to produce yet another comparison between CDMA and the other competing multiple access schemes. Rather, we aim to provide a thorough analysis for DS CDMA systems to provide a better understanding of the underlying factors that make CDMA so attractive. Furthermore, this study does not necessarily analyze the TIA/EIA-95 or any other CDMA based commercial system; we only aim to provide insight on the DS CDMA operation by analyzing basic DS CDMA systems.

The main objective of this thesis is to develop an accurate analysis scheme to calculate the performance of DS CDMA systems. The performance of such systems is usually given in terms of the bit error rate as a function of the number of active users. Throughout the thesis, we define the system capacity as the maximum number of users that can be accommodated simultaneously in such a way that every user is guaranteed an average bit error rate below or equal to a certain threshold. The dynamics, or time variation, of the error rate due to fading, which is related to the outage probability is not considered in this thesis. Developing an analysis scheme for DS CDMA is important for two reasons. First, it provides a better understanding of such systems and second, such an analysis scheme could be used to find just how accurate the popular approximations that are currently in use to estimate the system performance are and whether they have any limitations.

Once the analysis scheme is developed, we also investigate the degradation that is experienced in the performance of such systems as a result of all possible synchronization errors. The performance analysis is conducted for systems in AWGN and multipath fading channels. For multipath fading channels, CDMA has the advantage of being able to make use of the path diversity. Various methods of diversity combining may be used at the CDMA receiver. In this thesis, we analyze three such receivers.

The third generation wireless systems will not only provide voice communications but other services as well. Even though the proposed deployment of a multimedia wireless system is fast approaching, to the author's knowledge, there is not a significant number of published works on developing a mathematical model for a DS CDMA based multi-service system. In this thesis, we present such a mathematical model for a system that offers a number of services to its users. The analysis scheme developed for the simple DS CDMA systems is extended to find performance estimates for the multi-service system as well.

1.4 *Contributions of The Work*

The following list is a synopsis of the significant contributions presented in this thesis.

1. A Fourier series based analysis scheme is developed to find the capacity of coherent DS CDMA systems using [14, 15].

2. Performance of biphase and quadriphase spread, coherent DS CDMA systems in AWGN and multipath fading channels are calculated using the developed scheme.

3. Various diversity combining schemes are considered for the DS CDMA receiver in multipath fading channels. System capacities are calculated when selection diversity, equal gain combining and maximal ratio combining are employed at the receiver.

4. Various DS CDMA systems are considered. Capacities of synchronous, asynchronous and quasi-synchronous DS CDMA systems are calculated using the

developed scheme.

5. Effects of non-zero synchronization errors on the performance of coherent DS CDMA systems are found. Both constant and uniformly distributed chip timing errors and carrier phase errors are considered.

6. A number of the popular analysis schemes for DS CDMA systems are studied and their limitations are discussed. Scarcely populated DS CDMA systems, DS CDMA systems with unequal powered users are analyzed using the popular analysis schemes and the developed scheme for this purpose.

7. A mathematical model for multi-service DS CDMA systems is developed. Both the developed scheme and the popular analysis schemes from the literature have been used to analyze this system.

1.5 Presentation Outline

This thesis consists of seven chapters. A summary of the individual chapters is as follows.

In Chapter Two, system models for biphase and quadriphase spread DS CDMA systems are developed. Binary signaling and coherent reception is assumed. Synchronous, quasi-synchronous and asynchronous systems are considered. System models for both simple additive white Gaussian noise (AWGN) and multipath fading channels are generated. A brief overview of fading, multipath and diversity combining concepts as well as overviews of the TIA/EIA-95 standard and the third generation cdma2000 system proposal are also given in this chapter.

In Chapter Three, a literature survey on the analysis tools for DS CDMA systems is given. Methods that use approximations as well as bounds are presented. Popular approximations based on the Central Limit Theorem are given special attention.

Detailed derivations are given for these schemes and error probability expressions are generated for synchronous, asynchronous and quasi-synchronous systems.

In Chapter Four, a probability of error expression based on a Fourier series representation of the error function, $Q(x)$, developed by Beaulieu in [14] is derived for DS CDMA system in AWGN channels. The expression is then used to calculate system capacites. Impact of various chip timing and carrier phase errors on the system performance is studied.

In Chapter Five, the derivation of the probability of error expression in Chapter Four is expanded for DS CDMA systems in multipath fading environments. Three kinds of diversity combining schemes are considered at the receiver, namely, selection diversity, equal gain combining and maximal ratio combining. Relative performances of these schemes are compared and effects of non-zero synchronization errors on the system performance is evaluated.

In Chapter Six, the derived error probability expression is used to assess how accurate the popular Central Limit Theorem based approximations are in predicting the system performance when the scenario under investigation violates the requirements of the Central Limit Theorem. In this light, performances of scarcely populated systems, systems with dominant interferers, multi-service systems and systems in frequency non-selective Rayleigh fading channels are studied. A system model for the multi-service system is developed here based on the DS CDMA system model developed in Chapter Two.

Finally, in Chapter Seven, a summary is given of the material addressed in the thesis with concluding remarks. Also suggestions for further research topics are outlined.

Chapter 2

DS CDMA SYSTEM MODELS

Most scientific work begins with the derivation of a model of the system being considered. In this chapter, building on [124, 138, 156], we develop mathematical models of coherent DS CDMA systems in AWGN and multipath fading channels and present an overview of the fading and diversity fundamentals. Two spreading schemes are considered in this thesis, namely, biphase spreading and quadriphase spreading. System models for both schemes are developed in this chapter.

Without a doubt, the most important commercial DS CDMA system is the TIA/EIA-95 standard [226, 227] which is aimed to provide digital and personal communications services. This system is up and running in Hong Kong, Korea and in most parts of North America. For this reason, we present a brief summary of the TIA/EIA-95 standard physical layer air interface in this chapter as well. The third generation evolution of TIA/EIA-95 will provide means for higher and variable data rates. This system proposal is referred to as cdma 2000. A brief summary of cdma 2000 is also given here.

2.1 DS CDMA System in AWGN Channels

In DS CDMA systems, bandwidth spreading is accomplished by direct modulation of a data modulated carrier using a wideband spreading sequence. Here, the signals all occupy the full allocated bandwidth at all times. Interferers are therefore assumed to come from all directions. The correlation properties of the spreading codes of different users provide co-channel interference immunity in an ideal channel.

As already stated in Chapter 1, in DS CDMA, different types of spreading sequences are used for different systems. If the system of interest is synchronous, codes that provide perfect orthogonality, i.e. zero cross-correlation, at zero time delay are desired. Asynchronous systems on the other hand, aim to keep the cross-correlation amongst different users' spreading sequences at a minimum for all time delays. Pseudonoise (PN) sequences are suitable codes for asynchronous systems. PN sequences are generated with the basic properties of randomness, long period, ease of generation and difficulty of reconstruction from a subsegment [94]. Although there are various good codes proposed in the CDMA literature for both synchronous and asynchronous systems, the design issues to achieve more desirable sequences are still active fields of research [40, 112, 136, 143, 179, 187] and are beyond the scope of this thesis. The system that is considered here is one that employs PN spreading sequences with periods that exceed the data symbol duration. Such sequences are called long PN sequences [138, 235]. It is assumed throughout this thesis that the long PN sequences are made up of terms that are statistically independent of one another. The DS CDMA system under consideration uses binary phase shift keying (BPSK) to modulate the information signal. Both biphase spreading and quadriphase spreading schemes are considered in this thesis and the system models for each case are described separately below.

2.1.1 Biphase Spread DS CDMA System

The biphase spread system under consideration is shown in Figure 2.1. Referring to this figure, let us assume that there are K users transmitting signals in the DS CDMA system. Each transmitter transmits a signal in the form,

$$s_k(t) = \sqrt{2P_k}a_k(t - \mathcal{T}_k)b_k(t - \mathcal{T}_k)\cos(w_c t + \theta_k) \tag{2.1}$$

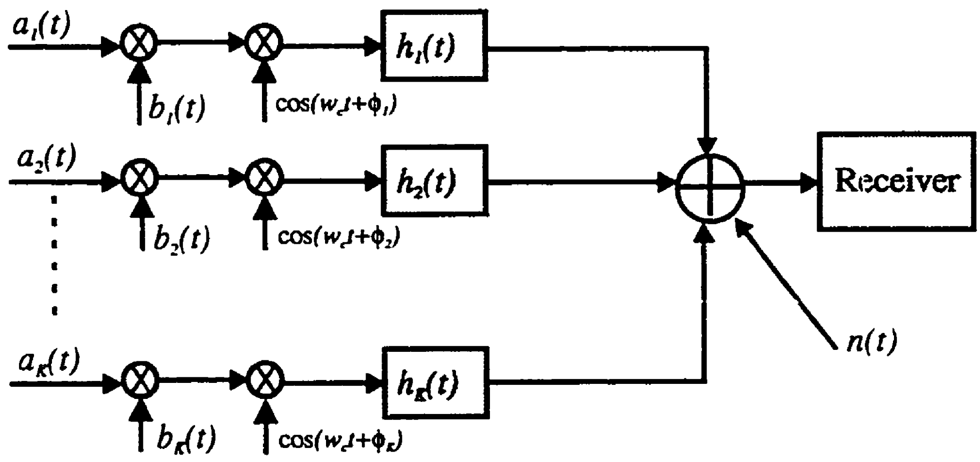

Figure 2.1: Biphase Spread DS CDMA System Model

where w_c is the common signaling frequency, $\mathcal{T}_k$ is the initial message starting time, θ_k is the initial phase offset and $a_k(t)$ and $b_k(t)$ are the data and spreading signals, respectively. The power of the transmitted signal $s_k(t)$ can be calculated as,

$$\begin{aligned} \wp_k &= \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} |s_k(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} 2P_k \cos^2(w_c t) dt \\ &= \lim_{T\to\infty} \frac{1}{2T} P_k \left[2T + \frac{1}{w_c}\sin(2w_c t)\right] = P_k. \end{aligned}$$

The random variables $\mathcal{T}_k$ and θ_k provide flexibility to the system modeling. If all of the active users' signals are required to be transmitted at the same time, the system is said to be a synchronous system. For a synchronous system, $\mathcal{T}_1 = \mathcal{T}_2 = \cdots = \mathcal{T}_K$. If the K signals are all transmitted by the same transmitter, clearly, achieving synchronous transmission is easily feasible. This scenario is synonymous with the downlink air interface of a commercial wireless communication system where a given base station transmits a number of signals, each intended for a different user, simultaneously. If the K signals originate from different transmitters, achieving synchronism is much more difficult. Here, a common master clock reference needs to be broadcast to all of the transmitters. The individual transmitters then need to adjust their timing and transmit according to this master clock. Note however that even when synchronous transmission is achieved for this case, reception of the K signals synchronously by a single receiver will not be possible since the individual signals will go through different channels imposing different time and phase offsets to the signals. On the other hand, if the users are allowed to transmit signals at arbitrary time references, the system is said to be asynchronous. Such systems are modeled with an independent, identically distributed $\mathcal{T}_k$ for all k.

The data signal $a_k(t)$ in (2.1) is a sequence of unit amplitude rectangular pulses of duration T. Similarly, $b_k(t)$ is a sequence of unit amplitude chips of duration T_c where $G = T/T_c$ is defined as the processing gain of the spread spectrum system. Mathematically, $a_k(t)$ can be expressed as,

$$a_k(t) = \sum_{j=-\infty}^{\infty} a_{k,j} \cdot p_T(t - jT). \tag{2.2}$$

Here $a_{k,j}$ is one symbol of the data modulation and is assumed to take on the values $\{\mp 1\}$ equally likely. $p_T(t)$ is the unit amplitude rectangular pulse of duration T.

Similarly, the user specific long PN sequence $b_k(t)$ in (2.1) has the form,

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_{k,i} \cdot \Psi(t - iT_c) \tag{2.3}$$

where $b_{k,i}$ is one chip of the long PN sequence and is assumed to take on values $\{\mp 1\}$ with equal probability. The sequence $\{b_{k,i}\}$ is modeled as a random binary sequence whose terms are statistically independent of one another. The unit amplitude chip waveform $\Psi(t)$ has duration $T_c = T/G$. Various chip waveforms have been considered in the literature [9, 114]. Throughout this thesis, we consider only DS CDMA systems with rectangular chip pulses of unit amplitude.

As can be seen from Figure 2.1, the k'th user's transmitted signal, $s_k(t)$, is transmitted through a channel, $h_k(t)$, whose low-pass equivalent transfer function is in the form,

$$h_k(t) = e^{j\phi_k} \cdot \delta(t - \tau_k). \tag{2.4}$$

Here, ϕ_k and τ_k are the phase and time delay terms introduced by the channel and $\delta(t)$ is the unit impulse function. In a multi-user DS CDMA system with K active users, the signals from all of the users arrive at the input of the receiver. Thus, the received signal both contains the desired user's signal and $K-1$ other users' signals as well as channel noise. It should be noted here that one of the basic assumptions throughout this thesis is that the noise that the channel in consideration introduces can be modeled as additive white Gaussian noise (AWGN). We consider both simple noise channels and multipath fading channels in this thesis and this assumption holds for both cases. Channels that introduce non-Gaussian noise have found limited interest in the literature. The interested reader is referred to [1, 2].

In (2.1), following [124, 156, 178], if the differences in message start times, $\mathcal{T}_k$, and initial phase offsets, θ_k, are incorporated into τ_k and ϕ_k, respectively, the total received signal can be written as,

$$r(t) = \sum_{k=1}^{K} \sqrt{2P_k} a_k(t-\tau_k) b_k(t-\tau_k) \cos(\omega_c t + \phi_k) + n(t) \tag{2.5}$$

where $n(t)$ is the additive white Gaussian noise (AWGN) introduced by the channel and has zero mean and variance of $N_0/2$. In (2.5), assuming that the transmitters are physically separated, both ϕ_k and τ_k can be assumed to be random variables uniformly distributed over $[0, 2\pi]$ and $[0, T]$, respectively.

Without any loss of generality, we assume that the signal from the first user, $a_1(t)$ is to be captured. A correlation receiver such as the one shown in Figure 2.2 is typically used to coherently filter the desired user's signal from all other users' signals which share the same channel. In such receivers, the received signal, $r(t)$, is multiplied by the spreading sequence of the desired user, mixed down to baseband, and integrated over one bit period. Figure 2.2 displays this sequence of operations in the reverse order. From a theoretical standpoint, these two modes of operation are identical, but in practice the despreading operation is always performed first. A

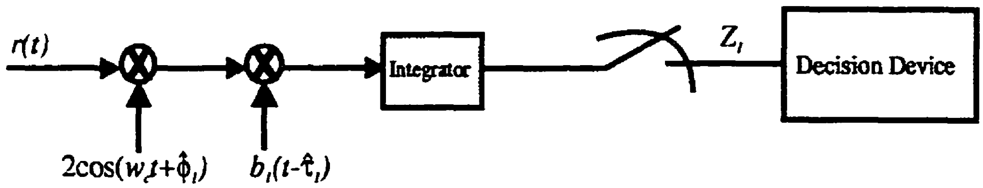

Figure 2.2: A Typical Coherent DS CDMA Receiver for the Biphase Spread System

system with synchronization errors will not be able to estimate the time delay and the phase delay corresponding to the desired user's signal correctly. If the receiver in Figure 2.2 estimates the time and phase delays of the first user's signal to be $\hat{\tau}_1$ and $\hat{\phi}_1$, the corresponding chip timing and carrier phase errors will be $(\tau_1 - \hat{\tau}_1)$ and $(\phi_1 - \hat{\phi}_1)$, respectively.

We also assume that the correlator receiver of Figure 2.2 is used to recover the data symbol $a_{1,0}$. For this reason, we consider the data symbol interval of $[0, T]$ in the analysis. In this case, from Figure 2.2, the input to the decision device for a receiver tuned in to recover the message of the first user is,

$$\begin{aligned} Z_1 &= 2\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} b_1(t-\hat{\tau}_1)r(t)cos(\omega_c t + \hat{\phi}_1)dt \\ &= D + M + N \end{aligned} \tag{2.6}$$

where

$$D = \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_1(t-\tau_1)b_1(t-\tau_1)b_1(t-\hat{\tau}_1)dt, \tag{2.7}$$

$$M = \sum_{k=2}^{K}\sqrt{2P_k}\cos(\phi_k - \hat{\phi}_1)\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_k(t-\tau_k)b_k(t-\tau_k)b_1(t-\hat{\tau}_1)dt, \tag{2.8}$$

$$N = 2\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} n(t)b_1(t-\hat{\tau}_1)\cos(\omega_c t + \hat{\phi}_1)dt. \tag{2.9}$$

Here, D is the term that consists of the desired signal plus self interference which is caused by the chip timing error, M is the multiple access interference and N is the channel noise. The factor $b_1(t)$ in (2.8) ensures that the multiple access interference energy remains spread out over the wide bandwidth. We assume that $\omega_c \gg 1/T$, so that the double frequency term is negligible in Z_1. In a practical receiver, the double frequency term may be suppressed by using a bandpass filter at an intermediate frequency. Since they arise from different physical sources, we also assume that $M, N, b_1(t), \tau_k$ and ϕ_k are all statistically independent of each other. Following these assumptions, we now simplify the terms, D, M and N.

2.1.1.1 Desired Signal Plus Self Interference

In (2.6), D is the desired signal plus self interference and is mathematically given in (2.7). Using (2.2) and the fact that $a_1(t)$ is constant over $[0, T]$, (2.7) can be rewritten as,

$$\begin{aligned} D &= \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,-1}\int_0^{\tau_1-\hat{\tau}_1} b_1(t)b_1(t-(\tau_1-\hat{\tau}_1))dt \\ &\quad + \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,0}\int_{\tau_1-\hat{\tau}_1}^{T} b_1(t)b_1(t-(\tau_1-\hat{\tau}_1))dt. \end{aligned} \tag{2.10}$$

Now, we can make use of (2.3) to get,

$$\begin{aligned} D &= \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,-1} \\ &\quad \cdot \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} b_{1,i}b_{1,j}\int_0^{\tau_1-\hat{\tau}_1} \Psi(t-iT_c)\Psi(t-jT_c-(\tau_1-\hat{\tau}_1))dt \\ &\quad + \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,0} \\ &\quad \cdot \sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty} b_{1,i}b_{1,j}\int_{\tau_1-\hat{\tau}_1}^{T} \Psi(t-iT_c)\Psi(t-jT_c-(\tau_1-\hat{\tau}_1))dt. \end{aligned} \tag{2.11}$$

We now define the following autocorrelation functions for the chip pulse, $\Psi(t)$,

$$R_\Psi(s) \triangleq \int_0^s \Psi(t)\Psi(t+T_c-s)dt, \tag{2.12}$$

$$\hat{R}_\Psi(s) \triangleq \int_s^{T_c} \Psi(t)\Psi(t-s)dt. \tag{2.13}$$

Since the synchronization error $(\tau_1 - \hat{\tau}_1)$ has to be smaller than the chip duration for the system to operate, the first term in (2.11) has a non-zero value only when

$i = 0$ and $j = -1$. Similarly, the the second term in (2.11) has non-zero values only when $i = 0, \ldots, G-1$ and $j = i$ or $j = i - 1$. Therefore, using (2.12) and (2.13), D can be rewritten as,

$$\begin{aligned} D &= \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,-1}b_{1,0}b_{1,-1}R_\Psi(\tau_1 - \hat{\tau}_1) \\ &+\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,0}\sum_{i=0}^{G-1}\hat{R}_\Psi(\tau_1 - \hat{\tau}_1) \\ &+\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,0}\sum_{i=1}^{G-1}b_{1,i}b_{1,(i-1)}R_\Psi(\tau_1 - \hat{\tau}_1). \end{aligned} \tag{2.14}$$

As stated before, in this thesis we consider only unit amplitude, rectangular shaped chips for the spreading sequences. In this case, the autocorrelation functions of the chip pulse shape in (2.12) and (2.13) become,

$$R_\Psi(s) = s, \tag{2.15}$$

$$\hat{R}_\Psi(s) = T_c - s. \tag{2.16}$$

where $R_\Psi(s) = \hat{R}_\Psi(s) = 0, |s| \geq T_c$ with $R_\Psi(s) = R_\Psi(-s)$ and $\hat{R}_\Psi(s) = \hat{R}_\Psi(-s)$. Substituting (2.15) and (2.16) into (2.14) yields,

$$\begin{aligned} D &= \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)\left[a_{1,-1}b_{1,0}b_{1,-1}\right](\tau_1 - \hat{\tau}_1) \\ &+\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,0}\sum_{i=0}^{G-1}\left[(T_c - (\tau_1 - \hat{\tau}_1)) + b_{1,i}b_{1,(i-1)}(\tau_1 - \hat{\tau}_1)\right] \\ &-\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)a_{1,0}b_{1,0}b_{1,-1}(\tau_1 - \hat{\tau}_1). \end{aligned} \tag{2.17}$$

Before proceeding, we present a lemma from [235] that will be useful in the analysis.

Lemma: *Suppose that $\{\alpha_i\}$ and $\{\beta_i\}$ are statistically independent random binary sequences. Let x and y denote arbitrary constants. Then, $\alpha_i\beta_j x$ and $\alpha_i\beta_k y$ are statistically independent random variables when $j \neq k$.*

Proof: Let $P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b)$ denote the joint probability that $\alpha_i\beta_j y = a$ and $\alpha_i\beta_k y = b$ where $|a| = |x|$ and $|b| = |y|$ since α_i and β_i, $\forall i$ are binary random variables. From the total probability theorem [166], it follows that,

$$\begin{aligned}
&P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b) \\
&\quad = P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b, \alpha_i = 1) + P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b, \alpha_i = -1)
\end{aligned}$$

From the independence of $\{\alpha_i\}$ and $\{\beta_i\}$ and the fact that they are random binary sequences, we obtain further simplification for $j \neq k$,

$$\begin{aligned}
&P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b) \\
&\quad = P(\alpha_i = 1)P(\beta_j x = a)P(\beta_k y = b) + P(\alpha_i = -1)P(\beta_j x = -a)P(\beta_k y = -b) \\
&\quad = \frac{1}{2}P\left(\beta_j = \frac{a}{x}\right)P\left(\beta_k = \frac{b}{y}\right) + \frac{1}{2}P\left(\beta_j = -\frac{a}{x}\right)P\left(\beta_k = -\frac{b}{y}\right)
\end{aligned}$$

Since β_j equals +1 or -1 with equal probability, $P(\beta_j = a/x) = P(\beta_j = -a/x)$ and $P(\beta_k = b/y) = P(\beta_k = -b/y)$. In other words, $P(\beta_j x = a) = P(\beta_j x = -a)$ and $P(\beta_k y = b) = P(\beta_k y = -b)$. Therefore,

$$P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b) = P(\beta_j x = a)P(\beta_k y = b)$$

Similarly,

$$\begin{aligned}
&P(\alpha_i\beta_j x = a)P(\alpha_i\beta_k y = b) \\
&\quad = \left(\frac{1}{2}P(\beta_j x = a) + \frac{1}{2}P(\beta_j x = -a)\right)\cdot\left(\frac{1}{2}P(\beta_k y = b) + \frac{1}{2}P(\beta_k y = -b)\right) \\
&\quad = P(\beta_j x = a)P(\beta_k y = b)
\end{aligned}$$

resulting in,

$$P(\alpha_i\beta_j x = a, \alpha_i\beta_k y = b) = P(\alpha_i\beta_j x = a)P(\alpha_i\beta_k y = b)$$

which satisfies the definition of statistical independence of $\alpha_i\beta_j x$ and $\alpha_i\beta_k y$.

We now define the following random variables,

$$\alpha_0 \triangleq a_{1,-1}b_{1,0}b_{1,-1}, \tag{2.18}$$

$$\alpha_i \triangleq a_{1,0}b_{1,i}b_{1,(i-1)}, \qquad i = 1, 2, \ldots, G-1. \tag{2.19}$$

Then, using the above lemma, α_i, $i = 0, 1, \ldots, (G-1)$ are iid random variables taking on values $\{\mp 1\}$ with equal probability. Thus, using (2.18) and (2.19), (2.17) can be rewritten as,

$$D = D_1 + D_2 \tag{2.20}$$

where

$$D_1 = \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)G(T_c - (\tau_1 - \hat{\tau}_1))a_{1,0}, \tag{2.21}$$

$$D_2 = \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)(\tau_1 - \hat{\tau}_1)\sum_{i=0}^{G-1}\alpha_i. \tag{2.22}$$

Here, D_1 is the desired information term and D_2 is the self interference due to the non-zero chip timing error. Note that if the system is free of synchronization errors,

$$D_1 = \sqrt{2P_1}Ta_{1,0} \tag{2.23}$$

$$D_2 = 0 \tag{2.24}$$

2.1.1.2 Multiple Access Interference

Now, we simplify the multiple access interference term, M defined in (2.8). In a multi-user DS CDMA receiver that is tuned in to receive the signal from the first user, $a_1(t)$, the signals that arrive from other users act as interference. The data modulation in an interference signal, $a_k(t)$, $k = 2, \ldots, K$, can be modeled as a random binary sequence [235]. The relationship between the desired signal and one of the interfering signals is illustrated in Figure 2.3. From this figure, it is clear that, only time delays modulo-T_c are significant, and thus, effectively $(\tau_k - \hat{\tau}_1)$, $k = 2, \ldots, K$ can be assumed to be uniformly distributed in the interval $[0, \lambda T_c]$. Here $\hat{\tau}_1$ represents the estimation error on the time delay of the desired user and is independent of τ_k. Therefore, without any loss of generality we incorporate $\hat{\tau}_1$ into the definition of τ_k. Then, τ_k can be modeled as a random variable that is uniform in $[0, \lambda T_c]$. Here, λ is a constant that may take on any value between 0 and 1 depending on the nature of the system. If the system

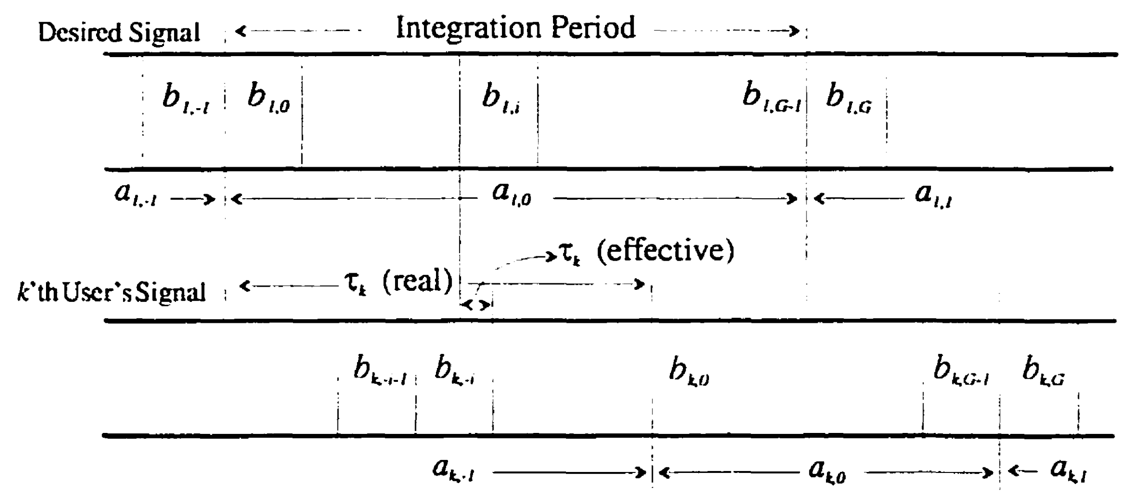

Figure 2.3: Timing of the Local PN Sequence for the First User and the k'th User.

is synchronous, then ideally, all of the users' signals arrive at the receiver at the same time. Then, for a synchronous system λ is set to 0. In this case τ_k is not random but, $\tau_k = \tau_1$. On the other hand, if the system is asynchronous, users transmit at will. In this case, users' signals arrive at the receiver at completely random times. Then, for an asynchronous system, λ is set to 1. Finally, for quasi-synchronous systems, the start epoch of every user is synchronized within a fraction of the chip duration [43]. Therefore, ideally, all users' signals arrive at the receiver within that specific fraction of the chip. Thus, any value of λ in the interval $(0, 1)$ will result in a quasi-synchronous system. For systems with tight synchronization requirements, λ will be set to a value close to 0. As the value of λ is increased, the system synchronism requirements become more relaxed.

Likewise, $(\phi_k - \hat{\phi}_1)$, $k = 2, \ldots, K$ in (2.8) are the phase offsets of the interfering signals. Here $\hat{\phi}_1$ represents the estimation error on the phase offset of the desired user and is independent of ϕ_k. Therefore, without any loss of generality we incorporate $\hat{\phi}_1$ into the definition of ϕ_k. Then, ϕ_k can be modeled as independent, identically distributed (iid) random variables uniformly distributed in the interval $[0, 2\pi]$.

For all practical purposes, all of the information signals that cause interference may be considered as random and thus be embedded into the long PN sequences [13, 138, 235]. Thus, we define,

$$q_k(t) \triangleq a_k(t) \cdot b_k(t) = \sum_{i=-\infty}^{\infty} q_{k,i}\Psi(t - iT_c) \tag{2.25}$$

where

$$q_{k,i} = a_{k,\lfloor \frac{i}{G} \rfloor} \cdot b_{k,i} \tag{2.26}$$

are a set of random variables that take on values $\{\mp 1\}$ with equal probability. In (2.26), $\lfloor x \rfloor$ represents the largest integer that is less than or equal to x. The random variables, $a_{k,i}$ and $b_{k,i}$ are defined in (2.2) and (2.3), respectively.

We can use (2.3) and (2.25) in (2.8) to get,

$$\begin{aligned} M &= \sum_{k=2}^{K} \sqrt{2P_k}\cos(\phi_k - \hat{\phi}_1) \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} b_1(t - \hat{\tau}_1)q_k(t - \tau_k)dt \\ &= \sum_{k=2}^{K} \sqrt{2P_k}\cos(\phi_k - \hat{\phi}_1) \sum_{i=0}^{G-1} \sum_{j=-\infty}^{\infty} b_{1,i}q_{k,j} \\ &\quad \cdot \int_{iT_c}^{(i+1)T_c} \Psi(t - iT_c)\Psi(t - jT_c - (\tau_k - \hat{\tau}_1))dt. \end{aligned} \tag{2.27}$$

M can be further simplified by observing the fact that only two j terms, $j = i$ and $j = i - 1$, result in non-zero values for the integral in (2.27). Furthermore, recall that $\hat{\tau}_1$ can be incorporated into the definition of τ_k and $\hat{\phi}_1$ can be incorporated into the definition of ϕ_k with no loss of generality. In this case, using (2.12) and (2.13), M can be written as,

$$\begin{aligned} M &= \sum_{k=2}^{K} \sqrt{2P_k}\cos(\phi_k) \sum_{i=0}^{G-1} \left[b_{1,i}q_{k,i}\hat{R}_\Psi(\tau_k) + b_{1,i}q_{k,i-1}R_\Psi(\tau_k)\right] \\ &= \sum_{k=2}^{K} \sqrt{2P_k}\cos(\phi_k) \sum_{i=0}^{G-1} \left[b_{1,i}q_{k,i}(T_c - \tau_k) + b_{1,i}q_{k,i-1}\tau_k\right]. \end{aligned} \tag{2.28}$$

We define the random variables,

$$v_{k,j} \triangleq b_{1,j}q_{k,j}, \quad k = 2, \ldots, K \text{ and } j = 0, \ldots, (G-1), \tag{2.29}$$

$$\gamma_{k,j} \triangleq b_{1,j}q_{k,j-1}, \quad k = 2, \ldots, K \text{ and } j = 0, \ldots, (G-1). \tag{2.30}$$

Following the lemma on page 35, it is clear that $v_{k,j}$ and $\gamma_{k,j}$ are iid random variables that take on values $\{\mp 1\}$ with equal probability. Then, using (2.29) and (2.30),

$$M = \sum_{k=2}^{K} \sqrt{2P_k}\cos(\phi_k) \sum_{j=0}^{G-1} \left[v_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\right] \tag{2.31}$$

is the multiple access interference term in its most simplified form.

We now study the statistical properties of the multiple access interference, M, given by (2.31). One of the analytical challenges presented by the nature of the spread spectrum multiple access interference on the communication channel is the inter-dependency of the $(K-1)$ multiple access interference terms that make up M and the chip-to-chip error dependency observed for every one of these $(K-1)$ multiple access interference terms. Although the data sequences, $a_k(t)$, and the spreading sequences, $b_k(t)$, in the DS CDMA system are all modeled as random and independent from chip to chip, for a given interfering signal, k, the chip-to-chip error dependency stems from the fact that the relative time delay, τ_k, and phase offset, ϕ_k associated with the k'th interfering signal remain essentially constant over the course of one symbol duration during which the receiver performs the correlation operation. The inter-dependency of the multiple access interference term, on the other hand, stems from the nature of the correlator receiver used in the DS CDMA system. The correlator receiver correlates every received signal to the same, desired spreading sequence over the course of one symbol. This in turn creates conditionally independent multiple access interference terms at the output of the correlator.

The chip-to-chip dependency of each of the $(K-1)$ multiple access terms in the K user DS CDMA system is obvious from (2.31). From this equation, it is clear that each multiple access interference term is made up of G terms, each representing interference caused by one chip of the signal in question. The time delay and phase offset associated with the specific multiple access interference term remain constant throughout the duration of all of these G chips. Therefore, these G terms are independent only when conditioned in the specific time delays and phase offsets. To see the chip-to-chip dependency, we look at the multiple access interference caused by the j'th chip of the k'th user, which we denote as, M_{kj}. From (2.31),

$$M_{kj} = \sqrt{2P_k}\cos(\phi_k)\Big[v_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\Big] \tag{2.32}$$

where, as stated before, $v_{k,j}$ and $\gamma_{k,j}$ are iid random variables that take on values $\{\mp 1\}$ with equal probability. Then, once conditioned on τ_k and ϕ_k, the random variables M_{kj} and M_{kl} are independent of one another.

Seeing the inter-dependency of the $(K-1)$ multiple access interference terms from (2.31) is somewhat more difficult. For this reason, we first rewrite (2.31) in the form,

$$M = \sum_{k=2}^{K} \sqrt{2P_k}w_k\cos(\phi_k) \tag{2.33}$$

where from (2.29), (2.30) and (2.31),

$$\begin{aligned} w_k &= \sum_{j=0}^{G-1}\left[v_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\right] \\ &= \sum_{j=0}^{G-1}\Big[b_{1,j}b_{k,j}a_{k,\lfloor\frac{j}{G}\rfloor}(T_c - \tau_k) + b_{1,j}b_{k,j-1}a_{k,\lfloor\frac{j-1}{G}\rfloor}\tau_k\Big] \\ &= \sum_{j=0}^{G-1} a_{k,0}b_{1,j}b_{k,j}(T_c - \tau_k) + \sum_{j=-1}^{G-2} a_{k,\lfloor\frac{j}{G}\rfloor}b_{1,j+1}b_{k,j}\tau_k \\ &= a_{k,0}\sum_{j=0}^{G-2} b_{k,j}\Big[b_{1,j}(T_c - \tau_k) + b_{1,j+1}\tau_k\Big] \\ &\quad + a_{k,0}b_{1,G-1}b_{k,G-1}(T_c - \tau_k) + a_{k,-1}b_{1,0}b_{k,-1}\tau_k. \end{aligned} \tag{2.34}$$

We now define the following random variables,

$$\varrho_k \triangleq \frac{\tau_k}{T_c}, \quad k = 2,\ldots,K. \tag{2.35}$$

It is clear from its definition that ϱ_k is a random variable uniformly distributed in the interval $[0, \lambda]$ where as stated before, the value of λ depends on whether the system is synchronous, quasi-synchronous or asynchronous. Using the definition of ϱ_k we get,

$$\begin{aligned} w_k &= T_c\left[a_{k,0}\sum_{j=0}^{G-2} b_{k,j}\{b_{1,j}(1-\varrho_k)+b_{1,j+1}\varrho_k\}\right. \\ &\left. +a_{k,0}b_{1,G-1}b_{k,G-1}(1-\varrho_k)+a_{k,-1}b_{1,0}b_{k,-1}\varrho_k\right]. \end{aligned} \quad (2.36)$$

To further simplify the expression for w_k, we now define the following random variables:

$$\zeta_{k,j} \triangleq \begin{cases} a_{k,0}b_{k,j}b_{1,j}, & j=0,\ldots,G-1 \text{ and } k=2,\ldots,K, \\ a_{k,-1}b_{k,-1}b_{1,0}, & j=G \text{ and } k=2,\ldots,K. \end{cases} \quad (2.37)$$

Following the lemma on page 35, it is clear that $\zeta_{k,j}$ are iid and take on values $\{\pm 1\}$ with equal probability. Using these variables (2.36) can be written as,

$$w_k = T_c\left[\sum_{j=0}^{G-2}\zeta_{k,j}\{(1-\varrho_k)+b_{1,j}b_{1,j+1}\varrho_k\}+\zeta_{k,G-1}(1-\varrho_k)+\zeta_{k,G}\varrho_k\right] \quad (2.38)$$

Following [155], we define two sets: A is the set of all non-negative integers, i, less than $G-1$ such that $b_{1,i}b_{1,i+1}=1$ and B is the set of all non-negative integers, i, less than $G-1$ such that $b_{1,i}b_{1,i+1}=-1$. In other words, A is the set that lists all possible number of non-transitions in signal amplitude at chip boundaries during one symbol time and similarly, B is the set of all possible number of transitions from one amplitude level to the other at chip boundaries during one symbol time. Furthermore, we assume that the cardinalities of the sets A and B are $|A|$ and $|B|$, respectively. During one symbol duration, there are G chips. From the definition of the two sets, A and B, it is clear that they are disjoint and,

$$|A|+|B|=G-1. \quad (2.39)$$

Using the two sets, A and B, w_k in (2.38) can be rewritten as,

$$w_k = T_c\left[\sum_{j\in A}\zeta_{k,j}+\sum_{j\in B}\zeta_{k,j}(1-2\varrho_k)+\zeta_{k,G-1}(1-\varrho_k)+\zeta_{k,G}\varrho_k\right]. \quad (2.40)$$

To further simplify the above equality we define the following random variables.

$$\Lambda_k \triangleq \sum_{j\in A} \zeta_{k,j}, \quad k = 2,\dots,K, \tag{2.41}$$

$$\Gamma_k \triangleq \sum_{j\in B} \zeta_{k,j}, \quad k = 2,\dots,K, \tag{2.42}$$

$$\Upsilon_k \triangleq \zeta_{k,G-1}, \quad k = 2,\dots,K, \tag{2.43}$$

$$\Omega_k \triangleq \zeta_{k,G}, \quad k = 2,\dots,K. \tag{2.44}$$

Substituting (2.41)-(2.44) into (2.40) we get,

$$w_k = T_c\left\{\Lambda_k + \Gamma_k(1-2\varrho_k) + \Upsilon_k(1-\varrho_k) + \Omega_k\varrho_k\right\}. \tag{2.45}$$

It is clear that the densities of $\Lambda_k, \Gamma_k, \Upsilon_k$ and Ω_k depend on the characteristics of the desired spreading sequence through the disjoint sets, A, B, $\{G-1\}$ and $\{G\}$.

From its definition in (2.41), it is clear that Λ_k has a distribution in the form [124],

$$Pr(\Lambda_k = j) = \binom{|A|}{\frac{j+|A|}{2}} 2^{-|A|}, \quad j \in \{-|A|, -|A|+2,\dots,|A|-2,|A|\}$$

and has a conditional mean and variance of

$$\mathrm{E}\{\Lambda_k|A\} = 0, \tag{2.46}$$

$$\sigma^2_{\Lambda_k|A} = \mathrm{E}\{\Lambda_k^2|A\} = |A|, \tag{2.47}$$

respectively. Similarly Γ_k has a distribution in the form [124],

$$Pr(\Gamma_k = j) = \binom{|B|}{\frac{j+|B|}{2}} 2^{-|B|}, \quad j \in \{-|B|, -|B|+2,\dots,|B|-2,|B|\}$$

and has a conditional mean and variance of

$$\mathrm{E}\{\Gamma_k|B\} = 0, \tag{2.48}$$

$$\sigma^2_{\Gamma_k|B} = \mathrm{E}\{\Gamma_k^2|B\} = |B|, \tag{2.49}$$

respectively. Finally, Υ_k and Ω_k are uniformly distributed in the set $\{\pm 1\}$ and they have

$$\begin{aligned} \mathrm{E}\{\Upsilon_k\} &= \mathrm{E}\{\Omega_k\} = 0. \quad (2.50) \\ \mathrm{E}\{\Upsilon_k^2\} &= \mathrm{E}\{\Omega_k^2\} = 1. \quad (2.51) \end{aligned}$$

Clearly, the densities of Λ_k and Γ_k depend on the desired spreading sequence through the cardinalities of A and B. The cardinalities in turn, are related to the aperiodic discrete autocorrelation function defined in [178] as,

$$C_j(\lambda) = \sum_{i=0}^{G-1-\lambda} b_{j,i} b_{j,i+\lambda}, \quad (2.52)$$

since the discrete aperiodic autocorrelation function of the first user's spreading sequence evaluated at a delay of 1 is equal to,

$$\begin{aligned} C_1(1) &= \sum_{i=0}^{G-2} b_{1,i} b_{1,i+1} \\ &= |A| - |B| \quad (2.53) \end{aligned}$$

with

$$\begin{aligned} \mathrm{E}\{C_1(1)\} &= 0, \quad (2.54) \\ \mathrm{E}\{C_1(1)^2\} &= G-1. \quad (2.55) \end{aligned}$$

From (2.53) it is clear that $C_1(1)$ is distributed as the sum of $G-1$ symmetric Bernoulli trials, so the density of $C_1(1)$ is given by,

$$Pr(C_1(1) = j) = \binom{G-1}{\frac{j+G-1}{2}} 2^{-(G-1)}, \quad j \in \{-(G-1), -(G-3), \ldots, G-3, G-1\}.$$

Since the random variables Λ_k, Γ_k, Υ_k and Ω_k are composed of disjoint sets of symmetric Bernoulli trials given a particular desired signature sequence, they are conditionally independent. Furthermore, for all $k \neq m$, Λ_k and Λ_m, Γ_k and Γ_m, Υ_k and Υ_m and Ω_k and Ω_m are independent of one another given the desired signature sequence. In fact, it is sufficient to condition the multiple access interference terms in M on only the aperiodic autocorrelation function of the spreading sequence of the desired signal. This is because, the probability density functions of Λ_k and Γ_k depend only on the parameters $|A|$ and $|B|$, and these parameters in turn depend on the first spreading sequence only through its aperiodic autocorrelation function evaluated at time delay 1, namely, $C_1(1)$. Note from (2.39) and (2.53) that,

$$|A| = \frac{G-1+C_1(1)}{2}, \tag{2.56}$$

$$|B| = \frac{G-1-C_1(1)}{2}, \tag{2.57}$$

with

$$\mathrm{E}\{|A|\} = \mathrm{E}\{|B|\} = \frac{G-1}{2} \tag{2.58}$$

and

$$\mathrm{E}\{|A|^2\} = \mathrm{E}\{|B|^2\} = \frac{G(G-1)}{4}. \tag{2.59}$$

In other words, if any one of the three quantities $|A|$, $|B|$ or $C_1(1)$ is known for a given processing gain, G, the other two are completely defined. Thus, the random variables $\Lambda_k, \Gamma_k, \Upsilon_k$ and Ω_k are independent given a specific value of $C_1(1)$. However, in general, the terms that make up the multiple access interference are not unconditionally independent. This can be shown using a counter-example. A general proof for this is given in [155].

Example: Consider a system with $G = 2$. Furthermore, assume that there are two multiple access interferers in the system. Then, from (2.31),

$$\begin{aligned} M &= M_1 + M_2 \\ &= \sqrt{2P_2}\cos(\phi_2)\left[(\nu_{2,0}+\nu_{2,1})(T_c-\tau_2)+(\gamma_{2,0}+\gamma_{2,1})\tau_2\right] \\ &\quad +\sqrt{2P_3}\cos(\phi_3)\left[(\nu_{3,0}+\nu_{3,1})(T_c-\tau_3)+(\gamma_{3,0}+\gamma_{3,1})\tau_3\right] \end{aligned}$$

Let the signal power be $P_k = 1/2$ for all users. Also, normalize the interference term to the chip duration so that $T_c = 1$.

Now, assume that the first interfering signal has the following information and spreading sequence values: $a_{2,0} = 1$, $b_{2,0} = 1$ and $b_{2,1} = 1$. Similarly, the second interfering signal has the following information and spreading sequence values: $a_{3,0} = 1$, $b_{3,0} = 1$ and $b_{3,1} = -1$. Furthermore, assume that the interfering signals are both time and phase aligned with the signal of the first user, i.e. $\phi_2 = \phi_3 = \tau_2 = \tau_3 = 0$. In this case,

$$\begin{aligned} M_1 &= b_{1,0} + b_{1,1}, \\ M_2 &= b_{1,0} - b_{1,1}. \end{aligned}$$

Since $G = 2$ is assumed,

$$C_1(1) = b_{1,0} b_{1,1}$$

is uniform on $\{\mp 1\}$.

We find the distribution of the multiple access interference, M, caused by the two interferers. We have shown that M_1 and M_2 are independent of one another when conditioned on $C_1(1)$. Therefore, the distribution of M can be written as,

$$\begin{aligned} f(M) &= f(M_1 + M_2) \\ &= f(M_1 + M_2 | C_1(1) = 1) P(C_1(1) = 1) \\ &\quad + f(M_1 + M_2 | C_1(1) = -1) P(C_1(1) = -1) \\ &= \left(f(M_1 | C_1(1) = 1) * f(M_2 | C_1(1) = 1) \right) P(C_1(1) = 1) \\ &\quad + \left(f(M_1 | C_1(1) = -1) * f(M_2 | C_1(1) = -1) \right) P(C_1(1) = -1) \end{aligned}$$

where "$*$" represents the convolution operation. When $C_1(1) = 1$, M_1 is uniform on the set $\{\mp 2\}$, and when $C_1(1) = -1$, $M_1 = 0$. Similarly, when $C_1(1) = 1$, $M_2 = 0$ and when $C_1(1) = -1$, M_2 is uniform on the set $\{\mp 2\}$. Averaging over the values of $C_1(1)$ at this point produces identical probability density functions for M_1 and M_2, with impulses at -2, 0, and 2 with amplitudes 0.25, 0.5, and 0.25, respectively.

The distribution of M conditioned on $C_1(1)$ is found by convolving the distribution of M_1 conditioned on $C_1(1)$ with the distribution of M_2 conditioned on $C_1(1)$. Therefore, when $C_1(1) = -1$, M has a probability density function with impulses at -4, 0, and 4, with amplitudes, 0.25, 0.5 and 0.25, respectively. Similarly, when $C_1(1) = 1$, $M = 0$. Then, by averaging over the values of $C_1(1)$, the probability density function for M with two interferers is found to consist of impulses at -4, 0, and 4 with amplitudes, 0.125, 0.75, 0.125, respectively.

If the individual M terms were unconditionally independent, the following equation would also provide the distribution of M,

$$f(M) = f(M_1) * f(M_2).$$

This convolution results in impulses at times -4, -2, 0, 2, and 4 with amplitudes 0.0625, 0.25, 0.375, 0.25, and 0.0625, respectively. As is clearly observed, this result is not equal to the actual probability density function for M with two interferers. Therefore, using this specific example, we conclude that the individual M terms are not unconditionally independent.

We now show that the individual multiple access interference terms, though only conditionally independent, are in fact uncorrelated. It is clear from (2.45) and (2.46)-(2.51) that,

$$E\{w_k|C_1(1)\} = 0. \tag{2.60}$$

We have shown that the multiple access interference terms are conditionally independent given $C_1(1)$. Thus,

$$E\{w_j w_k|C_1(1)\} = E\{w_j|C_1(1)\} \cdot E\{w_k|C_1(1)\} \tag{2.61}$$

for $j, k \in \{2, \ldots, K\}$ and $j \neq k$. Averaging both sides of (2.61) over $C_1(1)$ and making use of (2.60) gives,

$$E\{w_j w_k\} = E\{w_j\} \cdot E\{w_k\} = 0 \tag{2.62}$$

and therefore the interference produced by different transmitters are uncorrelated.

We also show that the G terms that represent the interference caused by the chips of a given multiple access interference term are uncorrelated as well. From (2.32), we can write,

$$\mathrm{E}\{M_{kj}|\tau_k,\phi_k\} = 0. \tag{2.63}$$

We have shown that the G terms that make up an individual multiple access interference term are conditionally independent given τ_k and ϕ_k. Thus,

$$\mathrm{E}\{M_{kj}M_{kl}|\tau_k,\phi_k\} = \mathrm{E}\{M_{kj}|\tau_k,\phi_k\}\cdot\mathrm{E}\{M_{kl}|\tau_k,\phi_k\} \tag{2.64}$$

for $j \neq k$. Averaging both sides of (2.64) over τ_k, ϕ_k and making use of (2.63) gives,

$$\mathrm{E}\{M_{kj}M_{kl}\} = \mathrm{E}\{M_{kj}\}\cdot\mathrm{E}\{M_{kl}\} = 0 \tag{2.65}$$

and therefore the interference terms produced by the chips of a given multiple access interference term though only conditionally independent, are in fact uncorrelated.

2.1.1.3 Channel Noise

Finally, let us consider the noise term, N described in (2.9). Recall that the channel noise $n(t)$ is modeled as an additive, zero mean, white Gaussian random process with a variance of $N_0/2$. Therefore, the channel noise at the output of the correlator receiver, N, is still a Gaussian random variable with zero mean and variance,

$$\begin{aligned}\sigma_N^2 &= 4\int_{\hat{\tau}_1}^{T+\hat{\tau}_1}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1}\mathrm{E}\{n(t_1)n(t_2)\}b_1(t_1-\hat{\tau}_1)b_1(t_2-\hat{\tau}_1)\\ &\quad\cdot\cos(\omega_c t_1+\hat{\phi}_1)\cos(\omega_c t_2+\hat{\phi}_2)dt_1dt_2\\ &= 2N_0\int_0^T\cos^2(\omega_c t)dt\\ &= N_0T\end{aligned} \tag{2.66}$$

since in practice, the symbol duration is selected to be much greater than the inverse of the signalling frequency, $T \gg \frac{1}{f_c}$ [156, 178, 235].

2.1.1.4 Summary

In summary, the input signal to the decision device in Figure 2.2 for a BPSK modulated, biphase spread DS CDMA system in an AWGN channel is,

$$Z_1 = D_1 + D_2 + M + N \tag{2.67}$$

where D_1 is the desired signal, D_2 is the self interference, M is the multiple access interference and N is the channel noise components. These terms are defined as,

$$D_1 = \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)G(T_c - (\tau_1 - \hat{\tau}_1))a_{1,0}, \tag{2.68}$$

$$D_2 = \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)(\tau_1 - \hat{\tau}_1)\sum_{i=0}^{G-1}\alpha_i, \tag{2.69}$$

$$M = \sum_{k=2}^{K}\sqrt{2P_k}\cos(\phi_k)\sum_{j=0}^{G-1}\left[\upsilon_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\right] \tag{2.70}$$

$$= \sum_{k=2}^{K}\sqrt{2P_k}\cos(\phi_k)T_c\Big(\Gamma_k + \Lambda_k(1 - 2\varrho_k) + \Upsilon_k(1 - \varrho_k) + \Omega_k\Big), \tag{2.71}$$

and N is a zero mean Gaussian random variable with a variance of N_0T.

2.1.2 Quadriphase Spread DS CDMA System

If the system of interest uses quadriphase spreading, each one of the K users will transmit a signal in the form,

$$s_k(t) = \sqrt{P_k}a_k(t - \mathcal{T}_k)\left[b_k^i(t - \mathcal{T}_k)\cos(\omega_c t + \theta_k) + b_k^q(t - \mathcal{T}_k)\sin(\omega_c t + \theta_k)\right] \tag{2.72}$$

as shown in Figure 2.4. In (2.72), P_k is the power of the signal transmitted by user k, $a_k(t)$ is the information bearing signal, $b_k^i(t)$ and $b_k^q(t)$ are the in-phase and quadrature spreading waveforms for the k'th user, respectively, and ω_c is the common signaling frequency. The random variables $\mathcal{T}_k$ and θ_k represent the user specific, initial time and phase offsets, respectively. The data modulation $a_k(t)$ is in the form of (2.2). Similar

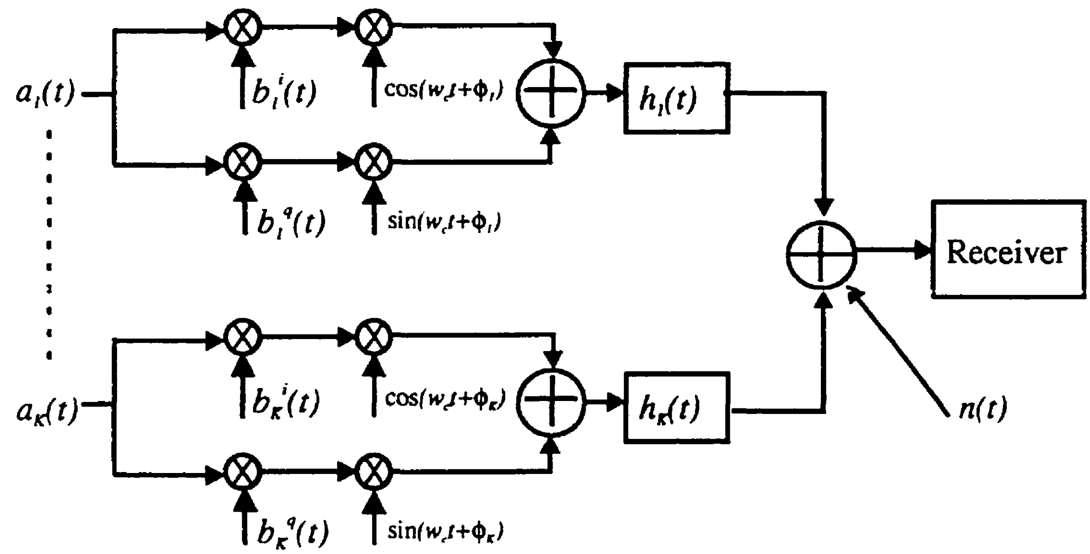

Figure 2.4: Quadriphase Spread DS CDMA System Model

to the biphase spreading case, the long PN sequences, $b_k^i(t)$ and $b_k^q(t)$ are represented as,

$$b_k^i(t) = \sum_{j=-\infty}^{\infty} b_{k,j}^i \cdot \Psi(t - jT_c) \tag{2.73}$$

and

$$b_k^q(t) = \sum_{j=-\infty}^{\infty} b_{k,j}^q \cdot \Psi(t - jT_c) \tag{2.74}$$

where $b_{k,j}^i$ and $b_{k,j}^q$ represent one chip of the in-phase and quadrature PN sequences, respectively, and they take on values $\{\mp 1\}$ independent of one another with equal probability. Like before, the rectangular chip waveform $\Psi(t)$ has duration $T_c = T/G$ where G is the processing gain of the spread spectrum system.

As in the case of the biphase spread system, the quadriphase spread system goes through a channel represented by (2.4). The channel introduces a timing delay of τ_k and a phase offset of ϕ_k as well as AWGN. In this case the total received signal becomes.

$$r(t) = \sum_{k=1}^{K} \sqrt{P_k} a_k(t-\tau_k) \left[b_k^i(t-\tau_k)\cos(\omega_c t + \phi_k) + b_k^q(t-\tau_k)\sin(\omega_c t + \phi_k) \right] + n(t). \quad (2.75)$$

In (2.75). the differences in message start times and initial phase offsets are incorporated into the values of τ_k and ϕ_k. Again, without any loss of generality, we assume that the signal from the first user, $a_1(t)$ is to be captured. A typical correlator receiver such as the one in Figure 2.5 is typically used to coherently filter the desired user's signal from all other users' signals that share the same channel in a quadriphase spread BPSK modulated DS CDMA system. In such receivers, the in-phase and the quadrature components of the received signal are first despread using the in-phase and quadrature PN sequences of the desired user and then stripped off their carriers in separate branches. Then, the signals in these two branches are integrated over one bit period. Subsequently, the outputs of the two integrators are summed and sampled at the bit rate. Figure 2.5 displays this sequence of operations in the reverse order. As stated before, from a theoretical standpoint, these two modes of operation are identical, but in practice the despreading operation is always performed first.

A system with synchronization errors cannot estimate the time and phase delays of the desired user's signal perfectly. If the receiver in Figure 2.5 estimates the time and phase delays of the first user's signal to be $\hat{\tau}_1$ and $\hat{\phi}_1$, the corresponding chip timing and carrier phase errors will be $(\tau_1 - \hat{\tau}_1)$ and $(\phi_1 - \hat{\phi}_1)$, respectively. As before, the relative time and phase offsets of the interfering signals, $(\phi_k - \hat{\phi}_1)$ and $(\tau_k - \hat{\tau}_1)$ may be modeled as iid random variables that are uniformly distributed in the intervals $[0, 2\pi]$ and $[0, \lambda T_c]$, respectively, where the value of λ specifies the nature of the system. With no loss of generality, we may incorporate $\hat{\phi}_1$ and $\hat{\tau}_1$ into the definitions of ϕ_k and $\tau_k, k = 2, \ldots, K$ to have τ_k and $\phi_k, k = 2, \ldots, K$ uniform in $[0, \lambda T_c]$ and $[0, 2\pi]$, respectively.

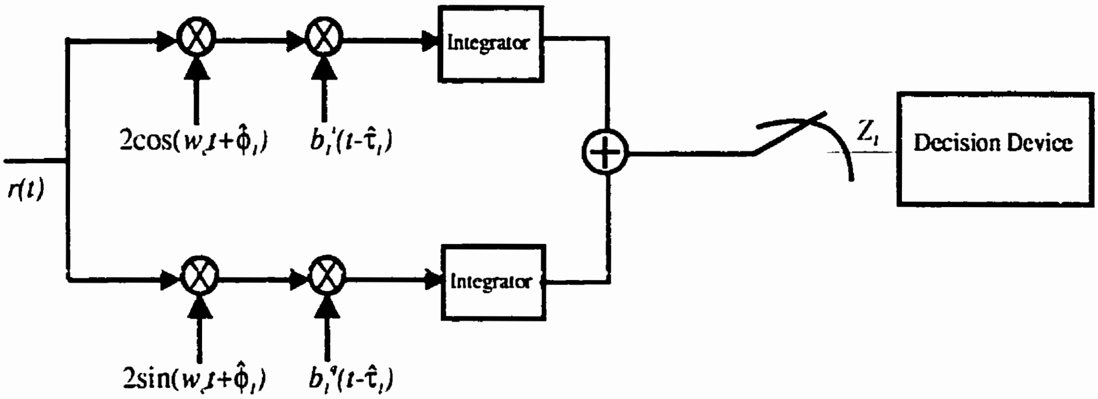

Figure 2.5: A Typical Coherent DS CDMA Receiver for the Quadriphase Spread System

From Figure 2.5, the input to the decision device for a quadriphase spread, BPSK modulated DS CDMA receiver tuned in to recover the message from the first user is written as,

$$\begin{aligned} Z_1 &= 2\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} r(t)\left[b_1^i(t-\hat{\tau}_1)\cos(\omega_c t+\hat{\phi}_1)+b_1^q(t-\hat{\tau}_1)\sin(\omega_c t+\hat{\phi}_1)\right]dt \\ &= D+M+N \end{aligned} \quad (2.76)$$

where D, M and N represent the desired signal plus self interference, multiple access interference and additive white Gaussian noise terms, respectively. The desired signal plus self interference term, D, may be written as,

$$\begin{aligned} D &= \sqrt{P_1}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_1(t-\tau_1)\cos(\phi_1-\hat{\phi}_1)b_1^i(t-\hat{\tau}_1)b_1^i(t-\tau_1)dt \\ &+\sqrt{P_1}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_1(t-\tau_1)\cos(\phi_1-\hat{\phi}_1)b_1^q(t-\hat{\tau}_1)b_1^q(t-\tau_1)dt \\ &+\sqrt{P_1}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_1(t-\tau_1)\sin(\phi_1-\hat{\phi}_1)b_1^i(t-\hat{\tau}_1)b_1^q(t-\tau_1)dt \\ &-\sqrt{P_1}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_1(t-\tau_1)\sin(\phi_1-\hat{\phi}_1)b_1^q(t-\hat{\tau}_1)b_1^i(t-\tau_1)dt. \end{aligned} \quad (2.77)$$

Similarly, the multiple access interference term, M, may be written as,

$$\begin{aligned}
M &= \sum_{k=2}^{K}\sqrt{P_k}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_k(t-\tau_k)\cos(\phi_k-\hat{\phi}_1)b_1^i(t-\hat{\tau}_1)b_k^i(t-\tau_k)dt \\
&+\sum_{k=2}^{K}\sqrt{P_k}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_k(t-\tau_k)\cos(\phi_k-\hat{\phi}_1)b_1^q(t-\hat{\tau}_1)b_k^q(t-\tau_k)dt \\
&+\sum_{k=2}^{K}\sqrt{P_k}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_k(t-\tau_k)\sin(\phi_k-\hat{\phi}_1)b_1^i(t-\hat{\tau}_1)b_k^q(t-\tau_k)dt \\
&-\sum_{k=2}^{K}\sqrt{P_k}\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} a_k(t-\tau_k)\sin(\phi_k-\hat{\phi}_1)b_1^q(t-\hat{\tau}_1)b_k^i(t-\tau_k)dt. \qquad (2.78)
\end{aligned}$$

Finally, the additive white Gaussian noise term, N, may be written as,

$$\begin{aligned}
N &= 2\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} n(t)b_1^i(t-\hat{\tau}_1)\cos(\omega_c t+\hat{\tau}_1)dt \\
&+2\int_{\hat{\tau}_1}^{T+\hat{\tau}_1} n(t)b_1^q(t-\hat{\tau}_1)\sin(\omega_c t+\hat{\tau}_1)dt. \qquad (2.79)
\end{aligned}$$

We now simplify these terms.

2.1.2.1 Desired Signal Plus Self Interference

In (2.76), D is the desired signal plus self interference and is mathematically given in (2.77). Using (2.2) and the fact that $a_1(t)$ is constant over $[0,T]$, (2.77) can be rewritten as,

$$\begin{aligned}
D &= \sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,-1}\int_0^{\tau_1-\hat{\tau}_1} b_1^i(t)b_1^i(t-(\tau_1-\hat{\tau}_1))dt \\
&+\sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,-1}\int_0^{\tau_1-\hat{\tau}_1} b_1^q(t)b_1^q(t-(\tau_1-\hat{\tau}_1))dt \\
&+\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)a_{1,-1}\int_0^{\tau_1-\hat{\tau}_1} b_1^i(t)b_1^q(t-(\tau_1-\hat{\tau}_1))dt \\
&-\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)a_{1,-1}\int_0^{\tau_1-\hat{\tau}_1} b_1^q(t)b_1^i(t-(\tau_1-\hat{\tau}_1))dt \\
&+\sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,0}\int_{(\tau_1-\hat{\tau}_1)}^{T} b_1^i(t)b_1^i(t-(\tau_1-\hat{\tau}_1))dt \\
&+\sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,0}\int_{\tau_1-\hat{\tau}_1}^{T} b_1^q(t)b_1^q(t-(\tau_1-\hat{\tau}_1))dt
\end{aligned}$$

$$+\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)a_{1,0}\int_{\tau_1-\hat{\tau}_1}^{T} b_1^i(t)b_1^q(t-(\tau_1-\hat{\tau}_1))dt$$
$$-\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)a_{1,0}\int_{\tau_1-\hat{\tau}_1}^{T} b_1^q(t)b_1^i(t-(\tau_1-\hat{\tau}_1))dt. \qquad (2.80)$$

Using (2.73),(2.74),(2.12) and (2.13), (2.80) can be written as,

$$\begin{aligned}
D &= \sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,-1}\left[b_{1,0}^i b_{1,-1}^i + b_{1,0}^q b_{1,-1}^q\right] R_\psi(\tau_1-\hat{\tau}_1) \\
&+\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)a_{1,-1}\left[b_{1,0}^i b_{1,-1}^q - b_{1,0}^q b_{1,-1}^i\right] R_\psi(\tau_1-\hat{\tau}_1) \\
&+\sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,0}2G\hat{R}_\psi(\tau_1-\hat{\tau}_1) \\
&+\sqrt{P_1}\cos(\phi_1-\hat{\phi}_1)a_{1,0}\sum_{j=1}^{G-1}\left[b_{1,j}^i b_{1,j-1}^i + b_{1,j}^q b_{1,j-1}^q\right] R_\psi(\tau_1-\hat{\tau}_1) \\
&+\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)a_{1,0}\sum_{j=1}^{G-1}\left[b_{1,j}^i b_{1,j-1}^q - b_{1,j}^q b_{1,j-1}^q\right] R_\psi(\tau_1-\hat{\tau}_1). \qquad (2.81)
\end{aligned}$$

In this thesis, we consider only unit amplitude, rectangular shaped chips. In this case, using (2.15) and (2.16), (2.81) can be rewritten as,

$$\begin{aligned}
D &= \sqrt{P_1}a_{1,-1}(\tau_1-\hat{\tau}_1)\left(b_{1,0}^i b_{1,-1}^i + b_{1,0}^q b_{1,-1}^q\right)\cos(\phi_1-\hat{\phi}_1) \\
&+\sqrt{P_1}a_{1,-1}(\tau_1-\hat{\tau}_1)\left(b_{1,0}^i b_{1,-1}^q - b_{1,0}^q b_{1,-1}^i\right)\sin(\phi_1-\hat{\phi}_1) \\
&+\sqrt{P_1}a_{1,0}(T-(\tau_1-\hat{\tau}_1))2G\cos(\phi_1-\hat{\phi}_1) \\
&+\sqrt{P_1}a_{1,0}(\tau_1-\hat{\tau}_1)\sum_{j=1}^{G-1}\left[b_{1,j}^i b_{1,j-1}^i + b_{1,j}^q b_{1,j-1}^q\right]\cos(\phi_1-\hat{\phi}_1) \\
&+\sqrt{P_1}a_{1,0}(\tau_1-\hat{\tau}_1)\sum_{j=1}^{G-1}\left[b_{1,j}^i b_{1,j-1}^q - b_{1,j}^q b_{1,j-1}^q\right]\sin(\phi_1-\hat{\phi}_1). \qquad (2.82)
\end{aligned}$$

We now define the following random variables,

$$\iota_0 \triangleq a_{1,-1}[b_{1,0}^i b_{1,-1}^i + b_{1,0}^q b_{1,-1}^q]. \qquad (2.83)$$

$$\iota_i \triangleq a_{1,0}[b_{1,i}^i b_{1,i-1}^i + b_{1,i}^q b_{1,i-1}^q], \qquad i = 1,\ldots,G-1 \qquad (2.84)$$

$$\epsilon_0 \triangleq a_{1,-1}[b_{1,0}^i b_{1,-1}^q - b_{1,0}^q b_{1,-1}^i]. \qquad (2.85)$$

$$\epsilon_i \triangleq a_{1,0}[b_{1,i}^i b_{1,i-1}^q - b_{1,i}^q b_{1,i-1}^i], \qquad i = 1,\ldots,G-1. \qquad (2.86)$$

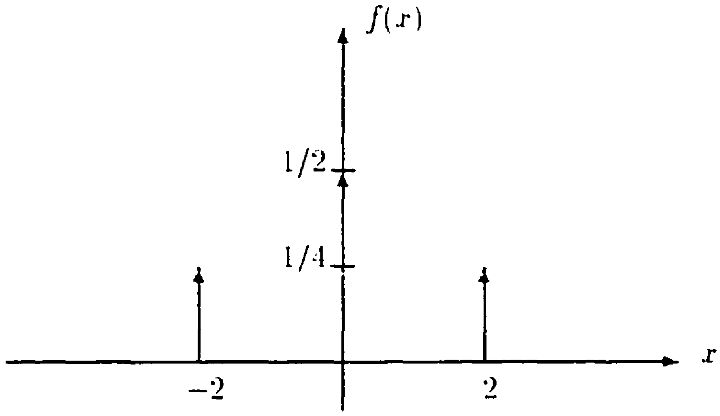

Figure 2.6: Probability density function for ι_i, ϵ_i and ξ_i.

It is obvious from the lemma given on page 35 that ι_i and ϵ_i are iid random variables and they take on values $\{0, \mp 2\}$ according to the probability density function given in Figure 2.6. Using these random variables, (2.82) can be rewritten as,

$$\begin{aligned} D &= 2\sqrt{P_1}a_{1,0}G\cos(\phi_1 - \hat{\phi}_1)(T_c - (\tau_1 - \hat{\tau}_1)) \\ &+ \sum_{j=0}^{G-1}\sqrt{P_1}\left(\cos(\phi_1 - \hat{\phi}_1)\iota_j + \sin(\phi_1 - \hat{\phi}_1)\epsilon_j\right)(\tau_1 - \hat{\tau}_1) \end{aligned} \quad (2.87)$$

where D consists of both the desired signal and the self interference term. If we define $D = D_1 + D_2$ where D_1 is the desired term and D_2 is the self interference term, the individual terms can be written as,

$$D_1 = 2\sqrt{P_1}a_{1,0}G\cos(\phi_1 - \hat{\phi}_1)(T_c - (\tau_1 - \hat{\tau}_1)), \quad (2.88)$$

$$D_2 = \sum_{j=0}^{G-1}\sqrt{P_1}(\cos(\phi_1 - \hat{\phi}_1)\iota_j + \sin(\phi_1 - \hat{\phi}_1)\epsilon_j)(\tau_1 - \hat{\tau}_1). \quad (2.89)$$

Note that, if the system is free of synchronization errors,

$$D_1 = 2\sqrt{P_1}Ta_{1,0} \quad (2.90)$$

$$D_2 = 0. \quad (2.91)$$

2.1.2.2 Multiple Access Interference

We now simplify M defined in (2.78). Similar to the biphase spreading case, for all practical purposes, all of the information signals that cause interference may be assumed as random and thus be embedded into the long PN sequence. Thus, we define,

$$q_k^i(t) \triangleq a_k(t)b_k^i(t) = \sum_{j=-\infty}^{\infty} q_{k,j}^i \Psi(t - T_c). \tag{2.92}$$

and

$$q_k^q(t) \triangleq a_k(t)b_k^q(t) = \sum_{j=-\infty}^{\infty} q_{k,j}^q \Psi(t - T_c). \tag{2.93}$$

where using the lemma given on page 35 it can be stated that $q_{k,j}^i$ and $q_{k,j}^q$ are independent random variables that take on values $\{\mp 1\}$ with equal probability. Using equations (2.73), (2.74), (2.92) and (2.93) in (2.78) we get,

$$\begin{aligned}
M &= \sum_{k=2}^{K} \sqrt{P_k} \cos(\phi_k - \hat{\phi}_1) \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} \left(b_1^i(t-\hat{\tau}_1) q_k^i(t-\tau_k) + b_1^q(t-\hat{\tau}_1) q_k^q(t-\tau_k) \right) dt \\
&\quad + \sum_{k=2}^{K} \sqrt{P_k} \sin(\phi_k - \hat{\phi}_1) \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} \left(b_1^i(t-\hat{\tau}_1) q_k^q(t-\tau_k) - b_1^q(t-\hat{\tau}_1) q_k^i(t-\tau_k) \right) dt \\
&= \sum_{k=2}^{K} \sqrt{P_k} \cos(\phi_k - \hat{\phi}_1) \sum_{i=0}^{G-1} \sum_{j=-\infty}^{\infty} \left(b_{1,i}^i q_{k,j}^i + b_{1,i}^q q_{k,j}^q \right) \\
&\qquad \cdot \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} \Psi(t - iT_c - \hat{\tau}_1) \Psi(t - jT_c - \tau_k) dt \\
&\quad + \sum_{k=2}^{K} \sqrt{P_k} \sin(\phi_k - \hat{\phi}_1) \sum_{i=0}^{G-1} \sum_{j=-\infty}^{\infty} \left(b_{1,i}^i q_{k,j}^q - b_{1,i}^q q_{k,j}^i \right) \\
&\qquad \cdot \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} \Psi(t - iT_c - \hat{\tau}_1) \Psi(t - jT_c - \tau_k) dt.
\end{aligned} \tag{2.94}$$

M can be further simplified by observing the fact that only two j terms, $j = i$ and $j = i - 1$, result in non-zero values for the integrals in (2.94). Furthermore, recall that $\hat{\tau}_1$ can be incorporated into the definition of τ_k and $\hat{\phi}_1$ can be incorporated into the definition of ϕ_k with no loss of generality. In this case, assuming rectangular chip shaping and by making use of (2.15) and (2.16), M can be rewritten as,

$$\begin{aligned}
M &= \sum_{k=2}^{K} \sqrt{P_k} \cos(\phi_k) \left[\sum_{m=0}^{G-1} b^i_{1,m} \left[q^i_{k,m}(T_c - \tau_k) + q^i_{k,m-1}\tau_k \right] \right. \\
&\left. + \sum_{m=0}^{G-1} b^q_{1,m} \left[q^q_{k,m}(T_c - \tau_k) + q^q_{k,m-1}\tau_k \right] \right] \\
&+ \sum_{k=2}^{K} \sqrt{P_k} \sin(\phi_k) \left[\sum_{m=0}^{G-1} b^i_{1,m} \left[q^q_{k,m}(T_c - \tau_k) + q^q_{k,m-1}\tau_k \right] \right. \\
&\left. - \sum_{m=0}^{G-1} b^q_{1,m} \left[q^i_{k,m}(T_c - \tau_k) + q^i_{k,m-1}\tau_k \right] \right] . \qquad (2.95)
\end{aligned}$$

We then define the following random variables,

$$\tilde{\upsilon}_{k,j} \triangleq b^i_{1,j} q^i_{k,j} + b^q_{1,j} q^q_{k,j}, \quad k = 2, \ldots, K \text{ and } j = 0, \ldots, (G-1), \qquad (2.96)$$

$$\tilde{\gamma}_{k,j} \triangleq b^i_{1,j} q^i_{k,j-1} + b^q_1 q^q_{k,j-1}, \quad k = 2, \ldots, K \text{ and } j = 0, \ldots, (G-1), \qquad (2.97)$$

$$\hat{\upsilon}_{k,j} \triangleq b^i_{1,j} q^q_{k,j} - b^q_{1,j} q^i_{k,j} \quad k = 2, \ldots, K \text{ and } j = 0, \ldots, (G-1), \qquad (2.98)$$

$$\hat{\gamma}_{k,j} \triangleq b^i_{1,j} q^q_{k,j-1} - b^q_1 q^i_{k,j-1}, \quad k = 2, \ldots, K \text{ and } j = 0, \ldots, (G-1). \qquad (2.99)$$

Once again, by making use of the lemma given on page 35, $\tilde{\upsilon}_{k,j}$, $\tilde{\gamma}_{k,j}$, $\hat{\upsilon}_{k,j}$ and $\hat{\gamma}_{k,j}$ are found to be iid random variables that take on values $\{0, \mp 2\}$ according to the probability density function given in Figure 2.6.

Using the definitions of these random variables, M in (2.95) can be rewritten as,

$$\begin{aligned}
M &= \sum_{k=2}^{K} \sqrt{P_k} \cos(\phi_k) \left((T_c - \tau_k) \sum_{m=0}^{G-1} \tilde{\upsilon}_{k,m} + \tau_k \sum_{m=0}^{G-1} \tilde{\gamma}_{k,m} \right) \\
&+ \sum_{k=2}^{K} \sqrt{P_k} \sin(\phi_k) \left((T_c - \tau_k) \sum_{m=0}^{G-1} \hat{\upsilon}_{k,m} + \tau_k \sum_{m=0}^{G-1} \hat{\gamma}_{k,m} \right) . \qquad (2.100)
\end{aligned}$$

2.1.2.3 Channel Noise

Finally, let us consider the channel noise term, N. Similar to the biphase spread system, at the output of the correlator receiver, N is still a Gaussian random variable with zero mean and variance equal to,

$$\sigma_N^2 = 4 \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} \int_{\hat{\tau}_1}^{T+\hat{\tau}_1} \mathrm{E}\{n(t_1)n(t_2)\}$$

$$
\begin{aligned}
& \cdot\left[b_1^i(t_1-\hat{\tau}_1)\cos(\omega_c t_1+\hat{\phi}_1)+b_1^q(t_1-\hat{\tau}_1)\sin(\omega_c t_1+\hat{\phi}_1)\right] \\
& \cdot\left[b_1^i(t_2-\hat{\tau}_1)\cos(\omega_c t_2+\hat{\phi}_1)+b_1^q(t_2-\hat{\tau}_1)\sin(\omega_c t_2+\hat{\phi}_1)\right] dt_1 dt_2 \\
= \; & 2N_0\int_0^T \left[\cos^2(\omega_c t)+\sin^2(\omega_c t)+b_1^i(t)b_1^q(t)\sin(\omega_c t)\cos(\omega_c t)\right] dt \\
= \; & 2N_0 T \qquad (2.101)
\end{aligned}
$$

since the symbol duration is much greater than the inverse of the signalling frequency, $T \gg \frac{1}{f_c}$.

2.1.2.4 Summary

In summary, the input signal to the decision device in Figure 2.5 for a BPSK modulated, quadriphase spread DS CDMA system in an AWGN channel is,

$$Z_1 = D_1 + D_2 + M + N$$

where, as before, D_1 is the desired signal, D_2 is the self interference, M is the multiple access interference and N is the channel noise components. These terms are defined as,

$$
\begin{aligned}
D_1 &= 2\sqrt{P_1}a_{1,0}G\cos(\phi_1-\hat{\phi}_1)(T_c-(\tau_1-\hat{\tau}_1)), \qquad (2.102) \\
D_2 &= \sum_{j=0}^{G-1}\sqrt{P_1}(\cos(\phi_1-\hat{\phi}_1)\iota_j+\sin(\phi_1-\hat{\phi}_1)\epsilon_j)(\tau_1-\hat{\tau}_1) \\
&\quad +\sum_{j=0}^{G-1}\sqrt{P_1}\sin(\phi_1-\hat{\phi}_1)\xi_j(T_c-(\tau_1-\hat{\tau}_1)), \qquad (2.103)
\end{aligned}
$$

and

$$
\begin{aligned}
M &= \sum_{k=2}^{K}\sqrt{P_k}\cos(\phi_k)\left((T_c-\tau_k)\sum_{m=0}^{G-1}\tilde{v}_{k,m}+\tau_k\sum_{m=0}^{G-1}\tilde{\gamma}_{k,m}\right) \\
&\quad +\sum_{k=2}^{K}\sqrt{P_k}\sin(\phi_k)\left((T_c-\tau_k)\sum_{m=0}^{G-1}\hat{v}_{k,m}+\tau_k\sum_{m=0}^{G-1}\hat{\gamma}_{k,m}\right), \qquad (2.104)
\end{aligned}
$$

with N as a zero mean Gaussian random variable with a variance of $2N_0T$.

2.2 DS CDMA System in Multipath Fading Channels

In this thesis, DS CDMA systems in multipath fading channels are studied as well as systems in AWGN channels. The DS CDMA system model described for the simple AWGN channel in the previous section changes slightly for the multipath fading case. This system is briefly described in this section. To facilitate a better understanding of the behaviour of DS CDMA systems in multipath fading channels, an overview of the notions of fading, frequency selectivity and diversity combining are given in this section as well.

2.2.1 Fading Channel Issues

2.2.1.1 Multipath Fading and Frequency Selectivity

The constantly changing physical characteristics of the mobile medium and the mobility of the user result in a time-variant mobile radio channel. Wave propagation in the mobile channel is characterized by three aspects: path loss, multipath and fading.

The path loss is an average propagation loss over wide areas. It is determined by macroscopic parameters, such as the distance between the transmitter and the receiver, the carrier frequency and the land profile. The simplest formula for path loss is,

$$L_P = Ar^{-\alpha} \tag{2.105}$$

where A and α are propagation constants that depend on the carrier frequency and the land profile and r is the distance between the transmitter and the receiver. In (2.105), L_P is defined as the loss in the signal power in dB as it is transferred through the mobile channel. More complicated formulae have been proposed in the literature [6]. The path loss effects have not been taken into account in this thesis.

In the time-varying mobile radio channel, the energy of a given transmitted signal arrives via several paths as a result of reflections or of inhomogeneities in the physical medium. For this reason, the mobile radio channel is a multipath channel. The random signals that propagate through different signal paths from the transmitter are superposed at the receiver to produce standing waves. When a receiver and/or a transmitter moves in the standing waves, the receiver experiences random variation in the signal level and in the phase, and also a Doppler shift.

The time-variant impulse responses of multipath channels are consequences of the constantly changing physical characteristics of the media. If we transmit a pulse signal over a time varying multipath channel, the received signal might appear as a train of pulses. Hence, one characteristic of a multipath medium is the time spread introduced in the signal which is transmitted through the channel. The difference between the maximum and minimum values of the time delay for which the average received signal power is essentially nonzero is called the multipath spread and its inverse characterizes the coherence bandwidth of the channel. The practical significance of the coherence bandwidth is primal since it describes the frequency selectivity characteristics of the channel. Two sinusoids with a frequency separation greater than the coherence bandwidth of the channel are affected differently by the multipath channel due to its time spreading characteristics. This in turn causes linear distortion of the received signal. If the bandwidth of the transmitted signal is smaller than or equal to the coherence bandwidth of the channel, all of the signal components undergo the same attenuation and phase shift by the channel. The channel for this specific signal is said to be frequency-nonselective. In this case, the multipath components of the received signal are not resolvable. If, on the other hand, the signal bandwidth is larger than the channel coherence bandwidth, different signal components are affected differently by the channel. Such channels are described as frequency-selective for the given information signal. In this case, the multipath components in the received signal are resolvable with a resolution in the time delay that is equal to the symbol duration

[176].

A third characteristic of the mobile channel is due to the time variations in the structure of the medium. When a steady-state, radio frequency wave is transmitted over a long path, the envelope of the received signal is observed to fluctuate in time as a result of the constantly changing physical characteristics of the media. This is called fading. There are two kinds of fading that are encountered in the mobile radio channel, namely, short-term fading and long-term fading. Short-term fading represents the variation of the received signal envelope around its mean. Long-term fading, also termed as shadow fading, on the other hand, corresponds to the variation of the mean itself. Many experiments have shown that the long term fading obeys the log-normal distribution.

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\{(x - \mu_x)^2/(2\sigma_x^2)\} \tag{2.106}$$

where x is the received power in dB, μ_x and σ_x are the mean and the variance in dB.

In the short-term fading channel, if the transmission of a given signal is repeated a number of times, certain changes in the received signal pulse train could be observed. Such changes might include changes in the sizes of the individual pulses, changes in the relative delays among the pulses and quite often, changes in the number of pulses observed in the received pulse train [176]. The time variations in the channel result in a Doppler broadening. The range of Doppler frequencies for which the average received signal power is nonzero is called the Doppler spread of the channel. The Doppler frequency for the n'th multipath component of the received signal can be calculated as

$$w_n = \beta v \cos \alpha_n \tag{2.107}$$

where v is the effective velocity of the medium (can be due to a moving transmitter, moving receiver, moving obstacles, or combinations thereof), α_n is the incident angle of the n'th multipath to the direction of the movement as shown in Figure 2.7 and β

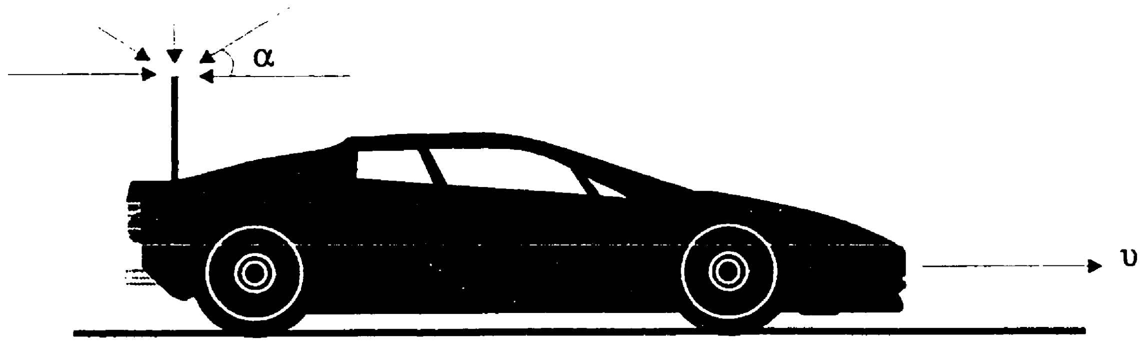

Figure 2.7: Incident Angles of the Multipaths to the Direction of the Movement

is the propagation constant expressed as,

$$\beta = \frac{2\pi}{\lambda} = \frac{w_c}{c} \tag{2.108}$$

where λ is the wavelength, w_c is the carrier frequency and c is the speed of light. The Doppler spread is then $[-\beta v, +\beta v]$. The inverse of the Doppler spread characterizes the coherence time. Like coherence bandwidth, coherence time is an important quantity for the mobile channel. Practically, if the information symbol duration is smaller than the coherence time of the channel of interest, the channel may be assumed to be time invariant for the duration of one symbol. Such channels are often referred to as slow fading channels. On the other hand, if the information symbol duration is larger than the channel coherence time, the channel characteristics can no longer be assumed to remain constant during one symbol duration. Such channels are referred to as fast fading channels. The fast fading within one symbol is often modeled as correlated [196]. In this thesis we only consider slow fading. A typical multipath fading scenario is given in Figure 2.8. As seen from the figure, it is possible to receive energy from a direct path from the transmitter as well as reflected paths. The direct path, if exists, is often referred to as the line-of-sight. Continuous physical changes

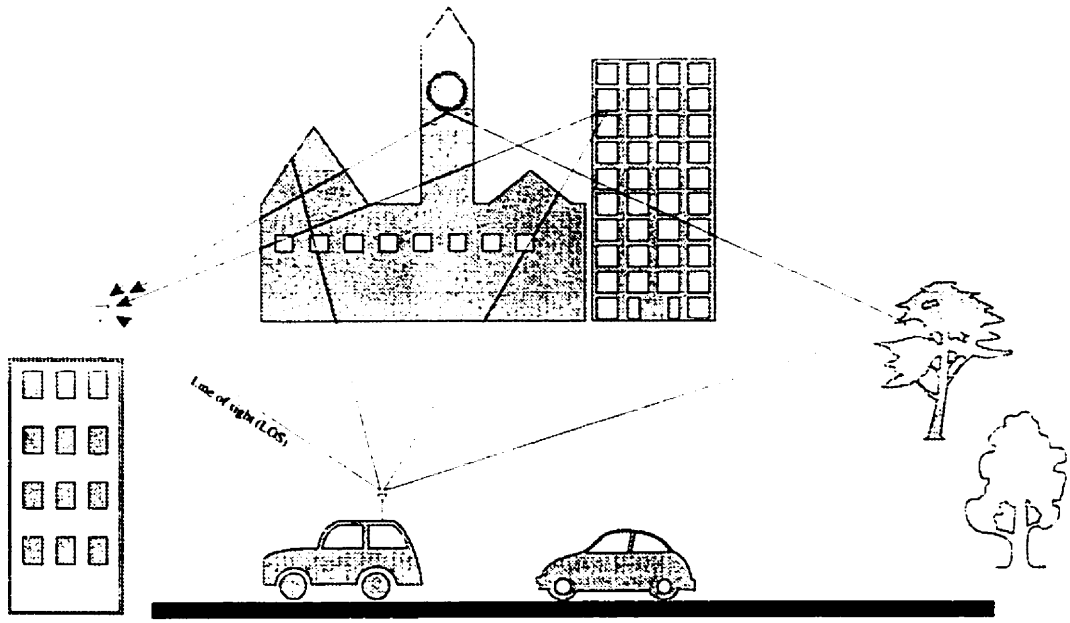

Figure 2.8: Multipath Fading Environment

in the channel cause small changes in the individual path lengths, but these may nevertheless equate to large phase changes for the radio frequency. It is the variation of the phase amongst the various paths that may ultimately change the polarity of the transmitted signals and thus may result in the interference to be constructive or destructive [208]. The combination of the constructive and the destructive signal components from different paths effectively cause multipath fading.

When the line-of-sight does not exist, even with only a few paths of roughly equal intensity contributing, Central Limit Theorem[1] arguments lead to the conclusion that the received waveform has all the characteristics of a very narrow band stationary Gaussian noise. It consists of Gaussian quadrature components characterized by a power spectral density of nonzero width, and with a corresponding Rayleigh distribution of the received envelope. Fading that fits to this model is called Rayleigh fading

[1]The Central Limit Theorem is described in Chapter 3 on page 110.

[176]. For systems, where a line-of-sight path exists, the envelope of the received signal has a Ricean distribution. Appropriately, such fading is referred to as Ricean fading. Intuitively, Ricean fading represents the best case scenario for mobile transmission whereas Rayleigh fading represents the worst case. Some researchers have suggested that envelope statistics on certain fading channels fit neither Rayleigh nor Ricean distributions but rather a Nakagami-m distribution where the special case of $m = 1$ gives the Rayleigh distribution [208]. Only Rayleigh fading is considered in this thesis.

While the physical channel most likely consists of paths that have considerable persistence and changing delay, the mathematical model for slow fading that is used more often is one of fixed delay with varying gain and phase offset at each delay. It is assumed that there is a continuum of multiple propagation paths between the transmitter and the receiver. Associated with each path is a propagation delay and an attenuation factor. Both of these factors are time varying as a result of the changes in the structure of the medium. The time varying impulse response of the fading channel can be expressed as.

$$h_k(\tau;t) = \sum_{l=-\infty}^{\infty} h_{k,l}(t)\delta(\tau - t_{kl}(t)). \tag{2.109}$$

In (2.109), the tap gains $h_{k,n}(t)$ are complex Gaussian random variables. When a wide-sense stationary channel with uncorrelated scattering is considered, the $h_{k,n}(t)$ can be assumed to be uncorrelated and, because they are Gaussian, also independent.

For a slowly fading channel, $h_{k,l}(t) = h_{k,l}$ and $t_{kl}(t) = t_{kl}$ during at least the entire duration of one symbol. Using the basic definition of frequency selectivity it can be stated that a time separation of at least T_c is necessary between two consecutive multipaths, $|t_{kl} - t_{km}|, \forall l \neq m$, for them to be resolvable [237]. Any two paths with path time delays less than T_c apart can be considered to give rise to the same path gain. Then, for a multipath spread of T_m, (2.109) can be truncated to have L terms with $L = \left\lfloor \frac{T_m}{T_c} \right\rfloor + 1$. Here $\lfloor x \rfloor$ denotes the largest integer that is less than or equal to x. To keep the analysis clear, we assume that the inequality $T_m < T$ is satisfied, thereby placing a ceiling on the data transmission rate [266].

Since $h_{k,l}$ are complex Gaussian random variables, the slowly fading, frequency selective channel model can equivalently be written as,

$$h_k(t) = \sum_{l=1}^{L} \beta_{kl} \exp\{j\vartheta_{kl}\}\delta(t - t_{kl}) \tag{2.110}$$

where β_{kl} is the path gain for the l'th path, ϑ_{kl} and t_{kl} are the phase delay and time delay terms for the l'th path, respectively.

When there is no line-of-sight, β_{kl} is a Rayleigh random variable and has a probability density function in the form,

$$f_{\beta_{kl}}(x) = \frac{x}{\rho_0} \exp\left(-\frac{x^2}{2\rho_0}\right) u(x) \tag{2.111}$$

where $u(x)$ is the unit step function. Here, $\mathrm{E}\{\beta_{kl}^2\} = 2\rho_0$. In (2.110), one should assume that $T_c < |t_{kl} - t_{km}| < T_m, \forall l \neq m$ [98]. Finally, ϑ_{kl} can be assumed to be uniform in $[0, 2\pi]$.

2.2.1.2 *Diversity Combining*

Even though frequency selective channels cause severe interference [176], having two or more closely similar copies of the desired signal at the receiver may sometimes be useful if appropriate receiver structures are used. Systems that are designed to utilize such scenarios are called diversity systems. Diversity systems are based on the observation that when two or more radio channels are sufficiently separated in space, frequency, time or polarization, the fading on each of these channels is more or less independent. Therefore, it is quite unlikely for all of the channels to fade simultaneously [19]. The objective of the diversity systems is to increase the realized signal-to-noise ratio at all times by making use of all of the signal components received from independent paths.

Frequency selectivity is not the only means to achieve multiple copies of the desired signal at the receiver. Artificial ways such as employing multiple antennas at the transmitter and/or receiver. transmitting the same signal on two or more carrier frequencies that are sufficiently separated etc. result in diversity systems as well. With CDMA techniques. the resolvable paths can be demodulated individually by a RAKE receiver [175] which exploits the excess redundancy due to the presence of independent channel outputs from the inherent and/or artificial multipaths. In a RAKE receiver. information obtained from each branch is combined in a certain way to minimize the interference and further mitigate the fading [77]. This is called diversity combining. Diversity receivers may be classified as either predetection or postdetection. In predetection diversity, signals from different branches are combined prior to detection or demodulation. whereas in postdetection diversity. the signals are first demodulated and then combined. In general. predetection combining will perform as well or better than postdetection combining [195].

Both linear and nonlinear combining schemes are possible. For linear demodulation. predetection and postdetection combining result in the same performance. For nonlinear demodulation, however, the postdetection combining will perform at best the same as predetection combining [195].

In general. in a diversity system, the receiver has one or more copies of the desired signal. For an L branch diversity system. the output of the diversity combining receiver can be written as [208].

$$f(t) = \sum_{i=1}^{L} a_i f_i(t) \tag{2.112}$$

where $f_i(t)$ is the i'th received replica of the desired signal corrupted by noise and interference and a_i is a weighting factor that is dependent on the type of the diversity combining scheme in use. There are various ways of performing diversity combining. Three of the most popular schemes used in coherent systems are considered in this thesis. These schemes are briefly described below. Further mathematical descriptions for the schemes are given Chapter 5 when their performance evaluations are performed using the analysis scheme developed in this thesis.

2.2.1.2.1 Selection Diversity

Upon receiving L copies of the desired signal, the selection diversity based RAKE receiver simply selects the one with the largest signal-to-noise ratio (SNR) and uses that specific component in the decision making process [19]. Other copies are discarded; they have no effect on the outcome of the demodulation. Thus the weighting factor, a_i, in (2.112) is given as,

$$a_i = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \tag{2.113}$$

where $\text{SNR}_j \geq \text{SNR}_i, \forall i$.

A sub-optimal selection diversity scheme is also described in the literature. Here, the branch with the largest received signal power is selected rather than looking at the SNR [87].

2.2.1.2.2 Equal Gain Combining

In the equal gain combining based RAKE receiver, all of the L copies of the received signal are time and phase synchronized and then added together. The weighting factor, a_i in (2.112) can be written as,

$$a_i = 1 \tag{2.114}$$

$\forall i$. Thus, all of the branches are taken into account with equal weight in the decision making.

2.2.1.2.3 Maximal Ratio Combining

Maximal ratio combining is optimal amongst linear diversity combining schemes in the sense that it achieves the maximum signal-to-noise ratio of the output signal [19]. Like equal gain combining, the ideal maximal ratio combining based RAKE receiver must align the time and phase offsets of the L incoming copies of the desired signal to perform perfect combining [225]. Additionally, the magnitudes of the L incoming copies of the desired signal are needed for maximal ratio combining. Once synchronized, the signal components from each multipath are weighted by a factor that is proportional to their corresponding SNR. Therefore, for maximal ratio combining, the weighting factor, a_i in (2.112) can be written as.

$$a_i = \frac{s_i}{\mathrm{E}\{n_i^2\}} \tag{2.115}$$

where s_i and $\mathrm{E}\{n_i^2\}$ are the RMS values of the i'th branch signal and mean-square noise power, respectively [19, 87]. The weighted signals from the branches are then added together.

The optimal performance for the maximal ratio receiver is hard to realize since the associated weights of the diversity branches are difficult to estimate [87].

2.2.2 DS CDMA System Model in Multipath Fading Channel

The DS CDMA system in the slowly fading multipath channel is in general similar to the one in the simple AWGN channel. Only the differences between the two system models are described here.

As in the AWGN case, the DS CDMA system in question is given in Figure 2.1. Once again, we assume there are K users present in the system. Every user transmits a signal in the form given in (2.1). This time though, the channel is a multipath, slowly fading channel that has a transfer function in the form,

$$h_k(t) = \sum_{l=1}^{L} \beta_{kl} \exp\{j\vartheta_{kl}\}\delta(t - t_{kl}) \tag{2.116}$$

where L is the number of resolvable multipaths in the system, β_{kl} is the path gain for the l'th path of the k'th user's signal, ϑ_{kl} and t_{kl} are the phase and time offsets introduced by the multipath channel on the l'th path of the k'th user's signal. β_{kl} is Rayleigh distributed with the probability density function given in (2.111). Each path's relative time delay, t_{kl}, satisfies $T_c < |t_{kl} - t_{km}| < T_m, \forall l \neq m$ [98]. The phase offsets, ϑ_{kl}, are assumed to be uniformly distributed in the region $[0, 2\pi]$. Note that the frequency-nonselective channel is a special case of the fading channel described by (2.116) with $L = 1$.

The total received signal can be written as,

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} \int_{-\infty}^{\infty} h_k(\varphi) s_k(t - \varphi) d\varphi + n(t) \\ &= \sum_{k=1}^{K} \sum_{l=1}^{L} \sqrt{2P_k} \beta_{kl} b_k(t - T_k - t_{kl}) a_k(t - T_k - t_{kl}) \cos(\omega_c t + \theta_k + \vartheta_{kl} - \omega_c \tau_{kl}) \\ &\quad + n(t) \end{aligned} \tag{2.117}$$

where $n(t)$ is the additive white Gaussian noise (AWGN) introduced by the channel. The net time delay, τ_{kl}, and the net phase offset, ϕ_{kl}, are obtained by summing their respective transmitter and channel parts such that,

$$\tau_{kl} = T_k + t_{kl}, \tag{2.118}$$

$$\phi_{kl} = \theta_k + \vartheta_{kl} - \omega_c \tau_{kl}. \tag{2.119}$$

It is easy to see that the initial message starting times of the K signals, T_k, are only modulo T significant. Therefore, following [165], we model τ_{kl} to be iid random variables, uniformly distributed over $[0, T + T_m]$. Similarly, we model ϕ_{kl} to be iid random variables, uniformly distributed over $[0, 2\pi]$.

As before, we want to capture the signal from the first user, namely, $a_1(t)$. For this purpose, the received signal goes through a RAKE receiver where the received signal is first despread independently for each multipath component by multiplying the signal with the spreading code of the first user delayed by an amount equal to the delay of the multipath component. At each RAKE finger, the signal is then stripped off its carrier and then passed through a correlator. Finally, the outcome of the correlators in all of the fingers are combined to achieve diversity combining. A typical RAKE receiver is illustrated in Figure 2.9. The figure displays the sequence of operations in each RAKE finger in the reverse order. We state once again that, from a theoretical standpoint, these two modes of operation are identical, but in practice the despreading operation is always performed first.

Similar to the system in the AWGN channel, the chip timing error and the carrier phase error for the j'th path in the RAKE receiver will be, $(\tau_{1j} - \hat{\tau}_{1j})$ and $(\phi_{1j} - \hat{\phi}_{1j})$, respectively.

For the analysis, we consider the data symbol interval of $[0, T]$ without any loss of generality. In this case, from Figure 2.9, the input to the decision device from the j'th path is,

$$
\begin{aligned}
Z_{1j} &= 2\int_{\hat{\tau}_{1j}}^{T+\hat{\tau}_{1j}} b_1(t-\hat{\tau}_{1j})r(t)\cos(\omega_c t + \hat{\phi}_{1j})dt \\
&= D_{1j} + I_{1j} + M_{1j} + N_{1j}
\end{aligned} \tag{2.120}
$$

where the individual terms can be expressed as,

$$
\begin{aligned}
D_{1j} &= \sqrt{2P_1}\beta_{1j}\cos(\phi_{1j} - \hat{\phi}_{1j}) \\
&\quad \cdot \int_{\hat{\tau}_{1j}}^{T+\hat{\tau}_{1j}} b_1(t-\tau_{1j})a_1(t-\tau_{1j})b_1(t-\hat{\tau}_{1j})dt
\end{aligned} \tag{2.121}
$$

$$
\begin{aligned}
I_{1j} &= \sum_{\substack{l=1 \\ l\neq j}}^{L} \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l} - \hat{\phi}_{1j}) \\
&\quad \cdot \int_{\hat{\tau}_{1j}}^{T+\hat{\tau}_{1j}} b_1(t-\tau_{1l})a_1(t-\tau_{1l})b_1(t-\hat{\tau}_{1j})dt
\end{aligned} \tag{2.122}
$$

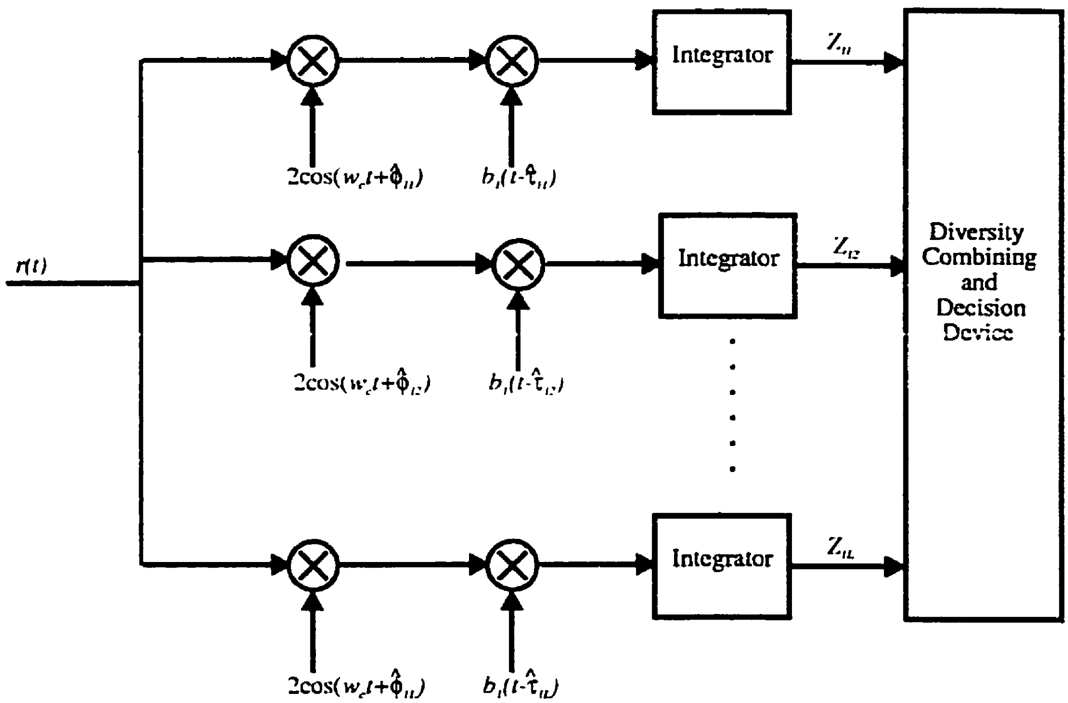

Figure 2.9: L-Branch Coherent RAKE Receiver for the Multipath Fading Channel

$$\begin{aligned} M_{1_j} &= \sum_{k=2}^{K}\sum_{l=1}^{L}\sqrt{2P_1}\beta_{kl}\cos(\varphi_{kl}-\hat{\phi}_{1_j}) \\ &\quad\cdot\int_{\hat{\tau}_{1_j}}^{T+\hat{\tau}_{1_j}} b_k(t-\tau_{kl})a_k(t-\tau_{kl})b_1(t-\hat{\tau}_{1_j})dt \end{aligned} \tag{2.123}$$

and

$$N_{1_j} = 2\int_{\hat{\tau}_{1_j}}^{T+\hat{\tau}_{1_j}} n(t)b_1(t-\hat{\tau}_{1_j})\cos(\omega_c t)dt \tag{2.124}$$

In (2.120)-(2.124), D_{1_j} is the term that consists of the desired signal plus self interference caused by synchronization errors. I_{1_j} is the intersymbol interference caused by the multipath. M_{1_j} is the multiple access interference and N_{1_j} is the additive white Gaussian noise term. We now simplify these terms. We start off with D_{1_j}.

2.2.2.1 *Desired Signal Plus Self Interference*

Using (2.2), (2.121) can be rewritten as,

$$D_{1_j} = \sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,-1}\int_0^{\tau_{1_j}-\hat{\tau}_{1_j}} b_1(t)b_1(t-(\tau_{1_j}-\hat{\tau}_{1_j}))dt$$
$$+\sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,0}\int_{\tau_{1_j}-\hat{\tau}_{1_j}}^{T} b_1(t)b_1(t-(\tau_{1_j}-\hat{\tau}_{1_j}))dt. \quad (2.125)$$

Now, for random PN sequences it is possible to further simplify (2.125). By making use of (2.3) and the fact that $(\tau_{1_j} - \hat{\tau}_{1_j}) < T_c$ in order for the spread spectrum system to operate, we get,

$$D_{1_j} = \sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,-1}b_{1,0}b_{1,-1}R_\psi(\tau_{1_j} - \hat{\tau}_{1_j})$$
$$+\sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,0}\sum_{m=0}^{G-1}\hat{R}_\psi(\tau_{1_j} - \hat{\tau}_{1_j})$$
$$+\sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,0}\sum_{m=1}^{G-1}b_{1,m}b_{1,(m-1)}R_\psi(\tau_{1_j} - \hat{\tau}_{1_j}). \quad (2.126)$$

For rectangular shaped chips, using (2.15) and (2.16), (2.126) becomes,

$$D_{1_j} = \sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,-1}b_{1,0}b_{1,-1}(\tau_{1_j} - \hat{\tau}_{1_j})$$
$$+\sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,0}G(T_c - (\tau_{1_j} - \hat{\tau}_{1_j}))$$
$$+\sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1_j})a_{1,0}\sum_{m=1}^{G-1}b_{1,m}b_{1,(m-1)}(\tau_{1_j} - \hat{\tau}_{1_j}). \quad (2.127)$$

As in the AWGN channel case, we define the following random variables,

$$\alpha_0 \triangleq a_{1,-1}b_{1,0}b_{1,-1}, \quad (2.128)$$

$$\alpha_i \triangleq a_{1,0}b_{1,i}b_{1,(i-1)}, \qquad i = 1, 2, \ldots, G-1. \quad (2.129)$$

Following the lemma given on page 35, $\alpha_i, i = 1, \ldots, (G-1)$ are iid random variables taking on values $\{\mp 1\}$ randomly. Thus (2.127) can be rewritten as,

$$D_{1_j} = D_{1_{ja}} + D_{1_{jb}} \quad (2.130)$$

where

$$D_{1_{ja}} = \sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1j})G(T_c - (\tau_{1_j} - \hat{\tau}_{1j}))a_{1,0} \quad (2.131)$$

and

$$D_{1_{jb}} = \sqrt{2P_1}\beta_{1_j}\cos(\phi_{1_j} - \hat{\phi}_{1j})(\tau_{1_j} - \hat{\tau}_{1j})\sum_{m=0}^{G-1}\alpha_m. \quad (2.132)$$

Here, $D_{1_{ja}}$ is the desired signal term and $D_{1_{jb}}$ is the self interference term caused by the non-zero chip timing error. Note that if the system is free of synchronization errors,

$$D_{1_{ja}} = \sqrt{2P_1}\beta_{1_j}Ta_{1,0}. \quad (2.133)$$

$$D_{1_{jb}} = 0. \quad (2.134)$$

2.2.2.2 Intersymbol Interference

Now, we simplify the intersymbol interference term, I_{1_j}, defined in (2.122). Once again, using (2.2), (2.122) can be rewritten as,

$$\begin{aligned} I_{1_j} &= \sum_{\substack{l=1 \\ l\neq j}}^{L}\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})\left[a_{1,-1}\int_0^{\tau_{1l}} b_1(t)b_1(t-\tau_{1l})dt\right. \\ &\left. +a_{1,0}\int_{\tau_{1l}}^{T} b_1(t)b_1(t-\tau_{1l})dt\right] \end{aligned} \quad (2.135)$$

As stated before, in the multipath fading channel, unlike in the AWGN case, τ_{1l}, $l = 1,\ldots,L$, is uniformly distributed in the interval $[0,T]$ while maintaining the inequality, $T_c < |\tau_{1l} - \tau_{1m}| < T_m, \forall l \neq m$ where T_m is the maximum delay spread of the channel [98, 266]. In this case, relative to the desired signal term, the intersymbol interference terms will always observe at least one chip offset.

We then define,

$$\Delta t_{1l} \triangleq \tau_{1l} \mod T_c \quad (2.136)$$

and

$$R \triangleq \frac{\tau_{1l} - \Delta t_{1l}}{T_c}. \quad (2.137)$$

Since τ_{1l} of the intersymbol interference terms are uniformly distributed over $[T_m, T_c]$, Δt_{1l} are random variables that have probability density functions (pdfs) that are dependent on the specific value of T_m. If T_m is an integer multiple of the chip duration, T_c, say $T_m = kT_c$, Δt_{1l} are uniform on $[0, T_c]$. Otherwise, T_m can be written as, $T_m \mod T_c = \epsilon$. Then, the pdfs of Δt_{1l} are in the form,

$$f_{\Delta t}(x) = \begin{cases} \frac{k}{(k-1)T_c+\epsilon}, & 0 < x < \epsilon \\ \frac{k-1}{(k-1)T_c+\epsilon}, & \epsilon < x < T_c \\ 0, & \text{elsewhere} \end{cases} \tag{2.138}$$

as shown in Figure 2.9. When T_m is not an integer multiple of T_c, the deviation from the uniform distribution will be very small since $T_m \gg T_c$, and hence, for our purposes, we can assume that Δt_{1l} are uniformly distributed over $[0, T_c]$.

We also define two random variables

$$\underline{\xi}_{li} \triangleq b_{1,i} b_{1,x} \tag{2.139}$$

$$\underline{\gamma}_{li} \triangleq b_{1,i} b_{1,(x-1)} \tag{2.140}$$

where x is an integer number that is dependent on the value of τ_{1l}. Provided that $x \neq i$, regardless of the value of x, $\underline{\xi}_{li}$ and $\underline{\gamma}_{li}$ are iid random variables taking on the values $\{\mp 1\}$ randomly. This observation is due to the lemma on page 35. Then,

$$\begin{aligned} I_{1j} = & \sum_{\substack{l=1 \\ l\neq j}}^{L} \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l}) a_{1,\mu} \sum_{i=0}^{R-1} \left[\underline{\gamma}_{li}\Delta t_{1l} + \underline{\xi}_{li}(T_c - \Delta t_{1l})\right] \\ & + \sum_{\substack{l=1 \\ l\neq j}}^{L} \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l}) a_{1,\mu}\underline{\gamma}_{lR}\Delta t_{1l} \\ & + \sum_{\substack{l=1 \\ l\neq j}}^{L} \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l}) a_{1,0}\underline{\xi}_{lR}(T_c - \Delta t_{1l}) \\ & + \sum_{\substack{l=1 \\ l\neq j}}^{L} \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l}) a_{1,0} \sum_{i=R+1}^{G-1} \left[\underline{\gamma}_{li}\Delta t_{1l} + \underline{\xi}_{li}(T_c - \Delta t_{1l})\right] \end{aligned} \tag{2.141}$$

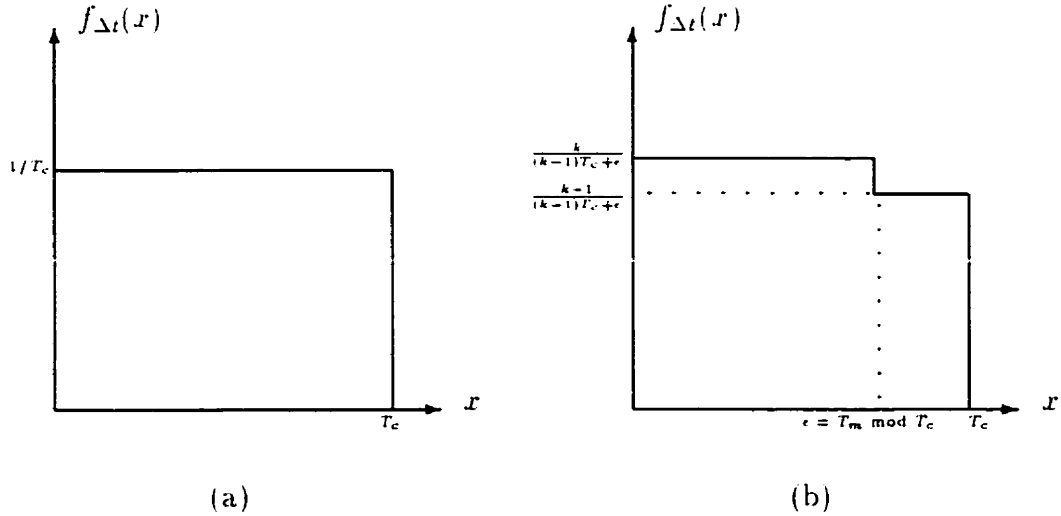

Figure 2.10: The probability density function of Δt_{1l} when (a) T_m is an integer multiple of T_c (b) T_m is not an integer multiple of T_c

We now define the following random variables.

$$\Xi_{li} \triangleq \begin{cases} a_{1,-1}\xi_{li} & i = 0,\ldots,R \\ a_{1,0}\xi_{li} & i = R+1,\ldots,G-1 \end{cases} \tag{2.142}$$

$$\varpi_{li} \triangleq \begin{cases} a_{1,-1}\gamma_{li} & i = 0,\ldots,R-1 \\ a_{1,0}\gamma_{li} & i = R,\ldots,G-1 \end{cases} \tag{2.143}$$

Once again, the lemma on page 35 can be used to show that Ξ_{li} and ϖ_{li} are iid random variables that take on values $\{\mp 1\}$ randomly. Then,

$$I_{1j} = \sum_{\substack{l=1 \\ l\neq j}}^{L} \sum_{i=0}^{G-1} \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})\left[\Delta t_{1l}\Xi_{li} + (T_c - \Delta t_{1l})\varpi_{li}\right] \tag{2.144}$$

2.2.2.3 Multiple Access Interference

Now, we simplify the multiple access interference term, M_{1j}, defined in (2.123). As before, for all practical reasons, all of the information signals that cause interference may be considered to be random and thus be embedded into the long PN sequences. Thus we define,

$$q_k(t) \triangleq a_k(t)b_k(t) = \sum_{i=-\infty}^{\infty} q_{k,i}\Psi(t - iT_c) \tag{2.145}$$

where, for our purposes, $q_{k,i}$ is a set of random variables that randomly take on the values $\{\mp 1\}$. Then, without any loss of generality, the random variables τ_{kl} can be assumed to be iid and uniformly distributed over the interval $[0, T_c]$. Substituting (2.145) into (2.123) we get,

$$\begin{aligned} M_{1_j} &= \sum_{k=2}^{K}\sum_{l=1}^{L}\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\sum_{j=0}^{G-1}\int_{jT_c}^{(j+1)T_c} b_{1,j} \\ &\quad \cdot \sum_{i=-\infty}^{\infty} q_{k,i}\Psi(t - jT_c)\Psi(t - iT_c - \tau_{kl})dt \\ &= \sum_{k=2}^{K}\sum_{l=1}^{L}\sum_{j=0}^{G-1}\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\left[b_{1,j}q_{k,(j-1)}\tau_{kl} + b_{1,j}q_{k,j}(T_c - \tau_{kl})\right]. \end{aligned} \tag{2.146}$$

We now define the following random variables,

$$\eta_{k_j} \triangleq b_{1,j}q_{k,(j-1)} \tag{2.147}$$

$$\kappa_{k_j} \triangleq b_{1,j}q_{k,j} \tag{2.148}$$

Once again, η_{k_j} and κ_{k_j} are iid random variables that take on values $\{\mp 1\}$ randomly. Thus,

$$M_{1_j} = \sum_{k=2}^{K}\sum_{l=1}^{L}\sum_{j=0}^{G-1}\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\left[\eta_{k_j}\tau_{kl} + \kappa_{k_j}(T_c - \tau_{kl})\right] \tag{2.149}$$

2.2.2.4 Channel Noise

Finally, the channel noise term, N_{1_j} is a Gaussian random variable with zero mean and N_0T variance as was the case with the AWGN channel [166].

2.2.2.5 Summary

In summary, the input signal to the decision device from the j'th path of the receiver is

$$Z_{1j} = D_{1ja} + D_{1jb} + I_{1j} + M_{1j} + N_{1j} \quad (2.150)$$

where D_{1ja} is the desired signal, D_{1jb} is the self interference due to chip timing errors, I_{1j} is the intersymbol interference due to the multipath, M_{1j} is the multiple access interference and N_{1j} is the channel noise terms. These terms are defined as,

$$D_{1ja} = \sqrt{2P_1}\beta_{1j}\cos(\phi_{1j} - \hat{\phi}_{1j})G(T_c - (\tau_{1j} - \hat{\tau}_{1j}))a_{1,0}, \quad (2.151)$$

$$D_{1jb} = \sqrt{2P_1}\beta_{1j}\cos(\phi_{1j} - \hat{\phi}_{1j})G(\tau_{1j} - \hat{\tau}_{1j})\sum_{m=0}^{G-1}\alpha_m, \quad (2.152)$$

$$I_{1j} = \sum_{\substack{l=1 \\ l\neq j}}^{L}\sum_{i=0}^{G-1}\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})\left[\Delta t_{1l}\Xi_{li} + (T_c - \Delta t_{1l})\xi_{li}\right], \quad (2.153)$$

$$M_{1j} = \sum_{k=2}^{K}\sum_{l=1}^{L}\sum_{j=0}^{G-1}\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\left[\eta_{kj}\tau_{kl} + \kappa_{kj}(T_c - \tau_{kl})\right], \quad (2.154)$$

and N_{1j} is the AWGN term with zero mean and N_0T variance.

Once all Z_{1j}'s are obtained, diversity combining is performed in the receiver and the decision variable Z is formed. Three diversity combining schemes, namely, selection diversity, maximal ratio diversity combining and equal gain diversity combining are considered in this thesis. The mathematical details of how Z is formed for each case are given in Chapter 5.

2.3 The TIA/EIA-95 Standard

As recently as 1985, a straightforward comparison of the capacity of CDMA to that of TDMA and FDMA for satellite applications suggested a reasonable edge for the latter two techniques [242]. Later, it was realized that by making use of some of the characteristics of CDMA, the system capacity could be increased at least to the level of the TDMA and FDMA capacity [68]. The main factor in achieving the capacity increase for the CDMA system was the realization of the fact that CDMA is interference limited whereas TDMA and FDMA are both bandwidth limited. Thus, any reduction in the interference would result in a direct increase in the CDMA system capacity. A number of techniques were suggested in [68, 69] to reduce the multiple access interference in CDMA based systems among which voice activity detection and spatial isolation were claimed to be sufficient to increase the system capacity to the level that was at least double that of the TDMA and FDMA based systems [69]. Later, actual field tests confirmed this theoretical edge of the use of CDMA over other multiple access schemes [39].

In a typical full duplex two-way communication, the duty cycle of each voice transmission is approximately 37% [250]. With a CDMA based system, it is possible to reduce the transmission rate when there is no speech, and thereby substantially reduce interference to other users. This in return, would approximately increase the system capacity by a factor of 8/3 [69]. It is more difficult to exploit the voice activity factor in either FDMA or TDMA systems because of the time delay associated with reassigning the channel resources during the speech pauses.

Similarly, with any spatial isolation through the use of directional cell antennas (e.g., the typical 120° sector antennas) the interference seen is simply divided by three because, on average, a specific antenna receives only in the direction of one third of the mobile stations. The capacity supportable by the total system is therefore increased by a factor of almost three. Accounting for side lobes, this is approximately 85% efficient [182], resulting in an efficient capacity gain of 2.55.

In 1992 QUALCOMM Inc. submitted a DS CDMA based common air interface standard proposal to the Telecommunications Industry Association (TIA) to provide a CDMA based PCS offering that uses voice activity detection and spatial isolation to support a wide range of service requirements in the 900 MHz band [182, 189].

This proposal was later adopted by the TIA as the TIA/EIA-95 standard under the name "Wideband Spread Spectrum Digital Cellular System Dual-Mode Mobile Station-Base Station Compatibility Standard " [226][2]. Inherent from its name, the TIA/EIA-95 standard proposes a dual-mode operation where both analog AMPS and digital CDMA are supported. The type of system the mobile station operates with depends on the availability of either system in the geographic area of the mobile station as well as its preference.

TIA/EIA-95 allows each user to use the same radio channel. The use of CDMA completely eliminates the need for frequency planning within a market. To facilitate a graceful transition from AMPS to digital CDMA, TIA/EIA-95 occupies 1.25 MHz of spectrum on each one way link, or 10% of the available cellular spectrum for a U.S. cellular provider. In practice, AMPS carriers must provide a 270 kHz guard band on each side of the spectrum dedicated for TIA/EIA-95 [226].

Unlike the other cellular standards, in TIA/EIA-95, the user data rate changes in real time to utilize the voice activity. The original speech coder in the TIA/EIA-95 standard is the 9600 bps Code Excited Linear Predictive (QCELP) coder[3]. This vocoder has the capability to detect voice activity and reduce the data rate to 1200 bps during silent periods. Intermediate data rates of 2400, 4800 and 9600 bps are also used for special purposes [226]. Therefore, initially a maximum data transmission rate of 9600 bps is supported by the TIA/EIA-95 standard. With a fully operational multimedia PCS, some of the available services might require higher transmission rates. For this reason, the third generation evolution path of TIA/EIA-95 is envisioned to provide wider band services at 2.5 MHz and 5 MHz [46]. TIA/EIA-95 provides for a portable subscriber terminal that can be used in a variety of different environments (e.g. at home, office, on the street and at different speeds).

[2]The TIA/EIA-95B release also supports service in the 2000 MHz band.
[3]A 14400 bps speech coder is now part of the TIA/EIA-95-B as a second option

The coverage area of the wireless system is partitioned into cells, each having its own base station. The digital operation of the TIA/EIA-95 standard uses CDMA for both directions of transmission. In what follows, the transmission from base stations to a given subscriber is referred to as the "Forward CDMA Channel" or "Downlink Channel" and the transmission from the subscriber to the base stations is referred to as the "Reverse CDMA Channel" or "Uplink Channel". TIA/EIA-95 uses different modulation and spreading techniques for the forward and reverse links. On the forward link, the base station synchronously transmits the user data for all mobiles in the cell by using a different spreading sequence for each mobile. A pilot code is also transmitted simultaneously and at a higher power level to allow all mobiles to use coherent carrier detection while estimating the channel characteristics. On the reverse link, all mobiles respond in an asynchronous fashion and have ideally a constant signal level due to power control applied by the base station. Due to the characteristics of the reverse link, non-coherent detection is used at the base station. Both forward and reverse link characteristics are summarized below.

2.3.1 Forward CDMA Channel

The TIA/EIA-95 standard specifies a forward link CDMA waveform design that uses a combination of frequency division, pseudorandom code division, and orthogonal signal multiple access techniques in the 869-894 MHz as well as 1800-2200 MHz bands. Frequency division is employed by dividing the available cellular spectrum of 25 MHz into nominal 1.25 MHz bandwidth channels. Normally, a cellular system would be implemented in a service area within a single radio channel until demand requires employment of additional channels.

Pseudorandom noise binary codes are used to distinguish signals received at a mobile station from different base stations. All CDMA signals in the system share a quadrature pair of PN codes. Signals from different cells and sectors are distinguished by time offsets from the basic universal code. In other words, time offsets of the two universal PN sequences form the base station identification. The PN codes used are generated by maximal length shift registers that produce a code with a period of 32768 chips [226]. The PN chip rate is 1.2288 MHz, or exactly 128 times the maximum 9600 bps information transmission rate. Thus, the processing gain of the system is set at 128.

Signals are transmitted from a given base station synchronously and they share a common PN code phase. They are distinguished at the mobile station receiver by using a signal specific binary orthogonal code based on Walsh functions. The Walsh function in question is 64 PN chips long and represents 64 different orthogonal codes. This provides near perfect isolation between multiple signals transmitted by the same base station.

The information to be transmitted is convolutionally encoded to provide error correction and detection capabilities at the mobile receiver. The code used has a constraint length of nine and a code rate of one half [226]. The encoded symbols are interleaved to combat fast fading and error bursts. To provide communications privacy, each data channel is scrambled with a user addressed long PN sequence. After the TIA/EIA-95 standard was accepted, it was claimed that word interleaving as opposed to bit interleaving would result in a capacity increase of 1-2 dB [28, 29, 133]. Later, this result was shown to be misleading since in actuality the advantage of word interleaving over bit interleaving is significantly smaller and is limited only to the cases of slow fading [251]. Furthermore, the extensive memory requirements of the word interleaving scheme make it a highly impractical system.

Since there are 64 different Walsh functions and a different Walsh function has to be used for every different transmitted signal, a maximum of 64 different signals can be sent simultaneously in the forward CDMA channel as shown in Figure 2.11.

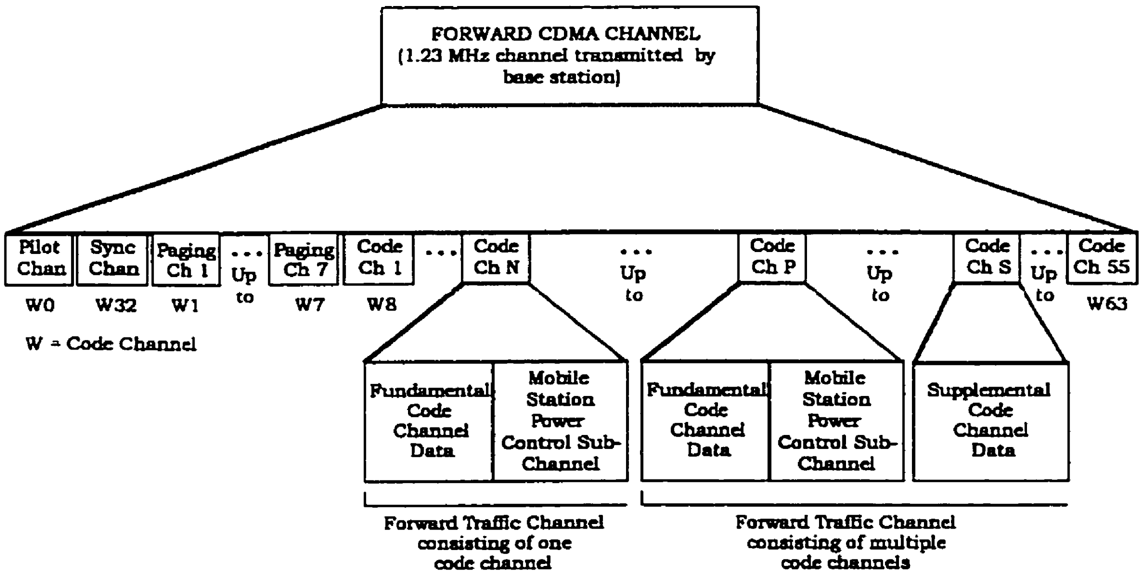

Figure 2.11: TIA/EIA-95 Forward Channelization [227]

Figure 2.11 shows all of the signals transmitted by a base station on a particular sector antenna. Out of the 64 forward code channels available for use, one channel is reserved for the pilot channel. The remaining 63 channels can be configured dynamically to satisfy the traffic demand. Of the 63, a maximum of 7 channels could be allocated as paging channels and a maximum of 1 as the synchronization channel which would leave a minimum of 55 channels for the cellular traffic. At times of heavy cellular traffic, all of the paging channels and the synchronization channel could be replaced as traffic channels making the available number of traffic channels to be 63. The TIA/EIA-95-B release allows for one Fundamental and up to seven Supplemental traffic code channels to be allocated to one user to achieve higher data rates [220, 221, 224]. Obviously each code channel has to select a different Walsh code from the set of 64 to provide orthogonality amongst themselves as well as traffic channels intended for other mobile stations and common channels (pilot, synchronization and paging channels).

The traffic channels contain power control information as well as the information data as shown in Figure 2.11. To minimize the average bit error rate for each user. TIA/EIA-95 forces each user to provide the same power level at the base station receiver. The base station reverse channel receiver estimates and responds to the signal strength for a particular mobile station. Since both the signal and the interference characteristics are continually varying, power control updates are sent by the base station every 1.25 msec (at a rate of 800 bps). Power control commands are sent to each subscriber unit on the forward control subchannel which instruct the mobile to raise or lower its transmitted power in 1 dB steps [185].

An important aspect of the forward link waveform design is the use of the pilot signal that is transmitted by each base station and is used as a coherent carrier reference for demodulation by all mobile station receivers. The pilot channel is used also to acquire timing and provide a means for signal strength comparisons between base stations to determine when to handoff. The pilot is transmitted at a relatively higher level than other types of signals which allows for extremely accurate tracking [226]. The pilot signal is unmodulated by information and uses the zero Walsh function (which consists of 64 zeroes). Thus. the pilot simply consists of the quadrature pair of PN codes that the base station uses. The mobile station can obtain synchronization with the best base station by searching out every possible time shift for the universal PN pair. The strongest signal's time offset corresponds to the time offset of the base station with which the communication would be established. Note that a subscriber could be served by a base station that is different than the closest base station. After synchronization. the pilot signal is used as a coherent carrier phase reference for demodulation of the other signals from the base station. The pilot channel is also used for initial power control by the mobile. which adjusts its output power inversely to the total signal power it receives [246. 252]. Power control is a basic requirement in CDMA systems that use correlator receivers and will be discussed in Section 2.3.3 in more detail.

Once the pilot channel is acquired, the subscriber starts demodulating the synchronization channel. The synchronization channel has a preassigned Walsh function (W32) that is known by all of the subscribers [226]. This channel conveys time of day and the time offset of the pilot PN code for the base station relative to true time. This allows the wireless handset to move its timing from that aligned with the pilot PN code to the true time. The true time, whose distribution in the system is provided by the satellite based, global positioning system (GPS), is assumed to be the same for all base stations.

Once true timing information is achieved, the subscriber demodulates the information sent by the base station by making use of its own Walsh function. Since, when perfectly synchronized, this channel will be completely orthogonal to the other 63 other channels that the base station might be sending, cochannel interference will not be severe (ideally it will be zero).

At both the base station and the subscriber, RAKE receivers are used to resolve and combine the multipath components observed due to the mobile channel characteristics. The aim in using RAKE receivers is to reduce the effects of fading. In almost all TIA/EIA-95 implementations, a three path RAKE receiver is used at the base station. The TIA/EIA-95 standard also provides base station diversity by providing soft handoffs. During a soft handoff, the mobile making the transition between cells maintains links with a number of base stations during the transition. The mobile combines the signals from the serving base stations in the same manner as it would combine signals associated with different multipath components.

In summary, the downlink CDMA channel is a broadcast channel in which a message consists of a signal centered on an assigned frequency, quadriphase modulated by a time shifted version of the universal PN code pair, biphase modulated by an assigned orthogonal Walsh function, and biphase modulated by the encoded, interleaved and scrambled digital information signal. The signaling operation is shown in Figure 2.12.

Mathematically, suppose that base station m wants to communicate with subscriber n. Then the information bearing code that the base station will send will be in the form

$$ENC_{PN_m}\{ENC_{Walsh_n}\{INT\{ENC_{con}\{INFO\}\}\}\}$$

where "ENC" stands for encoding, INT stands for interleaving, $INFO$ stands for the information to be sent. PN_m stands for the universal PN sequence time shifted according to the m'th base station id and finally $Walsh_n$ stands for the n'th Walsh function out of 64. Here, it should be noted that, for a voice signal of 9600 bps, the signal spreading is solely achieved by the use of the 64 Walsh functions and convolutional encoding. The universal PN sequence does not introduce any further spreading but merely reveals the base station identity and provides security.

Say, every channel has L paths due to diversity. Then, mathematically, the i'th base station in the system will transmit a signal in the form (excluding the pilot and synchronization channels),

$$s(t) = \sum_{r=0}^{62}\sum_{q=1}^{L}\sqrt{\hat{S}_{irq}}m_{ir}(t)w_r(t)\left[c_i^i(t)\cos(\omega t) + c_i^q(t)\sin(\omega t)\right] \tag{2.155}$$

where $\hat{S}_{irq}$ is the transmitted signal power from the i'th base station towards the r'th terminal through the q'th path, $m_{ir}(t)$ is the information signal from the i'th base station towards the r'th mobile terminal. $w_r(t)$ is the r'th Walsh function. $c_i^i(t)$ and $c_i^q(t)$ are the in-phase and quadrature spreading sequences for the i'th base station and ω is the common transmission frequency.

If we have N base stations in the network and all base stations are fully utilized (i.e. for every base station all 62 channels are used by the mobile terminals present in the system), the k'th mobile terminal tuned in to receive the signal from the l'th

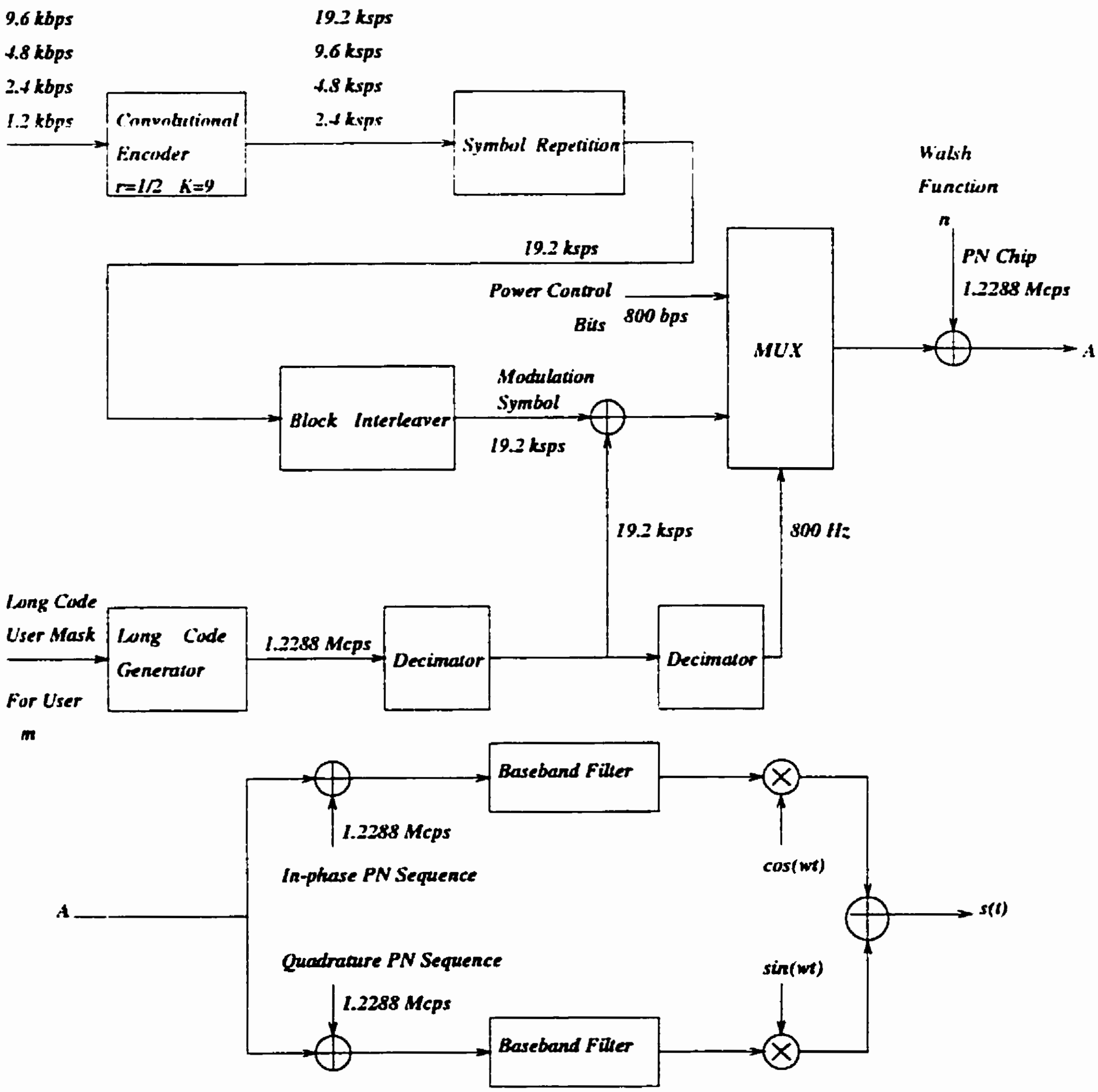

Figure 2.12: TIA/EIA-95 Forward Channel Signaling [227]

path between itself and base station i will receive a signal in the form.

$$
\begin{aligned}
d(t) \;=\; & \sqrt{S_{ikl}}\, m_{ik}(t-\tau_{ikl}) w_k(t-\tau_{ikl}) \\
& \qquad \cdot \left[c_i^{i}(t-\tau_{ikl})\cos(\omega t+\theta_{ikl}) + c_i^{q}(t-\tau_{ikl})\sin(\omega t+\theta_{ikl})\right] \\
& + \sum_{\substack{q=1 \\ q\neq l}}^{L} \sqrt{S_{ikq}}\, m_{ik}(t-\tau_{ikq}) w_r(t-\tau_{ikq}) \\
& \qquad \cdot \left[c_i^{i}(t-\tau_{ikq})\cos(\omega t+\theta_{ikq}) + c_i^{q}(t-\tau_{ikq})\sin(\omega t+\theta_{ikq})\right] \\
& + \sum_{\substack{r=0 \\ r\neq i}}^{62} \sum_{q=1}^{L} \sqrt{S_{irq}}\, m_{ir}(t-\tau_{irq}) w_r(t-\tau_{irq}) \\
& \qquad \cdot \left[c_i^{i}(t-\tau_{irq})\cos(\omega t+\theta_{irq}) + c_i^{q}(t-\tau_{irq})\sin(\omega t+\theta_{irq})\right] \\
& + \sum_{\substack{p=1 \\ p\neq i}}^{N} \sum_{r=0}^{62} \sum_{q=1}^{L} \sqrt{S_{prq}}\, m_{pr}(t-\tau_{prq}) w_r(t-\tau_{prq}) \\
& \qquad \cdot \left[c_p^{i}(t-\tau_{prq})\cos(\omega t+\theta_{prq}) + c_p^{q}(t-\tau_{prq})\sin(\omega t+\theta_{prq})\right] \\
& + n(t) \qquad\qquad (2.156)
\end{aligned}
$$

where S_{prq} is the received signal power from the q'th path of the p'th base station for the r'th subscriber. $m_{ir}(t)$. $c_i^i(t)$.$c_i^q(t)$ and ω are as above. θ_{prq} is the carrier phase offset for the q'th path of the p'th base station's message for the r'th subscriber. τ_{prq} is the time delay introduced by the q'th path from the p'th base station's to the r'th subscriber. $n(t)$ is the zero mean additive Gaussian noise and N is the total number of base stations serving. In (2.156). the first row is the message to be extracted. the second row is the self interference caused by the multipath. and the third and fourth rows are the cochannel interference.

It has been stated that use of a 1.25 MHz bandwidth would necessitate the use of artificial multipath to achieve adequate capacity due to the strong correlation between the inherent multipath signals in the wireless channel [90]. For smaller path loss exponents which are more likely in microcellular environments. artificial multipath diversity of order of as high as 4 may be needed. A wider bandwidth (of 10 MHz) would achieve greater efficiencies in terms of capacity per Hz [88, 89]. A wider band CDMA system is part of the third generation cdma 2000 system [46].

2.3.2 Reverse CDMA Channel

The reverse CDMA channel is composed of access channels and reverse traffic channels as seen in Figure 2.13 and operates in the 824-849 MHz and 1800-2200 MHz bands. The access channel is used by the mobile station to initiate communication with the base station and to respond to paging channel messages. The reverse CDMA channel may contain up to 32 access channels. At least one access channel exists for every paging channel on the corresponding forward channel.

The CDMA uplink channel is also a 1.25 MHz channel. This channel is a tougher point-to-point channel as opposed to the forward one. Here all users have the potential of fading differently into the base site receiver. Thus, a stronger error correcting code is needed. TIA/EIA-95 standard uses a rate one-third constraint length nine convolutional code for this purpose.

The reverse channel also uses the same 32768 length universal PN code pair but a fixed code time offset is used here. So, whenever the base station is to receive a signal, it tunes itself to the specific time shift reserved for the reverse link channel. Messages from different subscribers are distinguished by the time offset of a very long PN sequence (length $2^{42} - 1$). This time offset is in fact, the user's personal telephone number.

As stated before, the information that is to be transmitted is convolutionally encoded with a rate one-third code of constraint length nine. The encoded information is then interleaved over a 20 msec interval [226]. The interleaved information is grouped in six symbol groups to select one of the 64 different Walsh functions for transmission. If the symbol group in question is $(s_0, s_1, s_2, s_3, s_4, s_5)$, the Walsh function selection is

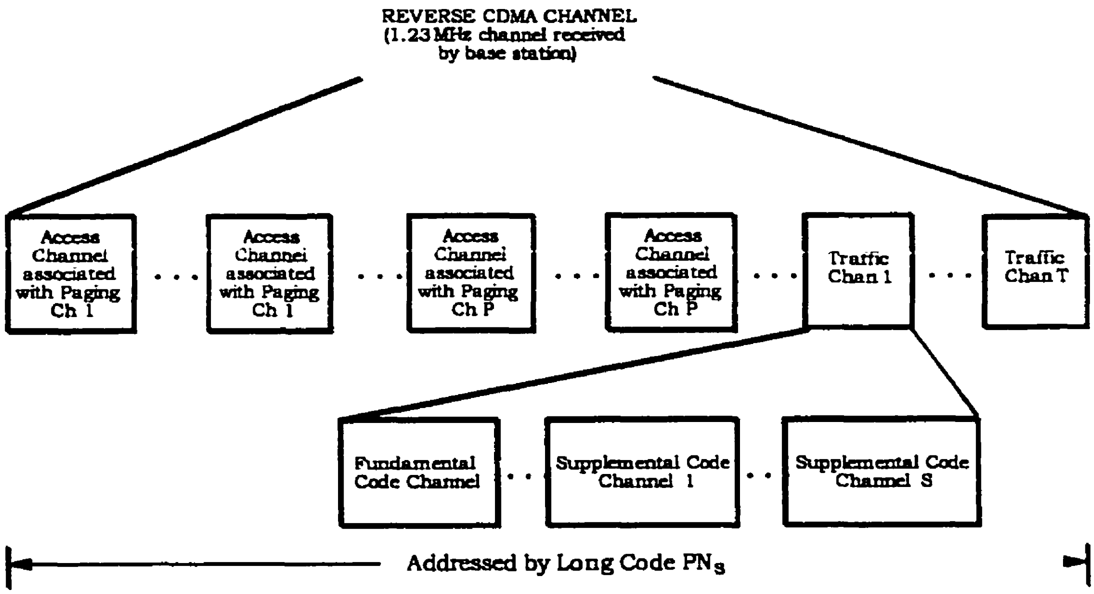

Figure 2.13: TIA/EIA-95 Reverse Channelization [227]

done using the equation [226].

$$\text{Walsh function Index} = s_0 + 2s_1 + 4s_2 + 8s_3 + 16s_4 + 32s_5 \tag{2.157}$$

The Walsh functions are then modulated with the time shifted PN code that is the user's telephone number and further offset modulated with the universal PN code that has the reverse link channel specific time offset. The use of Walsh functions is a simple way of obtaining 64-ary orthogonal modulation. Noncoherent demodulation is performed at the base station. It should be noted here that for the forward link, the Walsh functions are determined by the mobile station's assigned channel while on the reverse link, the Walsh function is determined by the information being transmitted. Similar to the forward link, on the TIA/EIA-95-B reverse link, each user may be allocated one Fundamental and up to seven Supplemental channels. These channels are distinguished from one another by use of long PN sequences with channel determined time offsets.

Thus, in summary, the reverse channel consists of a signal centered on an assigned frequency, offset quadriphase modulated by the universal PN code pair, biphase modulated by a long PN code with the user identity determined by the code phase, and biphase modulated by the Walsh encoded and convolutionally encoded digital information signal. The reverse channel signaling procedure is shown in Figure 2.14.

Mathematically, say user n wants to send $INFO$ to a base station then the transmitted signal will be in the form,

$$\mathrm{ENC}_{con}\{INFO\} \longrightarrow Walsh_k$$

$$\mathrm{ENC}_{PN_{rev}}\{\mathrm{ENC}_{PN_n}\{Walsh_k\}\}$$

where PN_{rev} is the universal PN code with the fixed time offset that is designated for the reverse channel, PN_n is the telephone number of user n and k is an integer number between 0 and 63 dependent on the information contents. Here, unlike the forward channel, Walsh functions are used in the modulation of the signal. For a voice signal of 9600 bps, the PN sequence, which is the only spreading sequence here, introduces a spreading of a factor of 4 beyond what is already achieved through convolutional coding and Walsh function orthogonal 64-ary modulation. Since the spreading factor is not large, inter-cell interference is not always cancelled. This yields collisions in 64-ary modulated signals.

Once again, say every channel has L paths due to diversity. Then, mathematically, every mobile terminal will send a signal in the form,

$$\begin{aligned} s_i(t) &= \sum_{q=1}^{L} \sqrt{\hat{S}_{iq}} \Big[w^j(t) p_i(t) c^i(t) \cos(\omega t) \\ &\quad + w^j(t - T_d) p_i(t - T_d) c^q(t - T_d) \sin(\omega t) \Big] \end{aligned} \tag{2.158}$$

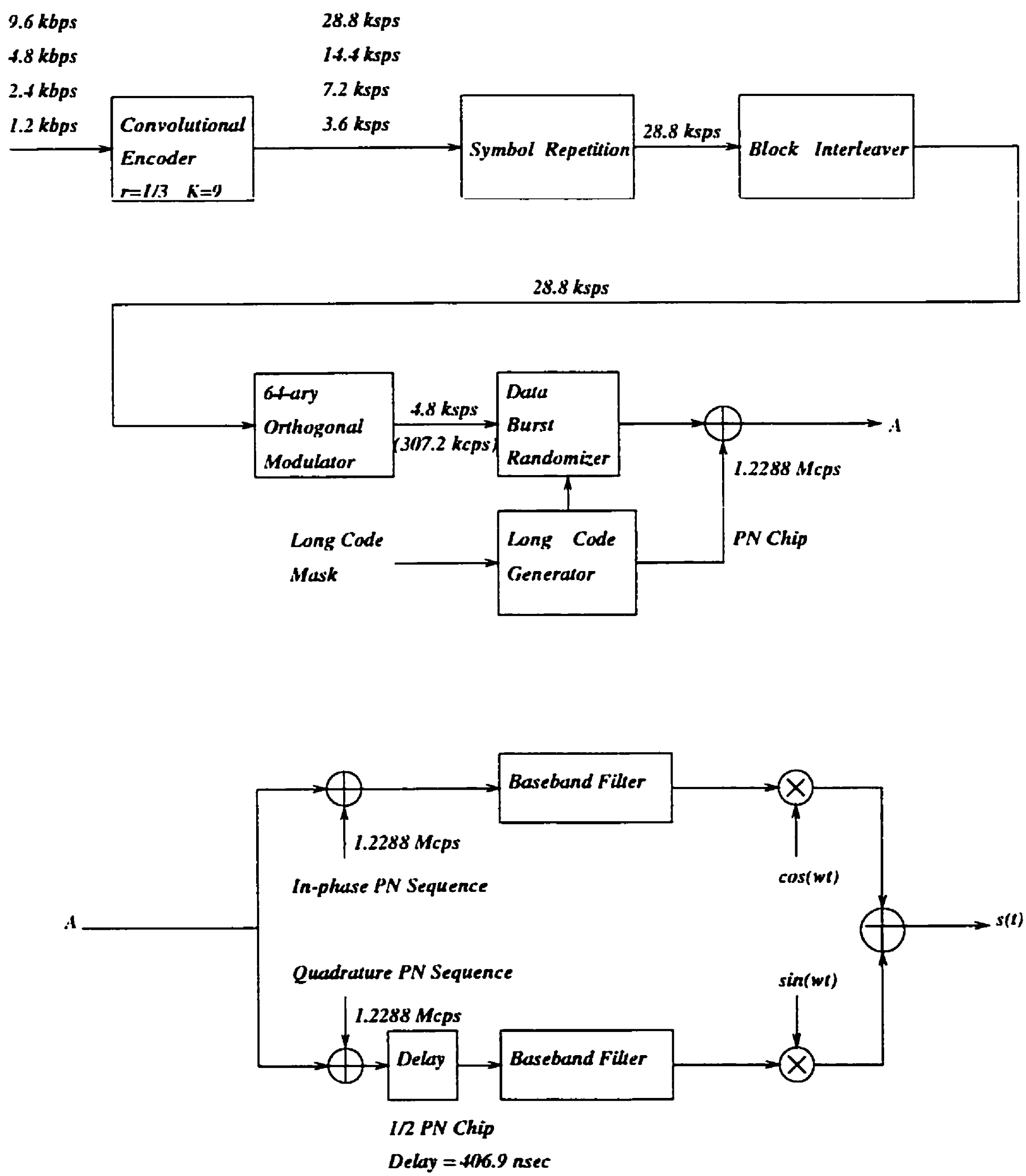

Figure 2.14: TIA/EIA-95 Reverse Channel Signaling [227]

where S_{iq} is the transmitted signal power from the i'th terminal through the q'th path. $w^j(t)$ is the j'th orthogonal Walsh function ($j = 1, \ldots, 64$) that corresponds to the symbol to be transmitted. $p_i(t)$ is the telephone number of the i'th user. $c^i(t)$ and $c^q(t)$ are the in-phase and quadrature spreading sequences for the uplink transmission. T_d is the offset time and ω is the common transmission frequency.

If we have K users in the system, the base station receiver will receive a signal in the form.

$$r(t) = \sum_{i=1}^{K} s_i(t - \tau_i) + n(t) \tag{2.159}$$

where τ_i is a random delay of user i and $n(t)$ is the additive white Gaussian noise.

2.3.3 Power Control

Every CDMA receiver at a given cell site operates by converting a selected CDMA signal from one of the mobile station transmitters into a signal that carries narrowband digital information. At the same time, the other signals that are not selected remain wide bandwidth noise signals. The processing gain of the system increases the signal-to-interference ratio from a negative value in dB to a level that allows operation with an acceptable bit error rate.

It is very desirable for the system operators to maximize the capacity of the CDMA system in terms of the number of simultaneous telephone calls that can be handled in a given system bandwidth. The system capacity is maximized if the transmit power of each mobile station is controlled so that its signal arrives at the cell site with the minimum required signal-to-interference ratio.

If a mobile station's signal arrives at the cell site with too low a value of received power, the bit error rate is too high to permit high quality communications. If the received power is too high, the performance of this mobile station is acceptable, but interference to all the other mobile station transmitters that are sharing the channel is increased, and may result in unacceptable performance to other users unless the capacity is reduced. This phenomenon is often referred to as the near-far problem [58].

Due to the threat of the near-far problem, power control is one of the most important system requirements for CDMA. To achieve high capacity, quality and other benefits, the CDMA mobile telephone system employs forward and reverse link power control. The objective of the power control process is to produce a nominal received signal power from each mobile station transmitter operating within the cell at the cell site receiver. Regardless of the mobile station's position or propagation loss, each mobile station's signal is aimed to be at the same level at the cell site. If all the mobile station transmitters within a cell site are so controlled, the total signal power received at the cell site from all the mobile stations is equal to the nominal received power times the number of mobile stations.

Power control is achieved as follows. Prior to any transmission, each of the subscribers monitors the total received signal power from the cell site. According to the power level it detects, it transmits an initial level which is as much below (above) a nominal level in decibels as the received pilot power level is above (below) its nominal level. Further refinements in power level in each subscriber can be commanded by the cell site depending on whether the block error rate observed by the base station receiver for the subscriber signal is above or below a threshold that is acceptable to maintain a pre-determined quality of service.

2.3.3.1 Reverse Channel Power Control

To maximize system capacity, CDMA requires that the received SNR from all wireless handsets be equal at the base station. This is commonly called the near-far problem in which wireless handsets closer to the base station have a smaller path loss and thus create more interference which significantly reduces the capacity. To solve this, the TIA/EIA-95 standard uses dynamic power control on the downlink pilot channel.

Two stages of power control are used: open loop and closed loop. In open loop power control, the wireless handset adjusts its transmitted power using the total received power in the receiver. The closer the wireless handset is to a base station, the stronger the received signal level and the lower the transmit power. In the closed-loop power control system however, the base station aims to ensure that the block error rate for the subscriber is maintained at a level that is acceptable using the minimum possible handset transmit power. If the block error rate is too good, the base station sends out a command to have the handset decrease its transmit power by 1 dB. Conversely, if the block error rate is not acceptable to sustain the required quality of service, it sends out a command to have the handset increase its transmit power by 1 dB.

The uplink channel in TIA/EIA-95 uses fast, closed power control to update the transmitted power every 1.25 msec. This power control scheme is intended to overcome the uplink near-far problem as well as multipath fading at low Doppler frequencies [89].

2.3.3.2 Forward Channel Power Control

The primary reason for providing power control for the forward channel is to accommodate disadvantaged links from the base station to the wireless handset. At times, it is possible for a wireless terminal to have a poor signal quality from each possible base station. To overcome this problem, the wireless handset measures the received signal quality and periodically transmits the results of the measurements to the base station that yields the highest correlation. In addition, if the quality is less than a desired minimum value, which may depend on the system load, then the wireless handset immediately transmits the results of the measurement. When the base station receives a report of poor quality, it evaluates the power needs of all the wireless handsets using the base station and decides whether to increase the code channel power to the wireless handset. The base station can either increase its total transmit power at the expense of increasing interference to wireless handsets using other base stations, or the base station can redistribute power from wireless handsets with good quality to the disadvantaged wireless handset.

The downlink channel in TIA/EIA-95 uses closed loop power control to follow the slow variations in the signal to interference ratio due to shadow fading and path loss (slow power control) or to adapt to both slow variations as well as fast variations due to fast multipath fading (fast power control). Power control obviously effects the individual signal power. The downlink signal power (S_{irj} in (2.156)) is defined to be [89].

$$S_{irj} = P_{irj} x_{ir}^2 \delta_{ir} \gamma_i^2 \alpha_{ij}^2 \quad (2.160)$$

where P_{irj} is the initial power, x_{ir}^2 is the power variation due to fast power control, δ_{ir} is the power variation due to slow power control, γ_i^2 is the power variation due to shadow fading and path loss and α_{ij}^2 is the power variation due to fast fading on the j'th path of the i'th transmitter signal.

2.3.3.3 Further Advantages of Power Control

An important gain from the power control algorithms is the reduction of the average transmitted power in both downlink and uplink signaling. Most of the time propagation conditions are benign. Narrowband systems must always transmit with enough power to override the occasional fades. CDMA uses power control to provide only the power required at the time, and thus reduces the average power by transmitting at high levels only during fades. This reduction means that the mobile stations also have reduced transmitter output requirements which reduces cost and allows larger power units to operate at larger ranges than the similarly powered analog or TIA/EIA-54 based digital cellular systems. Furthermore, a reduced transmitter output requirement increases coverage and penetration and may also allow a reduction in the number of cells required for coverage.

2.4 The cdma2000 System Proposal

While the evolution path to a 3G system is dependent on many factors, the TIA/EIA-95 operators have explicitly expressed desire to ensure that the evolution path towards cdma2000 should allow coexistence and integration of both 2G and 3G systems [32]. The TIA/EIA-95 community has developed a two phase plan to evolve the second generation system. The first phase provides for increased transmission rates without changing the infrastructure hardware. As described in the previous section, this is done by aggregating existing TIA/EIA-95 code channels on both the downlink and the uplink. This capability has been included in the TIA/EIA-95-B release of the standard [227]. The second phase is targeted towards 3G and IMT-2000. The TIA/EIA-95 based 3G system will offer enhancements within the 1.2288 Mcps chip rate as well as introducing a three-times chip rate of 3.6864 Mcps. Compared to the 2G system, the proposed 3G cdma2000 system has the following distinctive new features:

- fast power control in the downlink
- a coherent uplink using dedicated pilot
- interfrequency handoff
- optional multiuser detection use at the receivers

- additional pilot channel in the downlink for beamforming
- provision of transmit diversity for downlink

We now summarize the forward and reverse traffic channel configurations of the cdma2000 standard proposal.

2.4.1 Forward CDMA Channel

As stated above, the 3G system will support chip rates of 1.2288 Mcps, 3.6864 Mcps and higher. There exist two options as to how the information bits can be spread across the bandwidths larger than the basic 2G bandwidth of 1.25 MHz: multi-carrier and direct spread. The multi-carrier approach de-multiplexes symbols that are encoded, repeated, punctured and interleaved into N 1.25 MHz carriers. Each sub-carrier realizes spreading with a PN code rate of 1.2288 Mcps. Note that since de-multiplexing of the data is done past encoding, the multi-carrier system provides the information bits to be spread across the entire allocated bandwidth, not just one of the sub-carriers. Figures 2.15 and 2.16 show the building blocks of the multi-carrier forward link for the traffic channels.

The direct spread approach, on the other hand, transmits the symbols on a single carrier, which may be spread with a chip rate of 1.2288 Mcps, 3.6864 Mcps or higher, depending on the allocated bandwidth. The multi-carrier approach makes it easier to maintain backwards compatibility to TIA/EIA-95 while providing easier signal processing at the base station [32]. Most importantly, the multi-carrier approach enables an overlay to the existing TIA/EIA-95 carriers in frequency. On the other hand, the multi-carrier approach may potentially be complex since all of the existing subcarriers have to be demodulated simultaneously. The direct sequence approach has lower peak to average power ratio, thereby simplifying the design of the power

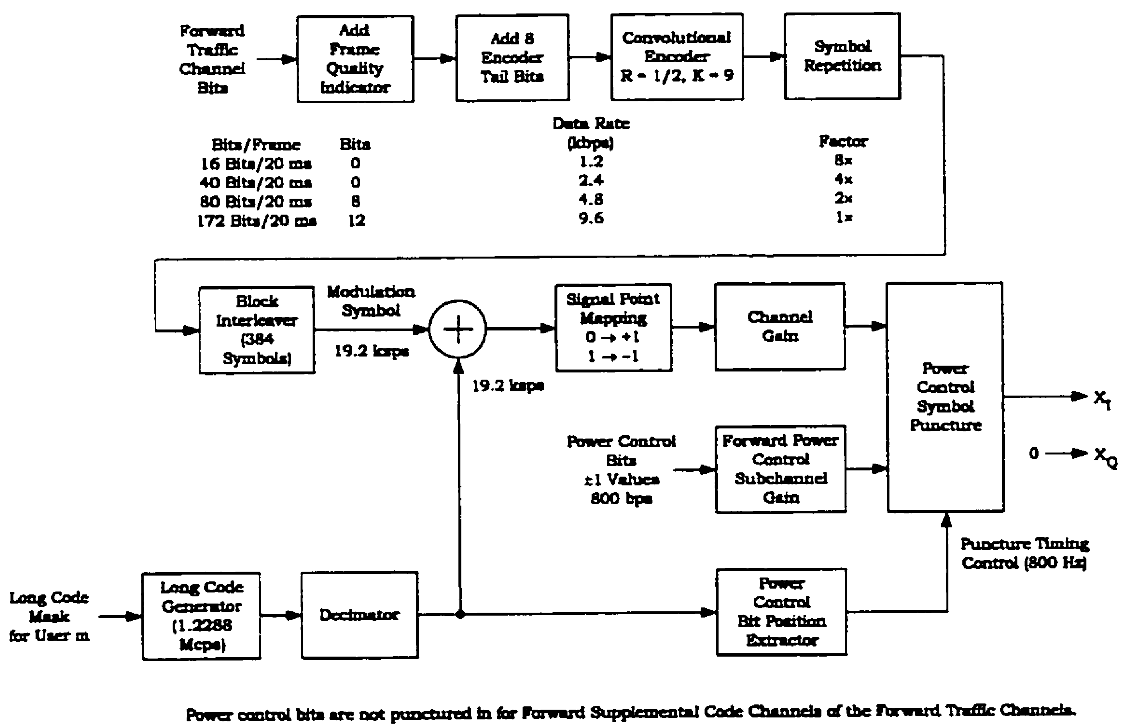

Figure 2.15: cdma2000 Forward Link - Part I [229]

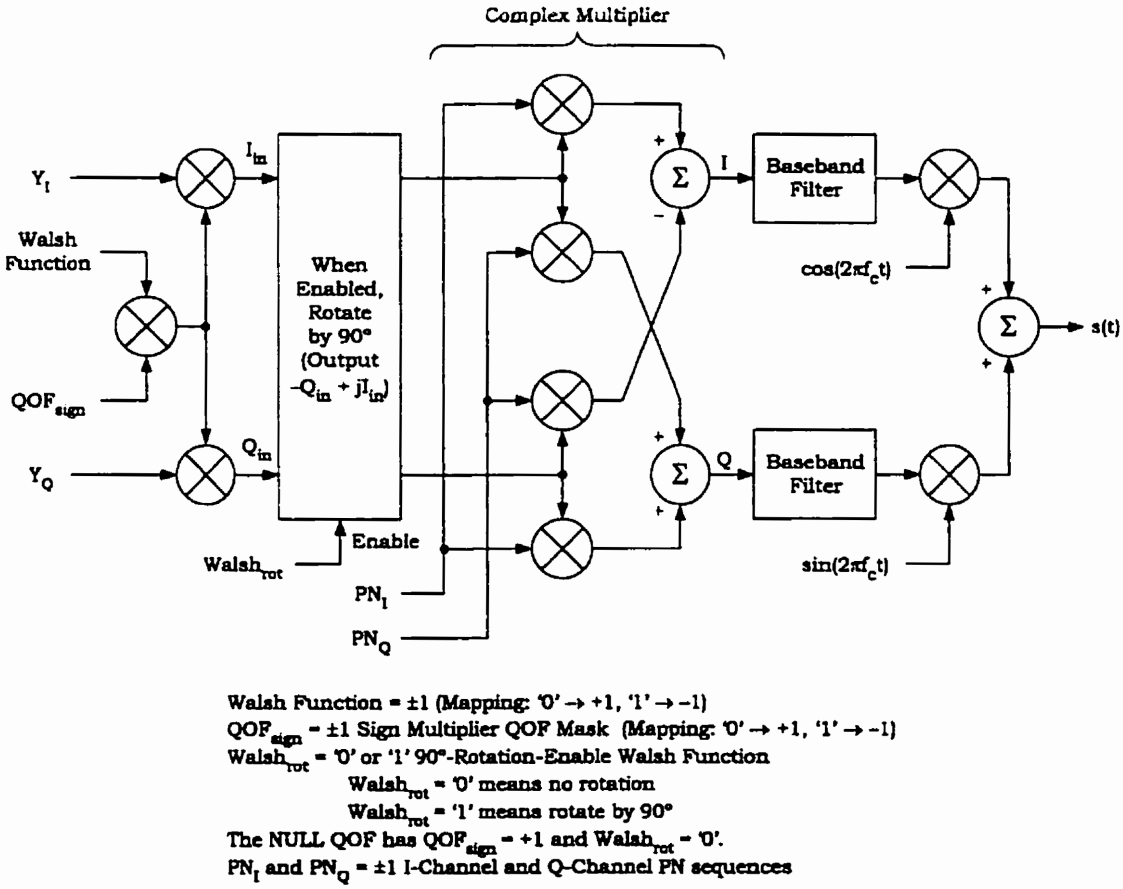

Figure 2.16: cdma2000 Forward Link - Part II [229]

amplifiers. Currently, the multi-carrier approach is favored over the direct-spread approach within the standards community.

The cdma2000 system still uses offsets of binary PN codes to distinguish signals received at a mobile station from different base stations. However, to achieve higher data rates without being Walsh code limited, the 3G system uses QPSK modulation. As shown in Figure 2.16, a complex quadriphase PN spreading follows this operation.

The cdma2000 system dedicated code channels are the Fundamental Channels, Supplemental Channels, Dedicated Auxiliary Pilot Channels and Dedicated Control Channels. Provision of multi-codes in the form of one Fundamental Channel and zero or more Supplemental Channels is still used in the 3G system. However, unlike TIA/EIA-95-B, where the individual Supplemental Channels always use the Walsh-64 alphabet for spreading, variable spreading is employed in the 3G system. That is, achieving higher transmission rates is accomplished both by using multiple code channels and by decreasing the Walsh spreading on the individual codes as the required transmission rate increases. It is possible to have parallel Supplemental Channels use different Walsh alphabets. In this case, the system has to ensure that the Walsh codes from the different alphabets are still orthogonal to one another. The 3G system may not require any rate determination for the Supplemental Channels. The number of Supplemental Channels and their corresponding transmission rates may be signaled explicitly to the mobile station by the base station.

From a performance point of view, use of multi-code transmission as well as variable spreading to achieve higher transmission rates does not offer any advantages over using a single code with variable spreading provided that the processing gain never becomes too small. However, in a system where a user is allowed to request multiple services simultaneously, division of the services into multiple codes, one per service, will clearly ease the management of the individual services.

Unlike the 2G system, the encoding of the Fundamental and Supplemental Channels may be different in the 3G system. Since the 3G system is supposed to provide a multitude of services with different quality of service requirements, the encoding scheme may change from service to service. The generic Fundamental and Supplemental Channels are encoded using constraint length 9 convolutional codes and Turbo codes with block lengths varying between 378 bits and 20730 bits depending on the data rate, respectively. As in the 2G system, the encoding process is followed by symbol repetition, puncturing and interleaving.

A new characteristic of the 3G system is the use of Spatial Division Multiple Access (SDMA) for very high rate mobile stations or mobile stations in high propagation loss environments. In these instances, a spot beam in the form of a Dedicated Auxiliary Pilot Channel is generated for the user in question. When using adaptive antennas in this mode, the channel response between the mobile station and the base station in angle and delay space is measured periodically and is used to adjust the pattern of the forward link antenna.

Another new characteristic of the 3G forward link is the use of transmit diversity. It is possible to achieve transmit diversity for both multi-carrier and direct spread options. In multi-carrier, antenna diversity may be implemented by using multiple antennas where a subset of the subcarriers is transmitted on each antenna. In direct spread, on the other hand, orthogonal transmit diversity may be employed where encoded bits are de-multiplexed into a number of parallel data streams and are transmitted via separate antennas. Each antenna uses a code that is orthogonal to the codes used by the rest of the antennas. Transmit diversity, obviously, increases system performance as it increases frequency diversity.

The Dedicated Control Channel in the cdma2000 forward link provides the mobile with power control information and layers one and two signaling, primarily to support the packet data services.

The 3G system forward link common code channels are the Pilot Channel, Common Auxiliary Pilot Channel, Paging Channel, Synchronization Channel and Common Control Channel. The Pilot, Synchronization and Paging Channels for the cdma2000 forward link are similar to their counterparts in the TIA/EIA-95 forward link. The Common Auxiliary Pilot Channel operates exactly the same way as the Dedicated Auxiliary Pilot Channel but is intended to service a multiple number of mobile stations located in a cluster within a cell to increase coverage and capacity in the cluster. The Common Control Channel is used for maintaining packet data services.

In the cdma2000 forward link, the power control is closed loop and is significantly faster than its second generation counterpart. The power control commands, sent by the mobiles on their Pilot Channels, are used to modify the transmission powers at the base station. Updates are performed at a rate of 800 Hz.

2.4.2 Reverse CDMA Channel

Perhaps the most significant difference between TIA/EIA-95 and cdma2000 reverse links is the presence of non-coherent versus coherent reception capabilities in the 2G and 3G base stations, respectively. The 3G reverse link, like its forward link counterpart, uses dedicated Pilot Channels that enable the base stations to coherently demodulate mobile stations' signals.

In the 3G reverse link, the encoded information symbols are transmitted on a single carrier, which is spread with a chip rate of 1.2288 Mcps, 3.6864 Mcps or higher, depending on the allocated bandwidth.

The 3G system still uses offsets of long binary PN codes to distinguish between subscribers. In the second-generation system, the long PN codes are also being used to distinguish between the Fundamental and Supplemental Channels of a given user. However, in the 3G system, Walsh codes are used for this purpose.

The 3G reverse link is composed of six different channels, four of which are dedicated channels. The dedicated channels, namely, the Pilot Channel, Fundamental Channel, Supplemental Channel and Dedicated Control Channel are separated by both Walsh functions and transmission of the Pilot and Dedicated Control Channels on the I channel and the Fundamental and Supplemental Channels on the Q channel. The separation of the channels in the QPSK modulation is intended to reduce the peak-to-average power ratio. The QPSK modulation is followed by complex quadriphase spreading. Figures 2.17 and 2.18 show the building blocks of the reverse link dedicated channels.

The 3G reverse link structure uses constraint length 9 convolutional codes for the Fundamental Channel. Like the forward link, the Supplemental Channel may choose a variety of different codes depending on the service, but the preferred coding scheme is Turbo coding with block lengths varying between 378 bits and 20730 bits depending on the data rate. Encoding procedure is followed by symbol repetition and puncturing to achieve the desired symbol rate and interleaving to combat fast fading.

The Pilot Channel in the 3G reverse link is used for initial acquisition, tracking, coherent reception and power control measurements. The Pilot Channel consists of a fixed reference value and includes time multiplexed forward link power control information.

The Fundamental Channel is a variable rate channel, which supports the existing TIA/EIA-95 transmission rates. It conveys signaling messages, voice and possibly part of the data stream. The Supplemental Channel is a high rate channel, typically used for data transmission. The Reverse Dedicated Control Channel is developed to support packet data services.

The reverse link also has two common channels, namely, the Access Channel and the Common Control Channel. Both of these channels are distinguished from one another by means of different offset long PN codes. These channels are used for the

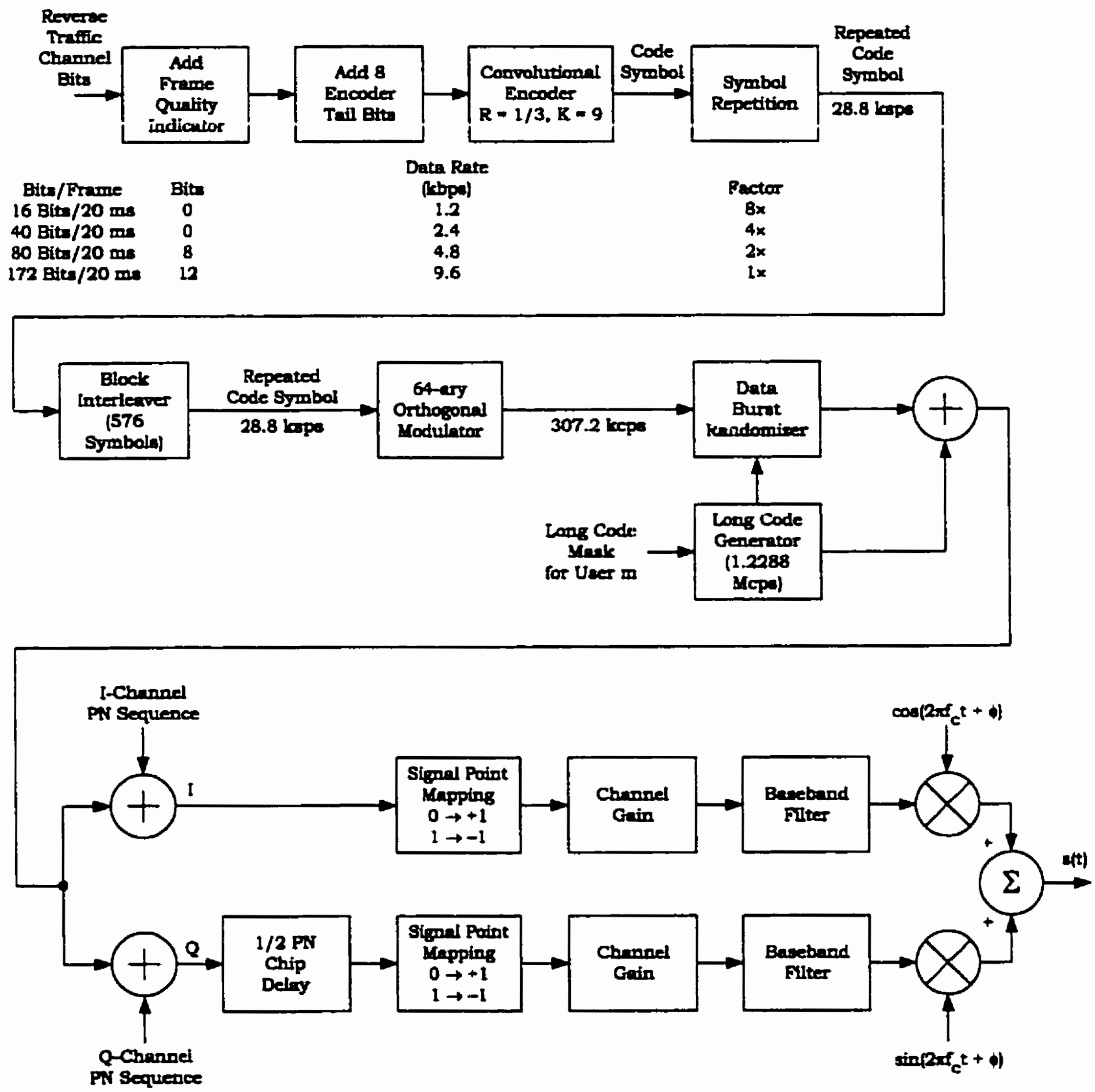

Figure 2.17: cdma2000 Reverse Link - Part I [229]

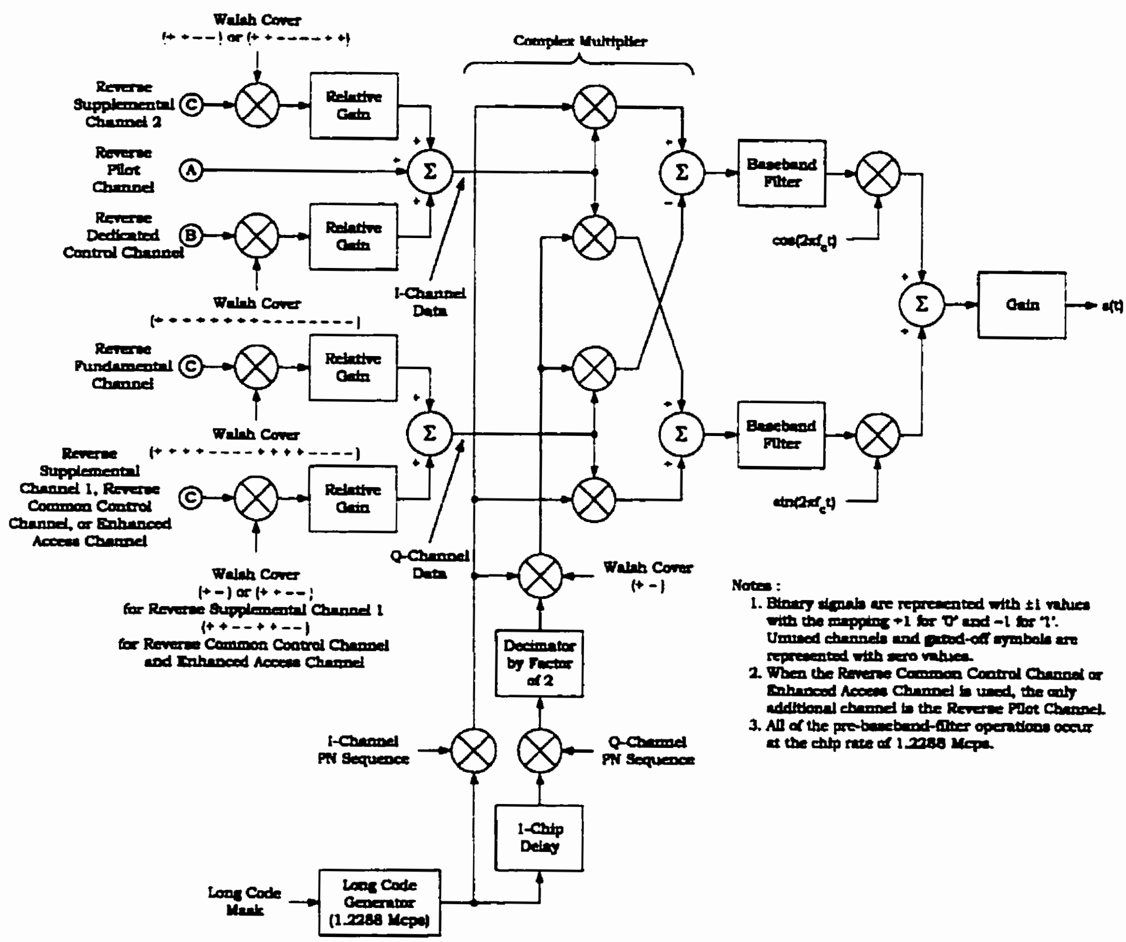

Figure 2.18: cdma2000 Reverse Link - Part II [229]

communication of layer three and media access control messages from the mobile station to the base station.

As in the case of TIA/EIA-95, the 3G reverse link power control consists of open and closed loop power control algorithms. The closed loop power update rate is 800 Hz.

2.5 Summary

In this chapter, mathematical models for DS CDMA systems are developed. Differences between the biphase and quadriphase spread systems are studied and behaviours of DS CDMA systems in simple AWGN and multipath fading channels are discussed. Brief overviews of the TIA/EIA-95 standard and its third generation evolution, cdma2000 are also given in this chapter.

Chapter 3

ANALYSIS OF DS CDMA SYSTEMS

Analysis of DS CDMA systems in AWGN and multipath fading channels is a well studied problem. To date, emphasis has been on approximations, bounds and simulation techniques since the exact calculation was believed to be computationally difficult [80]. This chapter presents a brief overview of some of the results documented in the literature.

3.1 Approximations

Throughout the years, various approximation techniques have been proposed in the literature to calculate error probabilities for DS CDMA systems. One particularly popular approximation has been the Standard Gaussian Approximation (SGA). Here, only the desired user's signal is considered and all other users which are simultaneously using the channel are treated as interfering additive Gaussian noise having uniform power spectrum over the band of interest. The SGA derives much of its appeal from the fact that it is very easy to compute regardless of the magnitude of the processing gain or the number of users present in the DS CDMA system. The SGA is based on the Central Limit Theorem which states that the sum of a large number of mutually independent random variables tends towards the normal distribution provided certain constraints are satisfied [206]. Thus, the SGA assumes that the interference terms are mutually independent and that there exist a large number of them. As we have shown in Chapter 2, in reality the multiple access interference terms in a DS CDMA system are independent only when conditioned on a set of specific operating conditions of every user as well as the discrete aperiodic autocorrelation function of the desired user's spreading sequence [124]. Thus, the SGA is not always accurate enough. This observation led to the proposal of another Central Limit Theorem based approximation, namely, the Improved Gaussian Approximation (IGA) [155, 156]. Here, the multiple access interference terms are conditioned on the particular operating conditions of each user so that they become mutually independent. The Central Limit Theorem is then invoked to find the conditional error probability. Finally, the total probability theorem [166] is used to numerically find the unconditional error probability. IGA is naturally more accurate than the SGA. However in [155], evaluation of the expressions used to describe the IGA technique requires significant computational time and complexity. Recently, this was overcome by a Taylor series based approximation [80]. Since SGA and IGA are the most popular techniques to approximate DS CDMA system performance in the literature, these techniques are studied in detail in this chapter. The IGA scheme has only been presented for the analysis of asynchronous DS CDMA systems in the literature [80, 131, 155, 156, 185, 212]. Here, we extend on this work and present an IGA scheme that is valid for asynchronous, synchronous and quasi-synchronous systems.

Other approximations have also appeared in the literature. Approximations based on the Gauss-Quadrature method, characteristic function of the multiple access interference and moments are among the alternative techniques proposed. These methods are briefly presented in this section as well.

3.1.1 The Standard Gaussian Approximation (SGA)

In a narrow-band communication system with additive noise present at a correlation receiver, a data bit error occurs when the integrated amplitude of the noise exceeds the integrated amplitude of the desired signal in the opposite direction, causing a decision error at the receiver. If the noise is a zero-mean Gaussian process, then the probability of bit error may be calculated by first finding the signal-to-noise ratio (SNR). Next, $Q(x)$, defined by,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\mu^2/2} d\mu \tag{3.1}$$

is used to evaluate the probability that a wrong decision is made at a receiver. In other words,

$$P(E) = Q(\sqrt{\text{SNR}}). \tag{3.2}$$

Additionally, if the noise is white, then its autocorrelation function is impulsive, and the process produces values which are uncorrelated (and hence independent) from instant to instant in time making the channel errors also independent from data bit to data bit.

In a practical DS CDMA system, there usually are a large number of active users at a given time. Every user transmits a PN modulated binary signal that is independent from the other signals. These signals become only conditionally independent once they are processed by the correlation receiver. The SGA approximates the multiple access interference caused by these signals as additive white Gaussian noise with variance equal to the sum of the individual interfering signal variances. Thus, the SGA not only approximates the interference terms as Gaussian distributed, but also ignores the conditional dependence of the individual multiple access interference terms. Along with the multiple access interference, the telecommunication channel usually introduces channel noise which is modeled as zero mean additive white Gaussian. Then, using the fact that the sum of two Gaussian random variables is still Gaussian with mean and variance equal to the sum of the individual means and variances, respectively [166], the equivalent noise as seen by the SGA can be modeled as zero-mean Gaussian with variance equal to the sum of the variances of the channel noise and the multiple access interference. Once such an approximation is made, the probability of error for the system in question can easily be found using (3.2).

To facilitate a better understanding of the SGA and its limitations, let us review the central limit theorem (CLT) here. This theorem is very well known and is given here without proof. Further details and proof of the CLT can be found in [51, 56, 206, 166].

Theorem: (The Central Limit Theorem) *Let $X_1, X_2, \ldots$ be mutually independent one-dimensional random variables with distributions $F_1, F_2, \ldots$ Assume*

$$E(X_k) = 0, \qquad Var(X_k) = \sigma_k^2, \quad \forall k = 1, 2, \ldots$$

and put

$$s_n^2 = \sigma_1^2 + \cdots + \sigma_n^2.$$

Assume that for each $t > 0$

$$\frac{1}{s_n^2} \sum_{k=1}^{n} \int_{|y| \geq t s_n} y^2 F_k\{dy\} \to 0$$

or, what amounts to the same, that

$$\frac{1}{s_n^2} \sum_{k=1}^{n} \int_{|y| < t s_n} y^2 F_k\{dy\} \to 1.$$

Then, for large n, the distribution of the normalized sum

$$S_n = \frac{X_1 + \cdots + X_n}{s_n}$$

tends to the normal distribution with zero expectation and unit variance.

This version of the CLT, first proposed by Lindeberg [51], guarantees that the individual variances σ_k^2 are small when compared to their sum s_n^2 in the sense that for given $\epsilon > 0$ and all n sufficiently large,

$$\frac{\sigma_k}{s_n} < \epsilon, \qquad k = 1, \ldots, n.$$

In other words, for the CLT to be reasonably accurate three important conditions need to hold: the random variables in the summation should be unconditionally independent from one another, the number of random variables in the sum should be large, and none of their corresponding variances should dominate the variance of the summation.

In Chapter 2, we have shown that not only the multiple access interference terms at the output of the correlator receiver are inter-dependent but also the interference terms from the chips of an individual multiple access interference are chip-to-chip dependent. This poses an analytical challenge on the evaluation of the DS CDMA system performance [155, 156]. For the analysis to progress with reasonable effort, it is tempting to assume somewhat independent multiple access interference terms and independent chip error events for each interferer within one symbol duration. The SGA does just that by first approximating the interference terms by white Gaussian random variables and then using an average interference variance for each term. This is simply because the $G \cdot (K-1)$ interference terms in (2.31), though only mutually independent given $C_1(1)$, τ_k and ϕ_k, are in fact uncorrelated. Therefore, when the SGA approximates these terms as Gaussian distributed, they are automatically assumed to be unconditionally independent as well because, if a group of random variables are uncorrelated and Gaussian distributed, they are also mutually independent [166].

We now derive the error probability expression for the coherently received DS CDMA system described in section 2.1.1 using the SGA.

In (2.68), suppose that $a_{1,0} = 1$ represents the binary symbol 1 and $a_{1,0} = -1$ represents the binary symbol 0. The decision device in Figure 2.2 produces the symbol 1 if the decision statistic, $Z_1 > 0$ and the symbol 0 if $Z_1 < 0$. An error occurs if $Z_1 < 0$ when $a_{1,0} = 1$ or if $Z_1 > 0$ when $a_{1,0} = -1$. Therefore,

$$P(E) = P(a_{1,0} = 1)P(Z_1 < 0 | a_{1,0} = 1) + P(a_{1,0} = -1)P(Z_1 > 0 | a_{1,0} = -1)$$

$$
\begin{aligned}
&= \frac{1}{2}\left(P(Z_1 < 0|a_{1,0} = 1) + P(Z_1 > 0|a_{1,0} = -1)\right) \\
&= P(Z_1 > 0|a_{1,0} = -1). \qquad (3.3)
\end{aligned}
$$

As stated before, the use of the SGA to determine the bit error rate of the DS CDMA system is based on the argument that the decision statistic, Z_1, given by

$$
\begin{aligned}
Z_1 &= \sqrt{2P_1}a_{1,0}T + \sum_{k=2}^{K}\sqrt{2P_k}\omega_k\cos(\phi_k) + N \\
&= D + M + N \qquad (3.4)
\end{aligned}
$$

as shown in (2.67)-(2.71), may be modeled as a Gaussian random variable. The first component in equation (3.4), $D = \sqrt{2P_1}a_{1,0}T$, is deterministic. The other two components of Z_1 are assumed to be zero-mean Gaussian random variables. Assuming that the additive channel noise is a bandpass Gaussian noise, it has already been stated that N is a zero-mean Gaussian random variable with variance N_0T. The Gaussian approximation then, derives an expression for the bit error rate based on the assumption that the multiple access interference term, M, in (3.4) may be approximated by a Gaussian random variable. From (2.71), M can be written as,

$$
\begin{aligned}
M &= \sum_{k=2}^{K}\sqrt{2P_k}w_k\cos(\phi_k) \\
&= \sum_{k=2}^{K}\sqrt{2P_k}T_c\{\Lambda_k + \Gamma_k(1-2\varrho_k) + \Upsilon_k(1-\varrho_k) + \Omega_k\varrho_k\}\cos(\phi_k). \qquad (3.5)
\end{aligned}
$$

Once M is assumed to be Gaussian distributed, from (3.2), (3.3) and (3.4), the SGA yields,

$$
\begin{aligned}
P(E)_{SGA} &= P(Z_1 > 0|a_{1,0} = -1) \\
&= Q\left[\frac{\sqrt{2P_1}T}{\sqrt{N_0T + \sigma_M^2}}\right] \qquad (3.6)
\end{aligned}
$$

where $\sqrt{2P_1}T$ is the desired signal term, N_0T and σ_M^2 are the variance of the AWGN and multiple access interference terms, respectively, and $Q(x)$ is the error function defined in (3.1).

To evaluate (3.6), we need to find the variance of the multiple access interference term, σ_M^2. We first find the variance conditioned on a specific set of operating conditions, namely, $K-1$ normalized time delays, $\{\varrho_2, \varrho_3, \ldots, \varrho_K\}$, $K-1$ phase offsets, $\{\phi_2, \phi_3, \ldots, \phi_K\}$, $K-1$ user power levels, $\{P_2, P_3, \ldots, P_K\}$, and finally the number of chip transitions during one symbol duration of the first user's spreading sequence, $|B|$, which is defined in (2.57). We define ν to be the conditional variance of M. We then get,

$$\begin{aligned}\nu &= \text{var}\{M|\varrho_2, \ldots, \varrho_K, \phi_2, \ldots, \phi_K, P_2, \ldots, P_K, |B|\} \\ &= \sum_{k=2}^{K} 2P_k \text{E}\{w_k^2|\varrho_k, |B|\}\cos^2(\phi_k) \qquad (3.7)\end{aligned}$$

where from (3.5),

$$\begin{aligned}&\text{E}\{w_k^2|\varrho_k, |B|\} \\ &= T_c^2\Big\{\text{var}\{\Lambda_k||B|\} + \text{var}\{\Gamma_k(1-2\varrho_k)|\varrho_k, |B|\} \\ &\quad + \text{var}\{\Upsilon_k(1-\varrho_k)|\varrho_k\} + \text{var}\{\Omega_k\varrho_k|\varrho_k\}\Big\}. \qquad (3.8)\end{aligned}$$

(3.8) can be evaluated by making use of the equations (2.46)-(2.59). In this case,

$$\begin{aligned}\text{E}\{w_k^2|\varrho_k, |B|\} &= |A| + |B|(1-2\varrho_k)^2 + (1-\varrho_k)^2 + \varrho_k^2 \\ &= T_c^2\left\{G + 2(2B+1)(\varrho_k^2 - \varrho_k)\right\} \qquad (3.9)\end{aligned}$$

Substituting (3.9) into (3.7) yields,

$$\nu = \sum_{k=2}^{K} 4P_kT_c^2\left[\frac{G}{2} + (2B+1)(\varrho_k^2 - \varrho_k)\right]\cos^2(\phi_k) \qquad (3.10)$$

The SGA uses the expected value of (3.10) in (3.6). Recall that ϱ_k is uniformly distributed over $[0, \lambda]$, ϕ_k is uniformly distributed over $[0, 2\pi]$ and $\text{E}\{|B|\} = (G-1)/2$. Then, averaging over the distributions of ϱ_k, ϕ_k and $|B|$ yields,

$$\begin{aligned}\sigma_M^2 &= \text{E}_{\varrho_k, \phi_k, |B|}\{\nu\} \\ &= \frac{G}{3}T_c^2(3 + 2\lambda^2 - 3\lambda)\sum_{k=2}^{K} P_k. \qquad (3.11)\end{aligned}$$

Now, if the system is asynchronous, $\lambda = 1$. Then, (3.11) simplifies to,

$$\sigma_{MI}^2 = \frac{2G}{3} T_c^2 \sum_{k=2}^{K} P_k. \qquad (3.12)$$

Similarly, if the system is synchronous, $\lambda = 0$. In this case, (3.11) simplifies to,

$$\sigma_{MI}^2 = G T_c^2 \sum_{k=2}^{K} P_k. \qquad (3.13)$$

Finally, quasi-synchronous systems have $\lambda \in (0, 1)$. For a given value of λ, the general expression of (3.11) can be used to evaluate the multiple access interference variance.

We can now substitute (3.11), (3.12) or (3.13) into (3.6) to find the expression for the average error probability. For an asynchronous system,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{1}{\frac{N_0}{2P_1T} + \frac{1}{3G}\sum_{k=2}^{K}\frac{P_k}{P_1}}}\right]. \qquad (3.14)$$

Similarly, for a synchronous system,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{1}{\frac{N_0}{2P_1T} + \frac{1}{2G}\sum_{k=2}^{K}\frac{P_k}{P_1}}}\right]. \qquad (3.15)$$

Finally, for a quasi-synchronous system,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{1}{\frac{N_0}{2P_1T} + \frac{1}{6G}(3 + 2\lambda^2 - 3\lambda)\sum_{k=2}^{K}\frac{P_k}{P_1}}}\right]. \qquad (3.16)$$

In typical mobile environments, communication links are interference limited and not noise limited. For an interference limited channel, the multiple access interference term variance would be significantly larger than the AWGN term variance. In this case, the generalized average error probability term for a DS CDMA system may be approximated by,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{6G}{(3 + 2\lambda^2 - 3\lambda)\sum_{k=2}^{K}\frac{P_k}{P_1}}}\right]. \qquad (3.17)$$

Furthermore, the probability of error expression can be further simplified if the individual power levels are identical, $P_1 = P_2 = \cdots = P_K = P$. This represents an ideal case scenario where the system maintains perfect power control. In this case,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{6G}{(3 + 2\lambda^2 - 3\lambda)(K - 1)}}\right]. \tag{3.18}$$

For an asynchronous system, the above expression becomes,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{3G}{(K - 1)}}\right] \tag{3.19}$$

which is the well-know error probability expression for DS CDMA systems that have appeared in the literature numerous times [13, 156, 178, 212].

Note that the SGA evaluates the error probability of a DS CDMA system assuming that the variance of the multiple access interference assumes its average value which in turn causes one to overlook the conditional independence of the multiple access interference terms. This sometimes yields quite optimistic results. An alternative expression for the error probability of the same system is presented in the next section. The alternative scheme averages the bit error rate over all possible operating conditions rather than evaluating it at the average operating medium, as was done in SGA.

3.1.2 Improved Gaussian Approximation

As stated before, the SGA does not always give accurate results. It has been stated in [156] and [185] that the SGA is reliable only when the number of active users in the system, K, is large. As shown in Chapter 2, the $(K - 1)$ multiple access interference terms are independent when conditioned on $C_1(1)$ which is defined in (2.53) and represents the discrete aperiodic autocorrelation function of the spreading sequence of the first user. The G terms that make up one of the $(K - 1)$ multiple access interference signals are also independent when conditioned on the relative time and phase offsets of the interfering signal, represented by ϱ_k and ϕ_k, respectively. When the variance of the multiple access interference term is averaged, as is the case in SGA, the information from the conditional independence is not utilized. This is the main reason why the SGA is not always reliable. In situations where the SGA is not appropriate, a more in-depth analysis which allows one to take the inter-dependency of the multiple access interference terms and the chip-to-chip dependency between the chips of every multiple interference term into account can be applied. The IGA is based on the observation that the $G \cdot (K - 1)$ multiple access interference terms in (2.31) are conditionally independent. Hence, from the central limit theorem, they tend towards a Gaussian distribution when conditioned on the delays and phases of all the interfering signals and on $|B|$ in (2.39) which represents the number of chip transitions during one symbol duration of the first user's spreading sequence. Thus, the IGA provides the bit error rate of a spread spectrum system using the equation,

$$\begin{aligned} P(E)_{IGA} &= \int_0^{\infty} Q\left(\frac{\sqrt{2P_1}T}{\sqrt{N_0 T + \nu}}\right) f_\nu(\nu) d\nu \\ &= \mathrm{E}_\nu\left\{Q\left(\frac{\sqrt{2P_1}T}{\sqrt{N_0 T + \nu}}\right)\right\} \qquad (3.20) \end{aligned}$$

where ν is the variance of the multiple access interference conditioned on ϱ_k, ϕ_k and $|B|$. Thus, when the IGA is employed, the error probability is simply an expectation of the well-known $Q(x)$ function, given in (3.1). Finding this expectation appears to be extremely complex since it involves the task of evaluating the conditional probability density function of the multiple access variance, $f_\nu(\nu)$. In [155, 156], first the conditional probability density function of a single user's signal variance is calculated. The density function for the variance of the multiple access interference is then a $(K - 2)$ fold convolution of the single user signal variance by itself since the DS CDMA system user signals are independent and identically distributed. [155, 156] proposes to perform this convolution numerically. Once a numerical expression for the density of ν is found, the integration in (3.20) can be performed numerically to find the error probability.

The requirement to perform a $(K-2)$ fold convolution numerically followed by another numerical integration is a rather time consuming approach especially for large values of G and K [80, 185]. A technique to speed up the evaluation time for the IGA by making use of a simple, Taylor series based approximation is presented in [80]. The simplified IGA expressions are based on the fact that using a Taylor series expansion, a function, $f(x)$, may be expressed around a point, μ as,

$$f(x) = f(\mu) + (x-\mu)f'(\mu) + \frac{1}{2}(x-\mu)^2 f''(\mu) + \cdots + \frac{1}{n!}(x-\mu)^n f^{(n)}(\mu) + \cdots \quad (3.21)$$

provided that $f'(x)$, $f''(x)$ and $f^{(n)}(x)$ which represent the first, second and n'th order derivatives of the function f evaluated at x, respectively, exist. In (3.21), if x is a random variable and μ is the expected value of x,

$$\begin{aligned} \mathrm{E}\{f(x)\} &= f(\mu) + \mathrm{E}\{(x-\mu)\}f'(\mu) + \frac{1}{2}\mathrm{E}\{(x-\mu)^2\}f''(\mu) + \cdots \\ &\quad + \frac{1}{n!}\mathrm{E}\{(x-\mu)^n\}f^{(n)}(\mu) + \cdots \\ &= f(\mu) + \frac{1}{2}\chi_2 f''(\mu) + \cdots + \frac{1}{n!}\chi_n f^{(n)}(\mu) + \cdots \\ &\simeq f(\mu) + \frac{1}{2}f''(\mu)\sigma^2 \end{aligned} \quad (3.22)$$

where $\chi_n = \mathrm{E}\{(x-\mu)^n\}$ represents the n'th central moment of the random variable x [166] and σ^2 is the variance of x. It is possible to ignore the higher order terms in (3.22) because the value of $\frac{1}{n!}$ dominates the term more and more as the term index increases.

We now substitute (3.22) into (3.20) to get,

$$P(E)_{IGA} \simeq Q\left(\frac{\sqrt{2P_1}T}{\sqrt{N_0T + \mu_\nu}}\right) + \frac{1}{2}Q''\left(\frac{\sqrt{2P_1}T}{\sqrt{N_0T + \mu_\nu}}\right)\sigma_\nu^2 \quad (3.23)$$

where μ_ν and σ_ν^2 represent the mean and the variance of the random variable ν, respectively.

Note that $Q(x)$ is a nonlinear function. In general, the derivatives of $Q(x)$ are not known. However, instead of starting with a Taylor series, as in (3.21), it is possible to use an expansion in central differences. Using differences does not necessarily represent another loss of accuracy in addition to truncating the series as is done in (3.22) [79] . Rather, using differences can be an opportunity for increasing accuracy. When the derivatives are replaced with the differences, (3.22) becomes,

$$\begin{aligned} \mathrm{E}\{f(x)\} &\simeq f(\mu) + \mathrm{E}\{(x-\mu)\}\frac{f(\mu+h)-f(\mu-h)}{2h} \\ &+\frac{1}{2}\mathrm{E}\{(x-\mu)^2\}\frac{f(\mu+h)-2f(\mu)+f(\mu-h)}{h^2}+\cdots \\ &\simeq f(\mu)+\frac{1}{2}\frac{f(\mu+h)-2f(\mu)+f(\mu-h)}{h^2}\sigma^2 \end{aligned} \tag{3.24}$$

resulting in

$$\begin{aligned} P(E)_{IGA} &\simeq \left[1-\frac{\sigma_\nu^2}{h^2}\right]Q\left(\frac{\sqrt{2P_1T}}{\sqrt{N_0T+\mu_\nu}}\right)+\frac{\sigma_\nu^2}{2h^2}Q\left(\frac{\sqrt{2P_1T}}{\sqrt{N_0T+\mu_\nu+h}}\right) \\ &+\frac{\sigma_\nu^2}{2h^2}Q\left(\frac{\sqrt{2P_1T}}{\sqrt{N_0T+\mu_\nu-h}}\right). \end{aligned} \tag{3.25}$$

Now, the difference, h can be chosen to yield good accuracy for the approximation. The value, $h=\sqrt{3\sigma^2}$ would have been exact if $Q(x)$ were a fifth order polynomial and ν were uniformly distributed [80]. Though neither are the case, [80] states that choosing $h=\sqrt{3\sigma^2}$ is still appropriate. In this case, (3.25) can be written as,

$$\begin{aligned} P(E)_{IGA} &= \frac{2}{3}Q\left(\frac{\sqrt{2P_1T}}{\sqrt{N_0T+\mu_\nu}}\right)+\frac{1}{6}Q\left(\frac{\sqrt{2P_1T}}{\sqrt{N_0T+\mu_\nu+\sqrt{3}\sigma_\nu}}\right) \\ &+\frac{1}{6}Q\left(\frac{\sqrt{2P_1T}}{\sqrt{N_0T+\mu_\nu-\sqrt{3}\sigma_\nu}}\right) \end{aligned} \tag{3.26}$$

Thus, we only need to find the mean and variance of the random variable ν to find the approximate error probability. The mean of ν was found in the previous section during the SGA analysis and is repeated here for completeness. From (3.11),

$$\mu_\nu = \frac{G}{3}T_c^2(3+2\lambda^2-3\lambda)\sum_{k=2}^{K}P_k. \tag{3.27}$$

The variance of ν can be found easily using,

$$\sigma_\nu^2 = \mathrm{E}\{\nu^2\} - \mu_\nu^2 \tag{3.28}$$

where

$$\begin{aligned}\mathrm{E}\{\nu^2\} &= \sum_{k=2}^{K}\sum_{l=2}^{K} 16 P_k P_l T_c^4 \cdot \mathrm{E}\{\cos^2(\phi_k)\cos^2(\phi_l)\} \\ &\cdot \mathrm{E}\left\{\left(\frac{G}{2} + (2|B|+1)(\varrho_k^2 - \varrho_k)\right)\left(\frac{G}{2} + (2|B|+1)(\varrho_l^2 - \varrho_l)\right)\right\}. \end{aligned}\tag{3.29}$$

It is straightforward to calculate the expectations in (3.29) to get,

$$\mathrm{E}\{\cos^2(\phi_k)\cos^2(\phi_l)\} = \begin{cases} \frac{1}{4} & k \neq l \\ \frac{3}{8} & k = l \end{cases} \tag{3.30}$$

and

$$\begin{aligned} &\mathrm{E}\left\{\left(\frac{G}{2} + (2|B|+1)(\varrho_k^2 - \varrho_k)\right)\left(\frac{G}{2} + (2|B|+1)(\varrho_l^2 - \varrho_l)\right)\right\} \\ &= \begin{cases} G^2\left[\frac{4\lambda^4 - 12\lambda^3 + 21\lambda^2 - 18\lambda + 9}{36}\right] \\ +(G-1)\left[\frac{4\lambda^4 - 12\lambda^3 + 9\lambda^2}{36}\right] & k \neq l \\ \\ G^2\left[\frac{12\lambda^4 - 30\lambda^3 + 40\lambda^2 - 30\lambda + 15}{60}\right] \\ +(G-1)\left[\frac{6\lambda^4 - 15\lambda^3 + 10\lambda^2}{30}\right] & k = l \end{cases} \end{aligned}\tag{3.31}$$

Substituting (3.29),(3.30),(3.31) and (3.27) into (3.28) we obtain,

$$\begin{aligned}\sigma_\nu^2 &= \sum_{k=2}^{K} P_k^2 T_c^4 \left[G^2\left(\frac{68\lambda^4 - 150\lambda^3 + 150\lambda^2 - 90\lambda + 45}{90}\right)\right. \\ &\quad \left. +(G-1)\left(\frac{6\lambda^4 - 15\lambda^3 + 10\lambda^2}{5}\right)\right] \\ &\quad + \sum_{k=2}^{K}\sum_{\substack{l=2\\ l\neq k}}^{K} P_k P_l T_c^4 (G-1)\left[\frac{4\lambda^4 - 12\lambda^3 + 9\lambda^2}{9}\right] \end{aligned}\tag{3.32}$$

If the system in question is asynchronous, $\lambda = 1$. In this case (3.32) simplifies to,

$$\sigma_\nu^2 = T_c^4\left[\frac{23G^2 + 18G - 18}{90}\sum_{k=2}^{K} P_k^2 + \frac{G-1}{9}\sum_{k=2}^{K}\sum_{\substack{l=2\\ l\neq k}}^{K} P_k P_l\right]. \tag{3.33}$$

This result has been presented in the literature in [80] and [185]. Similarly, if the system is synchronous, $\lambda = 0$. In this case,

$$\sigma_\nu^2 = T_c^4 \frac{G^2}{2} \sum_{k=2}^{K} P_k^2. \tag{3.34}$$

This result seems to be new. As stated before, quasi-synchronous systems have $\lambda \in (0,1)$. In this case, the variance is found using the equation (3.32). This result is new as well.

When perfect power control is achieved, $P_1 = \ldots = P_K = P$. In this case, for the asynchronous DS CDMA system, the mean and the variance expressions simplify to,

$$\mu_\nu = \frac{2G}{3} T_c^2 (K-1) P \tag{3.35}$$

and

$$\sigma_\nu^2 = T_c^4 P^2 \left[\frac{23G^2 + 18G - 18}{90}(K-1) + \frac{G-1}{9}(K-2)(K-1) \right]. \tag{3.36}$$

respectively.

Note that the equation (3.25) is only valid when

$$N_0 T + \mu_\nu > h \tag{3.37}$$

to ensure that the term inside the square-root operator at the denominator of the third term is positive [185]. Replacing μ_ν and σ_ν^2 with their values given in (3.35) and (3.36) and letting $h = \sqrt{3\sigma_\nu^2}$ yields,

$$\frac{N_0 T}{4} + \frac{N}{6} T_c^2 (K-1) P > \sqrt{\frac{3T_c^4 P^2}{4} \left[\frac{23N^2 + 18N - 18}{360}(K-1) + \frac{N-1}{36}(K-2)(K-1) \right]}. \tag{3.38}$$

Recall that ν is the conditional variance of the multiple access interference. Thus, from a practical point of view, this inequality states that the IGA is applicable only to cases where the multiple access interference is somewhat balanced. For systems where a single or a small group of interferers dominate and for systems where unequal number of users request services that have different bandwidth requirements, the above inequality may not hold for number of active users below a certain threshold. In this case, no result could be obtained from this version of the IGA. In such instances, the value of h can be lowered so that the inequality holds and thus (3.25) can be used. One should note however, that as h is decreased, the IGA performance approaches to that of the SGA [185].

3.1.3 Characteristic Function Method

An alternative method has been proposed by Geraniotis and Pursley in [59] to evaluate the error probabilities for DS CDMA systems. Unlike the SGA and IGA, this method does not rely on the Central Limit Theorem but rather on accurate evaluation and subsequent integration of the characteristic function of the multiple access interference. For this reason, the authors refer to this method as the Characteristic Function Method. In DS CDMA systems, the multiple access interference is a zero mean random variable with an even probability density function [59]. In this case, the corresponding characteristic function of the multiple access interference is real and even. Recall from (3.3) and (3.4) that,

$$\begin{aligned} P(E) &= P(D+M+N>0|a_{1,0}=-1) \\ &= \frac{1}{2}P(\sqrt{2P_1}T<-M-N)+\frac{1}{2}P(\sqrt{2P_1}T<M+N) \\ &= \frac{1}{2}\left[1-P(-\sqrt{2P_1}T<M+N<\sqrt{2P_1}T)\right] \\ &= \frac{1}{2}\left[1-\int_{-\sqrt{2P_1}T}^{\sqrt{2P_1}T} f_X(x)dx\right] \end{aligned} \tag{3.39}$$

where $X = M + N$ is the equivalent noise which is equal to the sum of the multiple access interference and the AWGN. Note that the characteristic function of X is real and even. In this case it is possible to write,

$$f_X(x) = \frac{1}{\pi}\int_0^{\infty} \Phi_X(\omega)\cos(\omega x)d\omega \tag{3.40}$$

using the basic definition of the characteristic function, $\Phi_X(\omega)$, of a random variable. Substituting (3.40) into (3.39) yields,

$$\begin{aligned} P(E) &= \frac{1}{2} - \frac{1}{\pi}\int_0^{\sqrt{2P_1}T}\int_0^{\infty} \Phi_X(\omega)\cos(\omega x)d\omega dx \\ &= \frac{1}{2} - \frac{1}{\pi}\int_0^{\infty} \frac{1}{\omega}\Phi_X(\omega)\sin(\sqrt{2P_1}T\omega)d\omega. \end{aligned} \tag{3.41}$$

Numerical evaluation of (3.41) constitutes the basis of the Characteristic Function Method. The technique presented in this thesis is based on the calculation of characteristic functions as well. However, unlike the Characteristic Function Method, no subsequent integration is necessary in this method. As will be evident in the following chapters, in certain instances, semi-analytical methods are necessary for the evaluation of the characteristic functions. The additional numerical integration operator present in the Characteristic Function Method greatly increases the computational complexity of the error probability evaluations especially for operating points that result in small error probabilities.

Geraniotis and Pursley consider both BPSK and QPSK modulated DS CDMA systems in AWGN channels in [59] using the Characteristic Function Method. Subsequently, they apply the Characteristic Function Method to calculate error probabilities for DS CDMA systems in multipath fading channels [60]. Both of these papers assume that the DS CDMA systems employ deterministic spreading sequences. In [62], Geraniotis and Ghaffari use the Characteristic Function Method to calculate error probabilities for DS CDMA systems that employ random spreading sequences in AWGN channels. In their work, they consider both rectangular and sinusoidal chip waveforms. In [61], Geraniotis extends this work further to include the effects of multipath Ricean fading. Finally in [201], Soroushnejad and Geraniotis use the

Characteristic Function Method to compare the performances of direct sequence and hybrid direct sequence-frequency hopping systems.

In all of the Geraniotis papers on the Characteristic Function Method, only numerical methods are used to calculate the characteristic functions of the multiple access interference. Recently, in [135], Liu and Despins make use of some of the integral equalities from [72] to provide faster, semi-analytical methods to evaluate these characteristic functions. However, integration of the characteristic functions is still performed numerically.

3.1.4 Gauss-Quadrature Method

Laforgia, Lavison and Zingarelli propose the use of Gauss-Quadrature rules to evaluate error probabilities for DS CDMA systems [111]. The moments of the multiple access interference are the driving force for the Gauss-Quadrature method. It is clear from (3.3) that, once conditioned on the multiple access interference, the error probability for the DS CDMA system can be written using the error function, $Q(x)$. The average error probability can be obtained numerically by averaging the conditional probability over the multiple access term using the Gauss-Quadrature method. This is accomplished by first evaluating the $N_m = 2N_c + 1$ moments of the multiple access interference which are applied in the evaluation of the N_c weights and nodes of the Gauss-Quadrature rule. The accuracy of this method is proportional to the number of moments considered, namely, N_m. The higher the value of N_m is, the higher the accuracy of the Gauss-Quadrature method would be. Clearly, this comes at the expense of increased computational complexity as it is increasingly more difficult to evaluate the higher order moments of the multiple access interference.

Laforgia, Luvison and Zingarelli use the Gauss-Quadrature method to evaluate error probabilities for DS CDMA systems in AWGN channels. Kavehrad in [97] extends this work to include the effects of frequency nonselective, Rayleigh fading. Subsequently Kavehrad and McLane in [98] use the Gauss-Quadrature method to evaluate error probabilities for DS CDMA systems in frequency selective, Rayleigh fading channels when selection diversity is employed at the receiver end to improve the performance.

3.1.5 Gram-Charlier Method

In [13], Bartucca and Biglieri propose the use of the Gram-Charlier expansion to express the unknown probability density function of the multiple access interference in terms of the derivatives of a known probability density function. The resulting expression for the multiple access interference probability density function is an infinite series where the series coefficients can be written using the Hermite polynomials. Once an expression for the probability density function is formed, the average error probability can be evaluated numerically. Bartucca and Biglieri admit that this technique provides satisfactory results when the number of users in the system is large and the signal-to-noise ratio is small, conditions for which the much simpler SGA and IGA give satisfactory results as well.

3.2 Bounds

Many techniques have been proposed in the literature for the calculation of upper and lower bounds for the error probability of multi-user spread spectrum systems.

3.2.1 Bounds Based on the Theory of Moment Spaces

One of the first notable papers in the area is by Yao where he finds upper and lower bounds for an asynchronous DS CDMA system in an AWGN channel using the theory of moment spaces [265]. Yao first defines generalized moments of a random variable as the expected value of a function of the random variable where the corresponding function is said to induce the moment. He then makes use of an isomorphism theorem from Game Theory which states, when a two-dimensional moment space is formed using two generalized moments induced by functions $k_1(z)$ and $k_2(z)$ of the given random variable z, the convex hull of the $k_1(z)$ versus $k_2(z)$ curve will be equal to the moment space itself. Yao, upon selecting the random variable to be the multiple access interference and one of the two functions to be the error function, $Q(x)$, so that the corresponding generalized moment is the error probability, replaces the convex hull of the $k_1(z)$ versus $k_2(z)$ curve with linear lines, and correspondingly finds upper and lower bounds. He tries various moments to find tight bounds. He investigates the second, fourth, single exponential and multiple exponential moments and concludes that by using the multiple exponential moment, arbitrarily tight bounds can be found at the expense of increased complexity. In [57], Gardner and Orr extend Yao's work to include the effects of multipath fading. The down side of the Moment Space method is that for tight bounds, rather complicated generalized moments are required.

3.2.2 Bounds Based on the Convexity of $Q(x)$

Pursley, Sarwate and Stark make use of the convexity properties of the error function, $Q(x)$, to find upper and lower bounds for the asynchronous DS CDMA system in an AWGN channel [180]. The authors first write the error probability conditioned on the multiple access interference in terms of the error function, $Q(x)$. The multiple access interference however, is a function of the relative time delays and carrier phases of all of the interfering signals. Thus, the unconditional error probability can be written in terms of a multidimensional integral. Pursley, Sarwate and Stark replace the multidimensional integral by a sequence of one dimensional integrals and then recursively bound the sequence of integrals to provide the upper and lower bounds on the error probability. The bounds derived in this work require the maximum multiple access interference to be less than the desired signal component at the output of the correlator. One major setback of this work is the exponential increase in the computational complexity with the increasing number of active users in the multi-user system, making it practically impossible to find values for most systems of current interest.

3.2.3 Bounds Based on Quantization of the Multiple Access Interference Distribution

Lehnert and Pursley find arbitrarily tight upper and lower bounds for the error probability of an asynchronous DS CDMA system in an AWGN channel by bounding the distribution of the multiple access interference [124]. The authors proceed by first dividing the interval containing all possible values of the total multiple access interference into a set of subintervals. For each interval, upper and lower bounds are found on the probability that the value of the multiple access interference lies in that interval. The set of upper and lower bounds corresponding to the set of subintervals form two vectors, one for the upper bounds and one for the lower bounds. These vectors are then used to obtain the bounds on the average probability of error for the DS CDMA system. The bounds become tighter as the size of the vectors grows or correspondingly, the number of subintervals increases. Lehnert and Pursley calculate these bounds for systems that employ pseudorandom spreading sequences in [124]. Subsequently, Lehnert reiterates this bounding technique in [126] to calculate upper and lower bounds for asynchronous DS CDMA systems that employ deterministic spreading sequences. Among all of the proposed bounds in the literature for the calculation of DS CDMA error probability, these bounds have become the most popular ones. Recently, Lam and Özlütürk in [112] and [164] use the same technique to calculate upper and lower bounds for the error probabilities of BPSK and MPSK modulated

DS CDMA systems that employ complex valued spreading sequences, respectively.

3.2.4 Bounds for the Convolutionally Coded DS CDMA System

Finally, Pursley and Taipale calculate the well-known union bound to upper bound the error probability for a convolutionally encoded DS CDMA system that uses Viterbi decoding at its receiver end [181].

3.3 Analyzing DS CDMA Systems Using Approximations and Bounds: A Literature Survey

The analysis tools described above have found extensive use in the literature towards the analysis of various DS CDMA systems in various channel conditions.

In one of the key papers on DS CDMA systems, Lunayach compares the performance of DS CDMA systems in the presence of white Gaussian noise with no synchronization errors [138]. He first compares spread spectrum systems, where the spreading sequence has a long period, with systems with spreading sequences that are one symbol duration long. He concludes that the short codes are acceptable for multiple access communications but to withstand intentional jamming, long period spreading sequences are required. He also compares biphase spreading with quadriphase spreading and concludes that quadriphase spreading achieves a 2-3 dB performance improvement for long spreading codes towards supressing a coherent jammer. For a system where there exist multiple access interference from other users but no coherent jammers, he concludes that the biphase and quadriphase spreading provide almost the same performance. Finally, he compares the BPSK and QPSK modulation schemes and concludes that as far as the bit error rate performance is concerned, quadrature modulation does not bring any improvement. In the analysis, he approximates the multiple access interference with a Gaussian random variable by making use of the central limit theorem. This approximation has been used in later publications extensively. In 1990, Sousa [203] rewords the conclusion that was drawn in [138] that the performance of a DS CDMA multiple access system is worse for the case of one strong interferer when compared to the performance of the system with many weak interferers.

Giubilei in [70] analyzes an asynchronous CDMA system by making use of some general information theory concepts such as the cut-off rate and efficiency. For an additive white Gaussian noise channel with BPSK modulation, a maximum efficiency of 2.16 bits/chip can be achieved. He finds that when a convolutional code of rate 1/8 is used, an efficiency of 0.877 bit/chip is achieved for a bit error rate of 10^{-3}.

Viterbi in [243] treats the DS CDMA system outlined in detail in Chapter 2 and introduces orthogonal convolutional codes to be used in this system. He shows that, ultimately, the capacity of the AWGN multiple access channel can approach the Shannon capacity in the limit of using arbitrarily long codes. In [68] and [69] Gilhousen et.al. and in [188] and [189] Salmasi et.al. present the DS CDMA system of Chapter 2 and claim that by making use of the voice activity factor and diversity, the capacity of CDMA would be greater than the capacity of TDMA or FDMA.

In [110], Kwong, Perrier and Prucnal show that for a BPSK DS CDMA system with no synchronization errors, synchronous CDMA always provides a lower probability of error than asynchronous CDMA. They make use of the Gaussian approximation for the multiple access interference following the work of Lunayach in [138].

In the important paper by De Gaudenzi, Elia and Viola [43], detection losses due to imperfect carrier frequency and chip timing synchronization are analytically derived for a QPSK modulated DS CDMA system in an AWGN channel. The authors show that synchronization errors affect the system performance drastically by comparing the fully synchronized system performance with an asynchronous system. For this reason, they propose a new multiple access scheme, which they call "bandlimited quasi-synchronous CDMA (BLQS CDMA)." This scheme, used in conjunction with the Gold codes, requires network terminal synchronization with an error of less than ±0.3 chips. Thus, the timing error between a receiver and a transmitter lies in the interval $[-0.3T_c, 0.3T_c]$ where T_c is the chip duration. The bandwidth efficiency is accomplished by using Nyquist shaped chips as opposed to rectangular pulse chips. In the analysis, they make use of the Gaussian approximation of the multiple access interference. One important remark about BLQS CDMA is that, unlike the traditional CDMA scheme, application of convolutional codes increases the symbol rate and consequently reduces the number of quasi-orthogonal spreading sequences for a given signal bandwidth. For this reason, the authors claim that Trellis coded modulation is a better alternative for BLQS CDMA. The network synchronization is accomplished by means of a special carrier called the master signal (similar to the pilot channel of the system described in Chapter 2), obtained by direct sequence modulating a precise frequency reference. This signal is then transmitted embedded in the CDMA signal structure. The presence of a reference available to all terminals in the network suggests the use of a master-slave synchronization technique.

In [152], Milstein, Rappaport and Barghouti evaluate the performance of a DS CDMA system for a flat Rayleigh fading channel with AWGN. The Gaussian approximation for multiple access interference is used in this paper as well. Under the assumption of perfect synchronization, the average probability of error of a user straddling the boundary between two adjacent cells is derived. The bit error rate for a mobile at the intersection of four cells is also evaluated. The bit error rate calculations are first done for perfect power control and then this constraint is relaxed to show the effects of imperfect power control.

Stüber and Kchao in their papers [101, 211] analyze a single cell and multiple cell DS CDMA system, respectively under the assumption of perfect synchronization. The effects of path loss, shadowing, multipath diversity, noise and multiple access interference are considered in the bit error rate calculations. They make use of the Gaussian approximation for multiple access interference. It is assumed that power control is perfect and accounts for path loss and shadowing. They make use of RAKE receivers to combat fading and use Gold codes as PN sequences and make use of BCH codes for error correction. They simulate the area averaged error probability as a function the number of mobiles per cell both for the uplink and the downlink channels to conclude that for the uplink channel, a significant improvement in performance can be obtained by using base station diversity and voice activity detection.

In [235], Torrieri gives a performance analysis of DS CDMA systems with both binary and quaternary spreading sequences assuming perfect synchronization. Like the majority of the papers in the literature, here, multiple access interference is approximated as a Gaussian process. Using Jensen's inequality, upper and lower bounds on the error probability are also found. These bounds are used to further approximate the bit error rate. Error rate calculations for multiple access, tone jamming, multipath interference and AWGN are performed.

In [248, 252], Viterbi, Viterbi and Zehavi state the experimentally found probability density function for the received signal power to interference power and state that this probability density function closely follows the log-normal distribution with mean and standard deviation of the normal component of 7 and 2.4, respectively. In [252], they find the blocking probability of the system of Chapter 2, assuming a Poisson arrival rate and an exponential service rate. They define the blocking probability as the probability that a new user will find all channels busy and hence be denied service. The average traffic load in terms of average number of users requesting service resulting in this blocking probability is called the Erlang capacity. The blocking probability of a FDMA or TDMA system depends on the number of servers present in the system. Hence, the Erlang capacity for these schemes are found by making use of the conventional M/M/S/S queue characteristics. However, CDMA is interference limited. The blocking probability of CDMA is defined by the authors as the event that the total interference to background noise level exceeds 10 dB. Erlang capacity is defined as the number of users resulting in a one percent blocking probability. By making use of the experimentally found density function for the received signal to interference ratio. they find the Erlang capacity for CDMA and compare this with that of TDMA and FDMA. They claim that CDMA results in an Erlang capacity that is 20 times that of the other multiple access schemes for a voice activity factor of 0.4.

In [140], Madhow and Pursley find the effect of multiple access interference on the chip timing acquisition of DS CDMA systems. They assume that there is no data modulation and that carrier frequency and phase are acquired perfectly. They assume that the multiple access interference is the only interference present in the system. A sliding correlator approach is used for acquisition. The acquisition based capacity which they define as the maximum number of users is a function of the acquisition window length. subject to the constraint that the probability of acquisition failure tends to zero as the acquisition window length goes to infinity. They conclude that this number is in the order of $N/\log N$. where N is the acquisition window length. The basic reason is that. unless the receiver has a very good idea of the delay of the target transmission. there are too many opportunities for the interfering signals to produce a false alarm. and this results in a reduction of capacity by $\log N$.

Jalali and Mermelstein in [89. 148] evaluate the capacity of the system described in Chapter 2 for the case where there are no synchronization errors. They consider fading, path loss. power control. diversity and bandwidth in the capacity evaluation. The analysis is done in two stages. First. for a user. the required signal to interference ratio for a given bit error rate (10^{-3} for voice transmission) is found. Then. in a grid of 11×11 squares. each grid representing a microcell. handsets are randomly distributed. The handsets. by using the received pilot signals from the base stations. decide on which base station to communicate with and establish power control accordingly. The center grid base station checks the signal to interference ratio and compares it with the value found in the first stage of analysis. This procedure is repeated for a number of different random distributions of handsets. If 98% of the configurations for a given number of handsets result in a signal to interference ratio higher than the value found in stage one, one more handset is included into the system and the procedure is repeated. This is continued until the above requirement is not met. For the bit error rate analysis, a Gaussian approximation for the multiple access interference is assumed and the above analysis is used to find the overall system capacity.

In [266], Yoon, Kohno and Imai discuss a canceller structure that cancels cochannel interference by creating replicas of the contributions of the cochannel interference embedded in the correlator outputs and by removing them for a improved hard data decision for a BPSK DS CDMA system in a slow fading, AWGN environment. In the canceller structure, after having completed acquisitions of the spreading sequence of a user, the receiver coherently demodulates and despreads the received signal to get the initial data estimates. Based on these estimates, the cancellation scheme essentially creates replicas of the contributions of the cochannel interference embedded in the correlator outputs and removes them for a second, improved hard data detection. Yoon, Kohno and Imai give bit error rate analysis of the canceller structure and conclude that this structure brings significant improvements to the system performance over the traditional reception techniques.

In [34], Chung, Chien, Samueli and Jain define a VLSI architecture for an asynchronous, BPSK modulated DS CDMA receiver for the AWGN channel. Coherent demodulation is considered. The system in question has an information data rate of 100 kbits/sec and a required bit error rate of at most 10^{-5}. The data signal is spread by a factor of 127 using maximal length Gold codes so that the chip rate for the system is 12.7 Mchips/sec. The allocated RF bandwidth for the system is assumed to be the 902-928 MHz band. The architecture uses a Costas loop for carrier recovery, a delay-locked loop for clock recovery and an energy detection scheme for PN acquisition. The authors conclude that these structures result in acceptable system performance and that it is feasible to implement an entire DS CDMA receiver on a single CMOS chip using approximately 50000 transistors.

In [9], Asano, Daido and Holtzman analyze the performance of a coherently received, asynchronous DS CDMA system that uses band-limited chip pulses as opposed to the ideal rectangular shaped chips. They investigate the use of various raised cosine filters to band-limit the system spectra. They make use of the Standard Gaussian Approximation to estimate the system bit error rate and conclude that bandwidth restriction by means of chip shaping results in performance degradation relative to the use of rectangular chips. For example, if an ideal low-pass filter is used to suppress the out-of-band emission caused by the use of rectangular chips, they conclude that the system capacity is reduced by a factor of 2/3 at any given system bit error rate requirement.

In [12], a DS CDMA based indoor mobile system that utilizes macro diversity is simulated. The simulation model presented in the paper is for a single floor indoor environment with an area of 60×60 meters and is covered by 6 base stations. The simulation accounts for the effects of imperfect power control, soft handoff, indoor channel propagation loss and the presence of internal light walls scattered around the floor. The telecommunication traffic is assumed to follow an exponential distribution and the mean value of the duration of an individual call and the mean time between two consecutive call set-ups for the same user are set at 100 seconds and 500 seconds, respectively. The authors find that these set values correspond to an average load of 6 active users per base station. The system is assumed to provide service at a transmission rate of 16.62 kbits/sec. The simulation is intended to obtain the cumulative distribution of the signal to interference ratio and forced link disconnections and the authors provide results for various system scenarios. The details of the simulation technique and speed are not presented in the paper.

In [77], Higashi and Matsumoto propose a new, adaptive diversity combining scheme for a differential PSK (DPSK) modulated DS CDMA system in a multipath fading environment. Here, the demodulated signals from various paths are unequally weighted and combined. The values of the weight coefficients are determined so that the mean square errors in the combiner output are minimized. To enable such a criterion, every signal that is to be transmitted is embedded by a known training sequence of 15 symbols every 48 or 96 information symbols. The weight coefficients are updated every time the receiver demodulates the 15 symbol long training sequence and compares it with its actual form. A lower bound on the performance of this system is given in the paper. The adaptive diversity combining scheme is compared to the conventional equal gain combining and it is shown that the proposed adaptive combining scheme outperforms the equal gain combiner at all signal to noise ratio levels. The authors also investigate the performance of the adaptive diversity combining scheme when a concatenated coding scheme that uses a cyclic redundancy code (CRC) together with a block code is used.

In [40] Da Silva and Sousa use an exhaustive search method to show that Walsh-Hadamard codes are the best codes amongst the set of orthogonal codes they consider for DS CDMA systems due to their relatively low cross-correlation properties even when perfect synchronization is not maintained. For this reason, the authors find the quasi-synchronous DS CDMA system performance for various types of orthogonal codes by making use of the Standard Gaussian Approximation and show that systems that employ Hadamard codes result in the lowest bit error rates.

Eng and Milstein in [49] use the Standard Gaussian Approximation to estimate the performance of a coherently received, asynchronous DS CDMA system in a multipath fading environment where individual path gains follow the Nakagami-m distribution.

The authors consider a RAKE receiver that utilizes maximal ratio combining and conclude that increasing the number of independent paths exhibits the usual diminishing returns characteristic of diversity systems, a phenomenon well-documented in the literature [87, 176]. They also conclude that, for a DS CDMA system with a processing gain of 127 and a 10 branch RAKE receiver, increasing the average received signal-to-noise ratio of the strongest path beyond 10 dB only slightly improves the bit error rate performance.

Finally, Fong, Bhargava and Wang in [53, 54] estimate the performance of a DS CDMA system that uses a concatenation of PN codes and Walsh-Hadamard codes for spectrum spreading using the Standard Gaussian Approximation and conclude that when perfect synchronization is achieved, the concatenation of the PN sequences with orthogonal Walsh-Hadamard codes provides a much lower multiple access interference level and thus a higher system capacity.

3.4 Summary

In this chapter, a literature survey on the analysis tools for DS CDMA systems is given. Methods that use approximations as well as bounds are presented. Furthermore, results from the literature on DS CDMA systems obtained by using the described analysis tools are presented.

Chapter 4

PERFORMANCE OF DS CDMA SYSTEMS IN AWGN CHANNELS

In this chapter, we present a performance analysis method to calculate the bit error rates of DS CDMA systems in AWGN channels. The method makes use of an accurate Fourier series expansion of $Q(x)$ which was originally derived in [14]. Using the series, we derive an analytical expression for the probability of bit error and find the system capacity from this expression for a given signal to noise ratio and maximum allowable bit error rate.

As evident from the previous chapter, there have been a fair number of papers on the calculation of error probabilities for DS CDMA systems in the literature. The approach taken in this thesis differs from the rest in a number of ways. First, the majority of these papers use approximations for the distributions of the multiple access and intersymbol interference encountered in the system while calculating error probabilities claiming that the exact calculation is computationally difficult [79]. In this thesis, however, we derive an accurate expression for the probability of error without approximating the multiple access or the intersymbol interference terms. Despite the lack of a simplifying approximation, the calculation of the expression is still very fast. In fact, a whole bit error rate versus signal to noise ratio curve of ten points can be generated in approximately two minutes on a Sun Sparc 10 work station. Furthermore, unlike the other papers, we do not neglect the presence of synchronization errors. In fact, we use the developed probability of error expression to investigate the performance loss in the system for various levels of synchronization errors. We show that a synchronization error, be it a chip timing error, a carrier phase error, or both, can be represented as an effective loss in the system processing gain.

We use the system models developed in Chapter 2 to derive the probability of error expressions. These models assume that correlator receivers are used in the system. It is well-known that such receivers are optimal in the minimum mean square error sense for an AWGN input [176]. We start this chapter with a brief introduction on Fourier series since it forms the backbone of the developed probability of error expressions. We then summarize [14, 15] to show how the Fourier series can be used to express the conventional error function, $Q(x)$. The series expression for $Q(x)$ is then used towards the generation of the probability of error expressions for DS CDMA systems in AWGN channels. Expressions for both biphase spread and quadriphase spread systems are generated in this chapter.

4.1 A Brief Overview of Fourier Series

In a memorable session of the French Academy on the 21st of December 1807, J. Fourier announced a thesis which inaugurated a new chapter in the history of applied mathematics. Fourier claimed that an arbitrary periodic function with period T could always be resolved into a sum of pure sine and cosine functions of frequencies $f_0 = 1/T$ and all its integral multiples. According to his theory, if $g(t)$ was a periodic function with period T, it could be written as,

$$g(t) = a_0 + 2\sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n \sin\left(\frac{2\pi nt}{T}\right)\right) \tag{4.1}$$

where the series coefficients, a_n and b_n could be calculated using,

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} g(t)dt \tag{4.2}$$

$$a_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\cos\left(\frac{2\pi nt}{T}\right) dt, \qquad n = 1, 2, \ldots \tag{4.3}$$

$$b_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\sin\left(\frac{2\pi nt}{T}\right) dt, \qquad n = 1, 2, \ldots \tag{4.4}$$

Clearly, the Fourier series of (4.1) can be put into a more compact form by making use of complex exponentials. In this case, we can rewrite (4.1) as,

$$g(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nt/T} \tag{4.5}$$

where

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} g(t)e^{-j2\pi nt/T}, \qquad n = 0, \pm 1, \pm 2, \ldots \tag{4.6}$$

In (4.1) and (4.5) the frequency $f_0 = 1/T$ is known as the Fundamental Frequency and the frequency nf_0 is known as the n'th Harmonic Frequency.

The claim of being able to represent any periodic function in terms of an infinite sum of pure sine and cosine functions was received by the referees of the thesis, Lagrange, Laplace and Legrendre with skepticism due to a "certain looseness of reasoning" in the development of the theory [20]. The referees found it very hard to believe that any superposition of the sine and cosine functions, which were analytical, could correctly represent an arbitrary function. Lacking the necessary tools in infinite series theory, Fourier could not justify his finding at the time. He started working at the Ecole Polytechnique and supervised a number of mathematicians. L. Dirichlet from Prussia was among them. Dirichlet was intrigued by Fourier's theory and he decided to work on it. He developed the first rigorous proof of the convergence of the Fourier series for a function subject to certain conditions. This allowed him to examine the "arbitrariness" of the periodic functions that allowed expansion in the Fourier series, and to put forward a set of conditions on such functions:

1. The function $g(t)$ had to be absolutely integrable, that is,

$$\int_{T/2}^{T/2} |g(t)|dt < \infty$$

2. The function $g(t)$ had to be single valued within one period interval.

3. The function $g(t)$ had to have at most a finite number of maxima and minima over one period interval.

4. The function $g(t)$ had to have at most a finite number of discontinuities over one period interval. Moreover, each of these discontinuities had to be finite.

Today, these conditions are commonly referred to as the Dirichlet Conditions [162].

While these refinements of Fourier's original discovery belonged to the realm of pure mathematics, the possibility of resolving all the ordinarily occurring functions of the physical world into pure sine and cosine terms had profound repercussions in the most varied areas of mathematical physics. Maybe the most fundamental of its impacts was in electrical engineering where the theory of electric networks with all its ramifications was simply inconceivable without the Fourier transform or Fourier series.

The main thrust of this work stems from this basic study. Throughout this dissertation, we use the Fourier series to describe certain physical phenomena that are encountered in wireless communication networks. Description of certain functions using their Fourier series enable us to pursue the mathematical analysis of rather complicated systems. Specifically, we use an approximate Fourier series and the approximation is made by making use of the well-known Chernoff bound. This theory was first developed by N. Beaulieu in his papers for AWGN channels with intersymbol interference [14, 15, 16].

4.2 Use of Fourier Series to Represent $Q(x)$

Noise in digital communication systems is frequently assumed to possess an amplitude probability density function that is Gaussian. Consequently, the error probability of

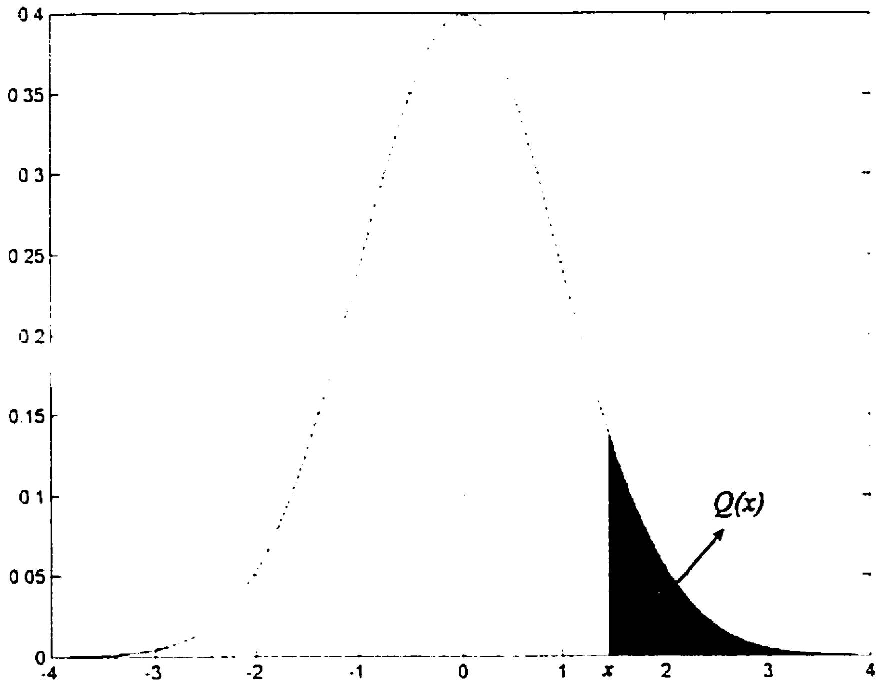

Figure 4.1: Definition of $Q(x)$.

many systems can be expressed in terms of $Q(x)$, where,

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} dt \tag{4.7}$$

is the area under the tail of the zero mean, unity variance Gaussian probability function as shown in Figure 4.1. As can be seen from Figure 4.2, $Q(x)$ is a strictly monotonically decreasing function.

In [14], Beaulieu derives a Fourier series approximation for $Q(x)$ which is based on the conventional Fourier series overviewed in Section 4.1 and the Chernoff bound. In this section we summarize this derivation. To apply the Chernoff bound to the Fourier series representation we first define two random variables, Y and Z. We assume Y is zero mean, unit variance Gaussian distributed and Z is a zero-one random variable

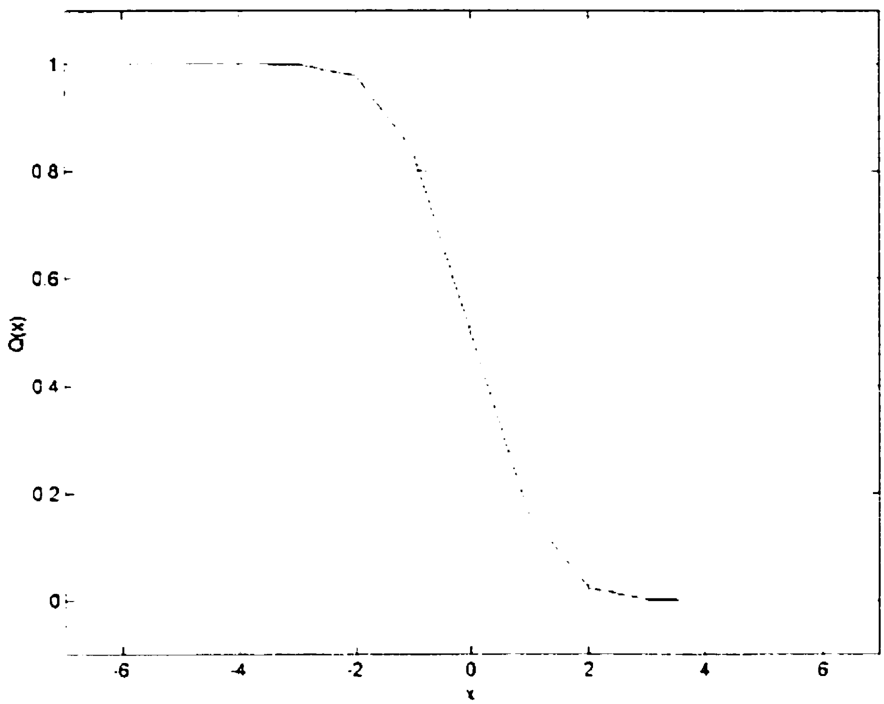

Figure 4.2: $Q(x)$.

dependent on Y such that,

$$Z = \begin{cases} 0, & y < x_0 \\ 1, & y \geq x_0 \end{cases} \tag{4.8}$$

where $x_0 \geq 0$ is a constant and y is a specific value of Y. The mean value of Z can then be written as,

$$E\{Z\} = P(Y \geq x_0) = Q(x_0). \tag{4.9}$$

In other words, once the random variables Y and Z are defined, $Q(x)$ can be defined in terms of these random variables. Next, we define a periodic square wave, $r(t)$, such that,

$$r(t) = \begin{cases} 1, & (2n)T/2 < t < (2n+1)T/2 \\ 0, & (2n+1)T/2 < t < (2n+2)T/2 \\ 1/2, & t = n(T/2) \end{cases} \tag{4.10}$$

We then find the Fourier series representation of $r(t)$. From (4.2)-(4.4), we find that,

$$a_0 = \frac{1}{2}, \tag{4.11}$$

$$a_n = 0, \tag{4.12}$$

$$\begin{aligned} b_n &= \frac{1}{2\pi n}(1 - \cos(\pi n)) \\ &= \begin{cases} 0, & n \text{ is even} \\ \frac{1}{\pi n}, & n \text{ is odd} \end{cases} \end{aligned} \tag{4.13}$$

resulting in the representation,

$$r(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{\sin\left(\frac{2\pi n}{T} t\right)}{n}. \tag{4.14}$$

Using Figure 4.3 one can easily see that,

$$Q(x_0) - Q(x_0 + \frac{T}{2}) < E\{r(Y - x_0)\} < Q(x_0) + Q(\frac{T}{2} - x_0). \tag{4.15}$$

Equation (4.15) can be rewritten as,

$$-Q(x_0 + \frac{T}{2}) < E\{r(Y - x_0)\} - Q(x_0) < Q(\frac{T}{2} - x_0). \tag{4.16}$$

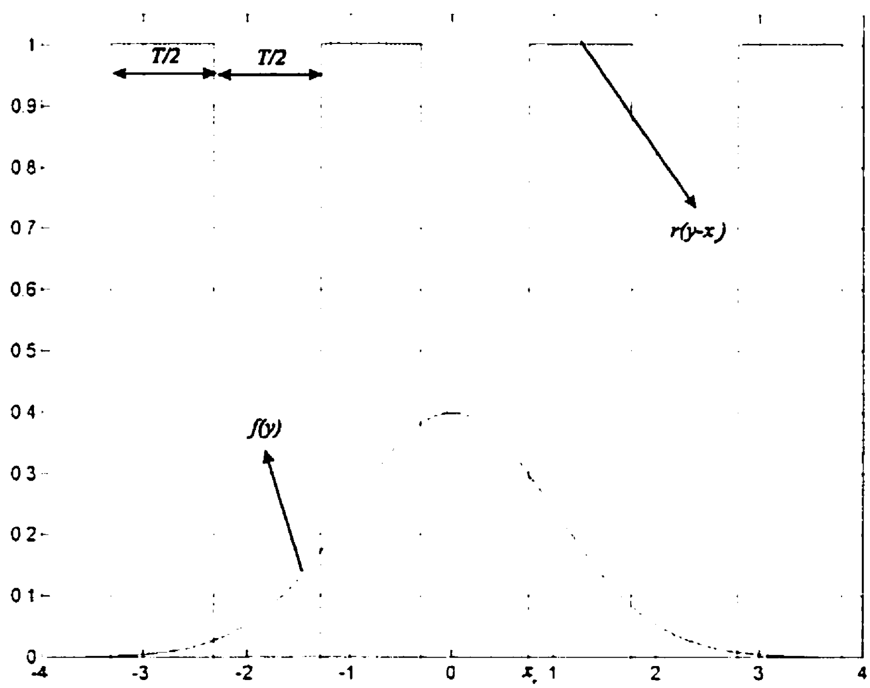

Figure 4.3: The Gaussian probability density function, $f(y)$ and the periodic square wave, $r(t)$.

Following [14, 15], we define,

$$\Delta_1 = Q(x_0 + \frac{T}{2}) \qquad (4.17)$$

$$\Delta_2 = Q(-x_0 + \frac{T}{2}) \qquad (4.18)$$

Using the definitions, (4.16) can be rewritten as,

$$-\Delta_1 < \mathrm{E}\{r(Y - x_0) - Q(x_0)\} < \Delta_2. \qquad (4.19)$$

Since $Q(x)$ is a strictly monotonically decreasing function, one can state that,

$$\lim_{T\to\infty} \Delta_1 = 0, \qquad (4.20)$$

$$\lim_{T\to\infty} \Delta_2 = 0. \qquad (4.21)$$

Thus, given a desired accuracy, ϵ, one can choose a T such that

$$|\mathrm{E}\{r(Y - x_0)\} - Q(x_0)| < \epsilon \qquad (4.22)$$

by ensuring that $\epsilon < \max\{\Delta_1, \Delta_2\}$. Assume that such a selection for T is performed. We have from (4.14),

$$\mathrm{E}\{r(Y - x_0)\} = \frac{1}{2} + \frac{2}{\pi}\sum_{\substack{n=1\\ n\ \mathrm{odd}}}^{\infty} \frac{\mathrm{E}\left\{\sin\left(\frac{2\pi n}{T}(Y - x_0)\right)\right\}}{n}. \qquad (4.23)$$

Since Y is defined as a zero mean, unit variance Gaussian random variable we have,

$$\mathrm{E}\left\{\sin\left(\frac{2\pi n}{T}(Y - x_0)\right)\right\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-y^2/2}\sin\left(\frac{2\pi n}{T}(y - x_0)\right) dy. \qquad (4.24)$$

Using the equality

$$\int_{-\infty}^{\infty} e^{-(ax^2+2bx+c)}\sin(px^2 + 2qx + r)dx =$$
$$\frac{\sqrt{\pi}}{\sqrt[4]{a^2 + p^2}}\exp\left\{\frac{a(b^2 - ac) - (aq^2 - 2bpq + cp^2)}{a^2 + p^2}\right\}$$
$$\cdot\sin\left(\frac{1}{2}\arctan\left(\frac{p}{a}\right) - \frac{p(q^2 - pr) - (b^2 p - 2abq + a^2 r)}{a^2 + p^2}\right), \qquad a > 0 \quad (4.25)$$

from page 485 in [72], (4.24) can be solved yielding,

$$\mathrm{E}\{r(Y - x_0)\} = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{e^{-n^2 w^2/2} \sin(n w x_0)}{n} \tag{4.26}$$

where $w = 2\pi/T$. (4.26) is a convergent series that is used the represent $Q(x)$ in the probability of error analysis in the remainder of this thesis. Since the Chernoff bound is invoked, (4.26) is strictly true only in the limit as T goes to ∞. However, Beaulieu in [14] states that with $T = 28$ and with 17 terms included in the sum, the magnitude of the relative error defined as

$$\text{Error} = \left| \frac{\frac{1}{2} - \frac{2}{\pi} \sum_{\substack{n=1 \\ n \text{odd}}}^{\infty} \frac{\exp(-n^2 w^2/2) \sin(n w x)}{n} - Q(x)}{Q(x)} \right| \tag{4.27}$$

is less than 6.3×10^{-7} for values of x less than 6 and less than 5×10^{-10} for values of x less than 4.85. Obviously, T can be increased to gain further accuracies. In this case, the number of terms than need to be taken into account in the sum should be increased as well.

4.3 Analysis of Biphase Spread DS CDMA Systems in AWGN Channels

The performance of DS CDMA systems is usually measured in terms of the probability of error as a function of the number of active users in the system for a given signal to noise ratio (SNR). In this section we derive a series expression for the probability of error for the biphase spread DS CDMA systems described in Section 2.1 and use this expression to evaluate the performance of the system.

4.3.1 Probability of Error Series Derivation

Recall from Chapter 2 that the system of interest uses binary signaling and that the k'th user's information and spreading sequences are defined as (2.2) and (2.3), respectively. Also recall that without any loss of generality, one can assume the symbol $a_{1,0}$ from the information signal of the first user, $a_1(t)$ is to be captured by the correlator receiver shown in Figure 2.2. In this case, the input signal to the decision device is given by (2.67).

Suppose that $a_{1,0} = 1$ represents the binary symbol 1 and $a_{1,0} = -1$ represents the binary symbol 0. The decision device in Figure 2.2 produces the symbol 1 if $Z_1 > 0$ and the symbol 0 if $Z_1 < 0$. An error occurs if $Z_1 < 0$ when $a_{1,0} = 1$ or if $Z_1 > 0$ when $a_{1,0} = -1$. Then, since $a_{1,0}$ is assumed to take on values $\{\mp 1\}$ randomly, the probability of error is simply equal to the probability of having $Z_1 > 0$ when $a_{1,0} = -1$. Recall from (2.67) that the input signal to the decision device is $Z_1 = D_1 + D_2 + M + N$ where D_1 is the desired signal, D_2 is the self interference term due to chip timing errors, M is the multiple access interference term and N is the AWGN term with zero mean and variance $N_0 T$. Then, the probability of error conditioned on the random variables D_2 and M is written as,

$$\begin{aligned} P(E|D_2, M) &= P(N > \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)G(T_c - (\tau_1 - \hat{\tau}_1)) - D_2 - M) \\ &= Q\left[\frac{\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)G(T_c - (\tau_1 - \hat{\tau}_1)) - D_2 - M}{\sqrt{N_0 T}}\right]. \end{aligned} \tag{4.28}$$

The random variables D_2 and M arise from different physical sources. Hence they are independent. Thus, using the total probability theorem [166], the unconditional error probability can be written as,

$$P(E) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(E|D_2, M) f_{D_2}(D_2) f_M(M) dD_2 dM. \tag{4.29}$$

The probability density functions $f_{D_2}(D_2)$ and $f_M(M)$ are difficult to determine. Instead, we proceed to rewrite the conditional error probability given in (4.28) using the Fourier series representation for $Q(x)$ derived in the previous section. We define $Q(x)$ to be,

$$Q(x) = \sum_{m=-\infty}^{\infty} c_m e^{jmwx} \tag{4.30}$$

where $w = 2\pi/T$ is the Fourier series fundamental frequency and c_m are the series coefficients. From (4.26), one can write c_m as,

$$c_m = \begin{cases} \frac{j}{\pi m} e^{-m^2 w^2/2}, & m > 0 \text{ and } m \text{ odd} \\ 0, & m > 0 \text{ and } m \text{ even} \\ \frac{1}{2}, & m = 0 \end{cases} \quad (4.31)$$

and $c_{-m} = -c_m, m > 0$. (4.30) becomes exact in the limit when w goes to zero.

If we substitute (4.28) and (4.30) into (4.29), we obtain,

$$\begin{aligned} P(E) &= \sum_{m=-\infty}^{\infty} c_m e^{\frac{jmw\sqrt{2P_1}\cos(\phi_1-\hat{\phi}_1)G(T_c-(\tau_1-\hat{\tau}_1))}{\sqrt{N_0 T}}} \\ &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jmw\frac{D_2+M}{\sqrt{N_0 T}}} f_{D_2}(D_2) f_M(M)\, dD_2\, dM. \end{aligned} \quad (4.32)$$

But, the characteristic function of a random variable t is defined as,

$$\Phi_t(w) = \mathrm{E}\{e^{jwt}\} = \int_{-\infty}^{\infty} e^{jwt} f_t(t)\, dt. \quad (4.33)$$

Then, (4.32) becomes,

$$\begin{aligned} P(E) &= \sum_{m=-\infty}^{\infty} c_m e^{\frac{jmw\sqrt{2P_1}\cos(\phi_1-\hat{\phi}_1)G(T_c-(\tau_1-\hat{\tau}_1))}{\sqrt{N_0 T}}} \\ &\cdot \Phi_{D_2}\left(-\frac{mw}{\sqrt{N_0 T}}\right) \cdot \Phi_M\left(-\frac{mw}{\sqrt{N_0 T}}\right). \end{aligned} \quad (4.34)$$

Thus, we no longer need to find the probability density functions of D_2 and M. Finding their characteristic functions will suffice. At this point, note that D_2 is a function of the synchronization errors, $(\tau_1 - \hat{\tau}_1)$ and $(\phi_1 - \hat{\phi}_1)$ but is independent of the relative timing and phase delays of interfering signals, $\tau_k, \phi_k, k = 2, \ldots, K$. M, on the other hand, is independent of the synchronization errors but is a function of the timing and phase delays of interfering signals. Therefore, the fact that the system is synchronous, quasi-synchronous or asynchronous will only effect the characteristic function of M in (4.34).

We now find the characteristic function terms in (4.34). We start with the characteristic function of the self interference term, D_2. To this end, we define,

$$d_i = \sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)(\tau_1 - \hat{\tau}_1)\alpha_i. \tag{4.35}$$

Then, from (2.69),

$$D_2 = \sum_{i=0}^{G-1} d_i. \tag{4.36}$$

Thus,

$$\begin{aligned}\Phi_{d_i}(w) &= \mathrm{E}\{e^{jw\sqrt{2P_1}\cos(\phi_1-\hat{\phi}_1)(\tau_1-\hat{\tau}_1)\alpha_i}\} \\ &= \cos(w\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)(\tau_1 - \hat{\tau}_1))\end{aligned} \tag{4.37}$$

Since the α_i's are iid random variables, from (4.36), there follows,

$$\begin{aligned}\Phi_{D_2}(w) &= \prod_{i=0}^{G-1} \Phi_{d_i}(w) \\ &= \left[\cos(w\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1)(\tau_1 - \hat{\tau}_1)\right]^G.\end{aligned} \tag{4.38}$$

We then find the characteristic function of the multiple access interference term, M. We first define,

$$m_k = \sqrt{2P_k}\cos(\phi_k)\sum_{j=0}^{G-1}\left[\beta_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\right]. \tag{4.39}$$

From (2.70),

$$M = \sum_{k=2}^{K} m_k. \tag{4.40}$$

Then,

$$\Phi_{m_k}(w) = \mathrm{E}\left\{\left[\frac{1}{2}\cos(w\sqrt{2P_k}\cos(\phi_k)T_c) + \frac{1}{2}\cos(w\sqrt{2P_k}cos(\phi_k)(T_c - 2\tau_k)\right]^G\right\} \tag{4.41}$$

Using the binomial expansion, it is possible to rewrite (4.41) as,

$$\begin{aligned}\Phi_{m_k}(w) &= \mathrm{E}\Big\{\frac{1}{2^G}\sum_{p}^{G}\binom{G}{p}\cos^p\left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_k)\right) \\ &\quad \cdot \cos^{G-p}\left(w\sqrt{2P_k}\cos(\phi_k)T_c\right)\Big\}.\end{aligned} \tag{4.42}$$

We assume that the processing gain of the system, G is even. Note that it is also possible to derive the characteristic function for odd G's. In this case, (4.42) can be rewritten as,

$$\begin{aligned}
&\Phi_{m_k}(w) = \\
&\mathrm{E}\Bigg\{ \frac{1}{2^G} \sum_{p=0}^{G/2} \binom{G}{2p} \cos^{2p} \left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_k) \right) \\
&\cdot \cos^{G-2p} \left(w\sqrt{2P_k}\cos(\varphi_k)T_c \right) \\
&+ \frac{1}{2^G} \sum_{p=0}^{G/2-1} \binom{G}{2p+1} \cos^{2p+1} \left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_k) \right) \\
&\cdot \cos^{G-2p-1} \left(w\sqrt{2P_k}\cos(\phi_k)T_c \right) \Bigg\}
\end{aligned} \tag{4.43}$$

It is possible to perform the expectation on τ_k using the equalities,

$$\int \cos^{2n}(x)dx = \frac{1}{2^{2n}}\binom{2n}{n} x + \frac{1}{2^{2n-1}} \sum_{k=0}^{n-1} \binom{2n}{k} \frac{\sin(2n-2k)x}{2n-2k} \tag{4.44}$$

$$\int \cos^{2n+1}(x)dx = \frac{1}{2^{2n}} \sum_{k=0}^{n} \binom{2n+1}{k} \frac{\sin(2n-2k+1)x}{2n-2k+1} \tag{4.45}$$

from page 132 of [72]. Recall from Chapter 2 that τ_k's are iid and uniform over $[0, \lambda T_c]$ where λ is a constant that may take any value from 0 to 1 depending on the nature of the system. If the system is synchronous, $\lambda = 0$, whereas $\lambda = 1$ for an asynchronous system. Quasi-synchronous systems have λ values between these two extremes. Performing the expectation on the τ_k's yields,

$$\begin{aligned}
&\Phi_{m_k}(w) = \\
&\mathrm{E}\Bigg\{ \frac{1}{2^G} \sum_{p=0}^{G/2} \binom{G}{2p} \cos^{G-2p} \left(w\sqrt{2P_k}\cos(\phi_k)T_c \right) \\
&\cdot \frac{1}{\lambda T_c} \int_0^{\lambda T_c} \cos^{2p} \left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_k) \right) d\tau_k \\
&+ \frac{1}{2^G} \sum_{p=0}^{G/2-1} \binom{G}{2p+1} \cos^{G-2p-1} \left(w\sqrt{2P_k}\cos(\phi_k)T_c \right) \\
&\cdot \frac{1}{\lambda T_c} \int_0^{\lambda T_c} \cos^{2p+1} \left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_k) \right) d\tau_k \Bigg\}
\end{aligned} \tag{4.46}$$

Now, if the system is synchronous, $\lambda = 0$ and thus $\tau_k = \tau_1$ resulting in a $\Phi_{m_{k,j}}(w)$.

$$\begin{aligned}
\Phi_{m_k}(w) = \\
& \mathrm{E}\Bigg\{ \frac{1}{2^G} \sum_{p=0}^{G/2} \binom{G}{2p} \cos^{G-2p}\left(w\sqrt{2P_k}\cos(\phi_k)T_c\right) \cos^{2p}\left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_1)\right) \\
& + \frac{1}{2^G} \sum_{p=0}^{G/2-1} \binom{G}{2p+1} \cos^{G-2p-1}\left(w\sqrt{2P_k}\cos(\phi_k)T_c\right) \\
& \cdot \cos^{2p+1}\left(w\sqrt{2P_k}\cos(\phi_k)(T_c - 2\tau_1)\right) \Bigg\} \qquad (4.47)
\end{aligned}$$

If the system is not synchronous, with some algebraic manipulations, one can write (4.46) as,

$$\begin{aligned}
\Phi_{m_k}(w) &= \mathrm{E}\Bigg\{ \sum_{p=0}^{G/2} \binom{G}{2p} \binom{2p}{p} \frac{\cos^{G-2p}\left(w\sqrt{2P_k}cos(\phi_k)T_c\right)}{2^{G+2p}} \\
& + \sum_{p=0}^{G} \sum_{q=0}^{\lfloor\frac{p-1}{2}\rfloor} \binom{G}{p} \binom{p}{q} \frac{\cos^{G-p}\left(w\sqrt{2P_k}\cos(\phi_k)T_c\right)}{2^{G+p-1}} \\
& \cdot \sin\left((p-2q)w\sqrt{2P_k}\cos(\phi_k)\lambda T_c\right) \\
& \cdot \frac{\cos\left((p-2q)w\sqrt{2P_k}\cos(\phi_k)(\lambda-1)T_c\right)}{(p-2q)w\sqrt{2P_k}\cos(\phi_k)\lambda T_c} \Bigg\}. \qquad (4.48)
\end{aligned}$$

If the system in question is asynchronous, the random variable τ_k can take on values from the interval $[0, T_c]$. In other words, $\lambda = 1$. In this case, (4.48) becomes

$$\begin{aligned}
\Phi_{m_k}(w) &= \mathrm{E}\Bigg\{ \sum_{p=0}^{G/2} \binom{G}{2p} \binom{2p}{p} \frac{\cos^{G-2p}\left(w\sqrt{2P_k}cos(\phi_k)T_c\right)}{2^{G+2p}} \\
& + \sum_{p=0}^{G} \sum_{q=0}^{\lfloor\frac{p-1}{2}\rfloor} \binom{G}{p} \binom{p}{q} \frac{\cos^{G-p}\left(w\sqrt{2P_k}\cos(\phi_k)T_c\right)}{2^{G+p-1}} \\
& \cdot \frac{\sin\left((p-2q)w\sqrt{2P_k}\cos(\phi_k)\lambda T_c\right)}{(p-2q)w\sqrt{2P_k}\cos(\phi_k)T_c} \Bigg\} \qquad (4.49)
\end{aligned}$$

In (4.47)-(4.49) the remaining expectation is on the distribution of the random variable ϕ_k. From Chapter 2, we know that ϕ_k are iid random variables uniformly distributed over the interval $[0.2\pi]$. This expectation is performed using numerical integration. A simple trapezoidal rule with only 20 intervals provides accurate results in a reasonably fast manner.

Once the characteristic functions of m_k are found, since the $\beta_{k,j}$'s and $\gamma_{k,j}$'s in (2.70) are iid random variables, from (4.40), there follows,

$$\Phi_M(w) = \prod_{k=2}^{K} \Phi_{m_k}(w) = (\Phi_{m_k}(w))^{K-1}. \tag{4.50}$$

Alternatively, one can find the characteristic function of M by making some independence assumptions as was done in [213] at the expense of some degradation in the accuracy of the technique. If one assumes that the term $\cos(\phi_k)\left[\beta_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\right]$ vary independently from chip to chip, in other words, if the chip-to-chip dependence of the DS CDMA system as described in detail in Chapter 2 is ignored, it is possible to find a closed form expression for the characteristic function of M. Under the chip-to-chip independence assumption we first define,

$$m_{k,j} = \sqrt{2P_k}\cos(\phi_k)\left[\beta_{k,j}(T_c - \tau_k) + \gamma_{k,j}\tau_k\right] \tag{4.51}$$

so that

$$M = \sum_{k=2}^{K}\sum_{j=0}^{G-1} m_{k,j}. \tag{4.52}$$

Since the $m_{k,j}$ are assumed to be iid, (4.52) would result in

$$\Phi_M(w) = \left(\Phi_{m_{k,j}}(w)\right)^{G(K-1)} \tag{4.53}$$

Therefore, we first find the characteristic function of $m_{k,j}$.

$$\begin{aligned}
\Phi_{m_{k,j}}(w) &= \mathrm{E}\{e^{jw\sqrt{2P_k}\cos(\phi_k)\left[\beta_{k,j}(T_c-\tau_k)+\gamma_{k,j}\tau_k\right]}\} \\
&= \int_0^{\lambda T_c}\int_0^{2\pi}\frac{1}{8\pi\lambda T_c}e^{jw\sqrt{2P_k}\cos(\phi_k)T_c}d\phi_k d\tau_k \\
&\quad + \int_0^{\lambda T_c}\int_0^{2\pi}\frac{1}{8\pi\lambda T_c}e^{-jw\sqrt{2P_k}\cos(\phi_k)T_c}d\phi_k d\tau_k \\
&\quad + \int_0^{\lambda T_c}\int_0^{2\pi}\frac{1}{8\pi\lambda T_c}e^{jw\sqrt{2P_k}\cos(\phi_k)[2\tau_k-T_c]}d\phi_k d\tau_k \\
&\quad + \int_0^{\lambda T_c}\int_0^{2\pi}\frac{1}{8\pi\lambda T_c}e^{-jw\sqrt{2P_k}\cos(\phi_k)[2\tau_k-T_c]}d\phi_k d\tau_k.
\end{aligned} \tag{4.54}$$

We know from page 360 of [4] that the zero-order Bessel function of the first kind satisfies the equality.

$$J_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{jz\cos(\theta)}d\theta \tag{4.55}$$

In that case, (4.54) can be written as.

$$\begin{aligned}\Phi_{m_{k,j}}(w) &= \frac{1}{2\lambda T_c}\int_0^{\lambda T_c}\left(J_0(w\sqrt{2P_k T_c}) + J_0(w\sqrt{2P_k}(2\tau_k - T_c))\right)d\tau_k \\ &= \frac{1}{2}J_0(w\sqrt{2P_k T_c}) + \frac{1}{2\lambda T_c}\int_0^{\lambda T_c} J_0(w\sqrt{2P_k}\,[2\tau_k - T_c])d\tau_k \end{aligned}\tag{4.56}$$

Now, if the system is synchronous, $\lambda = 0$ and thus $\tau_k = \tau_1$ giving,

$$\Phi_{m_{k,j}}(w) = \frac{1}{2}J_0(w\sqrt{2P_k T_c}) + \frac{1}{2}J_0(w\sqrt{2P_k}(2\tau_1 - T_c)). \tag{4.57}$$

If the system is not synchronous, we can use the equality on page 666 of [72].

$$\int_0^{\xi} J_0(\nu)d\nu = \xi J_0(\xi) + \frac{\pi\xi}{2}\left\{J_1(\xi)H_0(\xi) - J_0(\xi)H_1(\xi)\right\} \tag{4.58}$$

to solve the integral in equation (4.56). Here $J_0(\xi)$ and $J_1(\xi)$ are zeroth and first order Bessel functions of the first kind and $H_0(\xi)$ and $H_1(\xi)$ are zeroth and first order Struve functions, respectively [4]. Note that both $J_0(x)$ and $H_1(x)$ are even functions and $J_1(x)$ and $H_0(x)$ are odd functions of x.

We define $\mu_k = w\sqrt{2P_k}\,[2\tau_k - T_c]$. Then,

$$\begin{aligned}\Phi_{m_{k,j}}(w) &= \frac{1}{2}J_0(w\sqrt{2P_k T_c}) \\ &+\frac{1}{4w\sqrt{2P_k}\lambda T_c}\left[\int_0^{w\sqrt{2P_k}T_c} J_0(\mu_k)d\mu_k + \int_0^{(2\lambda-1)w\sqrt{2P_k}T_c} J_0(\mu_k)d\mu_k\right]\end{aligned}\tag{4.59}$$

since $J_0(x)$ is an even function [4]. We then define $\kappa_k = w\sqrt{2P_k}T_c$. In this case,

$$\begin{aligned}\Phi_{m_{k,j}}(w) &= \frac{2\lambda+1}{4\lambda}J_0(\kappa_k) + \frac{2\lambda-1}{4\lambda}J_0((2\lambda-1)\kappa_k) \\ &+\frac{\pi}{8\lambda}\left[J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k)\right] \\ &+\frac{(2\lambda-1)\pi}{8\lambda}\cdot\Big[J_1((2\lambda-1)\kappa_k)H_0((2\lambda-1)\kappa_k) \\ &-J_0((2\lambda-1)\kappa_k)H_1((2\lambda-1)\kappa_k)\Big].\end{aligned}\tag{4.60}$$

Since the chip to chip independence is assumed the $m_{k,j}$ are modelled to be iid random variables. Then,

$$\begin{aligned}\Phi_M(w) &= \prod_{k=2}^{K}\prod_{j=0}^{G-1}\Phi_{m_{k,j}}(w) = \prod_{k=2}^{K}\left(\Phi_{m_{k,j}}(w)\right)^G \\ &= \prod_{k=2}^{K}\left(\frac{2\lambda+1}{4\lambda}J_0(\kappa_k) + \frac{2\lambda-1}{4\lambda}J_0((2\lambda-1)\kappa_k)\right. \\ &\quad + \frac{\pi}{8\lambda}\left[J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k)\right] \\ &\quad + \frac{(2\lambda-1)\pi}{8\lambda}J_1((2\lambda-1)\kappa_k)H_0((2\lambda-1)\kappa_k) \\ &\quad \left.-\frac{(2\lambda-1)\pi}{8\lambda}J_0((2\lambda-1)\kappa_k)H_1((2\lambda-1)\kappa_k)\right)^G \end{aligned} \tag{4.61}$$

And if $P_k = P_1, \forall k$ (which would be the case under perfect power control), m_k's are iid random variables as well. In that case (4.61) simplifies to,

$$\begin{aligned}\Phi_M(w) &= \left(\frac{2\lambda+1}{4\lambda}J_0(\kappa_k)\right. \\ &\quad + \frac{2\lambda-1}{4\lambda}J_0((2\lambda-1)\kappa_k) + \frac{\pi}{8\lambda}\left[J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k)\right] \\ &\quad + \frac{(2\lambda-1)\pi}{8\lambda}J_1((2\lambda-1)\kappa_k)H_0((2\lambda-1)\kappa_k) \\ &\quad \left.-\frac{(2\lambda-1)\pi}{8\lambda}J_0((2\lambda-1)\kappa_k)H_1((2\lambda-1)\kappa_k)\right)^{G(K-1)} \end{aligned} \tag{4.62}$$

Note that (4.62) is for a quasi-synchronous system. An asynchronous system would have $\lambda = 1$. In this case, (4.62) simplifies to,

$$\Phi_M(w) = \left(J_0(\kappa_k) + \frac{\pi}{4}\left[J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k)\right]\right)^{G(K-1)}. \tag{4.63}$$

Similarly, using (4.57) and (4.61), the characteristic function of M for a fully synchronous system can be written as,

$$\Phi_M(w) = \left[\frac{1}{2}J_0(\kappa_k) + \frac{1}{2}J_0((\frac{2\tau_1}{T_c} - 1)\kappa_k)\right]^{G(K-1)}. \tag{4.64}$$

Having found the characteristic functions of D_2 and M for synchronous, quasi-synchronous and asynchronous systems, we can now write the infinite Fourier series for $P(E)$.

$$P(E) = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2 w^2/2} \sin\left(\frac{mw\sqrt{2P_1}\cos(\phi_1 - \hat{\phi}_1) G(T_c - (\tau_1 - \hat{\tau}_1))}{\sqrt{N_0 T}} \right)$$
$$\cdot \Phi_{D_2}\left(-\frac{mw}{\sqrt{N_0 T}}\right) \Phi_M\left(-\frac{mw}{\sqrt{N_0 T}}\right) \qquad (4.65)$$

where the characteristic function of D_2, $\Phi_{D_2}(w)$, is given in (4.38) and the characteristic function of M, $\Phi_M(w)$ can be found using one of two methods: exact calculation using semi-analytical expressions or making a chip to chip independence assumption towards obtaining closed form expressions. If the exact calculations are carried, $\Phi_M(w)$ is found using (4.48) for a quasi-synchronous system, (4.49) for an asynchronous system and (4.47) for a fully synchronous system. For all three cases (4.50) is eventually used to calculate the characteristic function of M. If the approximate characteristic function for M is invoked then (4.62) is used for a quasi-synchronous system, (4.63) for an asynchronous system and (4.64) for a fully synchronous system.

The infinite Fourier series in (4.65) is absolutely convergent. Recall from Section 4.2 that $w = 2\pi/T$ is the Fourier series frequency. Thus the above expression for the probability of error is strictly true in the limit as w goes to zero. In our case, it is sufficient to assume $w = \pi/25$ to correctly calculate error probabilities greater than or equal to 10^{-7} with negligible error. As discussed in Section 4.2, the accuracy of the technique is clearly bounded by the truncation of the infinite series. In our calculations, taking the first 51 terms in the series into consideration was sufficient to achieve a level of accuracy much higher than what is necessary for all of the cases considered in the thesis. Beaulieu in [15] states that in practice, it is not necessary to search for suitable values of w and the number of terms one should include in the truncated Fourier series. Beaulieu points out that by taking only 16 terms into account and setting $w = \pi/15$, error probabilities in the order of 8×10^{-10} can be calculated to six or more significant figures accuracy.

Note that equation (4.65) is an error probability expression for a system that has a given level of chip timing and carrier phase errors. If these errors are random, we need to integrate (4.65) over the probability density functions of the errors, $\tau_\epsilon = (\tau_1 - \hat{\tau}_1)$ and $\phi_\epsilon = (\phi_1 - \hat{\phi}_1)$ to get the error probability.

$$P(E) = \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2 w^2/2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{\phi_\epsilon}(\phi_\epsilon) f_{\tau_\epsilon}(\tau_\epsilon) \Phi_M\left(-\frac{mw}{\sqrt{N_0 T}}\right)$$
$$\Phi_{D_2}\left(-\frac{mw}{\sqrt{N_0 T}}\right) \sin\left(\frac{mw\sqrt{2P_1}\cos(\phi_\epsilon)G(T_c - \tau_\epsilon)}{\sqrt{N_0 T}}\right) d\tau_\epsilon d\phi_\epsilon. \quad (4.66)$$

The error probability in this case can be found by evaluating the integrals in (4.66) numerically.

4.3.2 System Analysis

The probability of error, as can be seen in (4.65), is a function of the signal to noise ratio,

$$\text{SNR} = \frac{P_1 T}{N_0}, \quad (4.67)$$

the number of users present in the system, K, the synchronization errors, τ_ϵ and ϕ_ϵ, and the processing gain G. Thus, it is possible to find the system capacity for different values of signal to noise ratio, synchronization errors and processing gain. When G, SNR, τ_ϵ and ϕ_ϵ are fixed, we can evaluate the error probability for increasing number of users. The capacity of the system is simply the maximum number of users that will still yield an error probability below a certain threshold.

If we assume a user data transmission at 9600 bits/sec over a bandwidth of 1.2288 MHz, a processing gain of 128 is observed. In this case, using the derived series for the probability of error, we find that for a symbol signal to noise ratio of 20 dB and perfect synchronization, a maximum of 39 users can be accommodated in the asynchronous system. Each one of these users is guaranteed to have an error probability less than or equal to 10^{-3}. We use both the exact and the approximate representations of

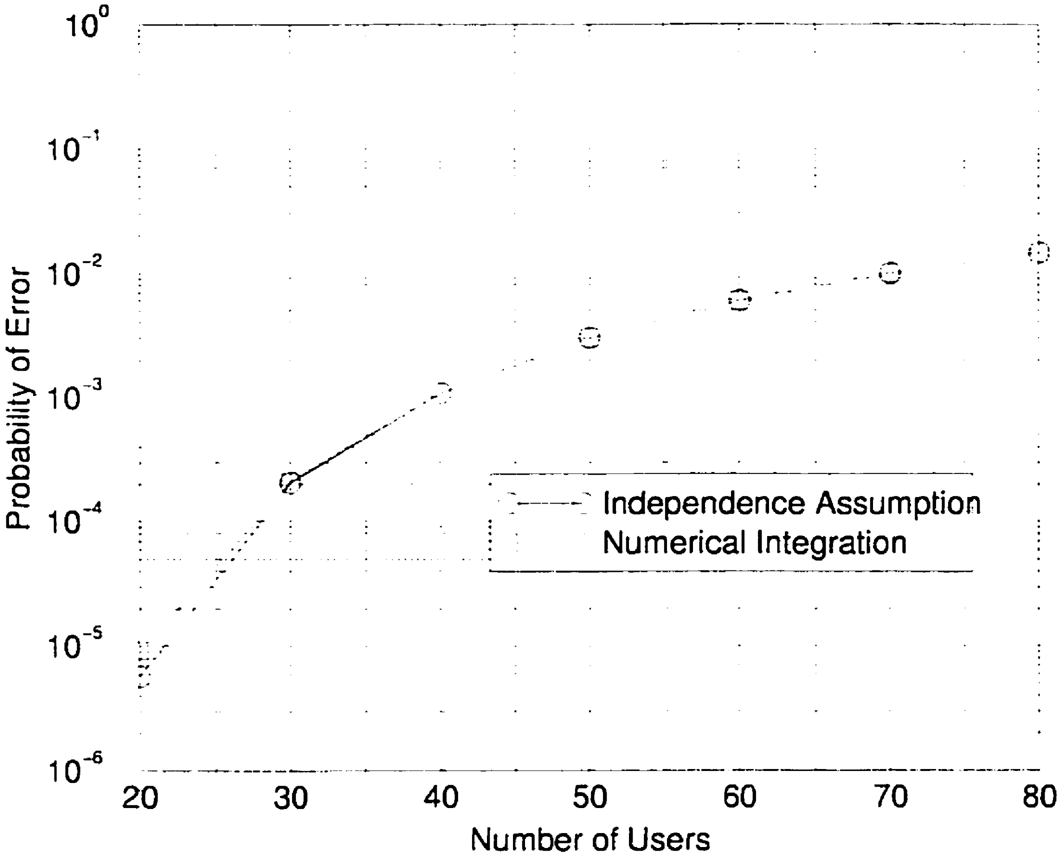

Figure 4.4: Comparison of the two methods in evaluating the system capacity for a system operating at 20 dB SNR.

the characteristic function of the multiple access interference to compute the system capacities. System performance as a function of the number of active users is shown in Figure 4.4 for systems operating at an SNR of 20 dB. As can be seen from the figure, the probability of error numbers calculated using the two methods are only slightly different: the chip to chip independence assumption in the multiple access interference results in slightly optimistic values. The difference, however, is never large enough to grant a discrepancy in the system capacity calculated using these methods. We find that when an error probability of 10^{-3} is desired, the two methods give exactly the same value for the system capacity.

Synchronization errors effect the system capacity significantly. For example, a chip timing error of 10% will reduce the asynchronous system capacity by approximately 20%. From (4.65), it can be seen that the synchronization errors effect the system performance by potentially reducing the energy of the desired signal component of the received signal and by introducing self interference. We have numerically found that the effects of the introduced self interference, in comparison to the desired signal energy reduction, is negligibly small. Thus, a 10% error in chip timing will approximately result in a 10% reduction in the desired signal energy level. This corresponds to a 20% reduction in the signal to noise ratio which results in approximately 20% reduction in the system capacity.

System capacity as a function of signal to noise ratio is plotted for different levels of chip timing errors for an asynchronous system in Figure 4.5. Here, we assume that each user requires a bit error rate of 10^{-3} or less. Similarly, capacity versus signal to noise ratio for different levels of carrier phase errors is graphed in Figure 4.6. In a practical system, both chip timing and carrier phase errors will be present. If we define the capacity loss of a system as the difference between the capacity when there are no synchronization errors and the capacity when synchronization errors are present, our calculations show that the capacity loss from the presence of both chip timing and carrier phase errors is approximately the sum of individual losses for all values of signal to noise ratio. Figure 4.7 shows this additive property for a chip timing error of $T_c/10$ and a carrier phase error of $\pi/10$. The capacity losses for various levels of synchronization errors are listed in a tabular form in Table 4.1. These values are for a DS CDMA system that employs voice transmission at 9600 bit/sec and that has a processing gain of 128. The corresponding maximum allowable bit error rate is, as before, 10^{-3}. The signal to noise ratio is set at 20 dB.

The reduction in the desired signal energy level can alternatively be interpreted

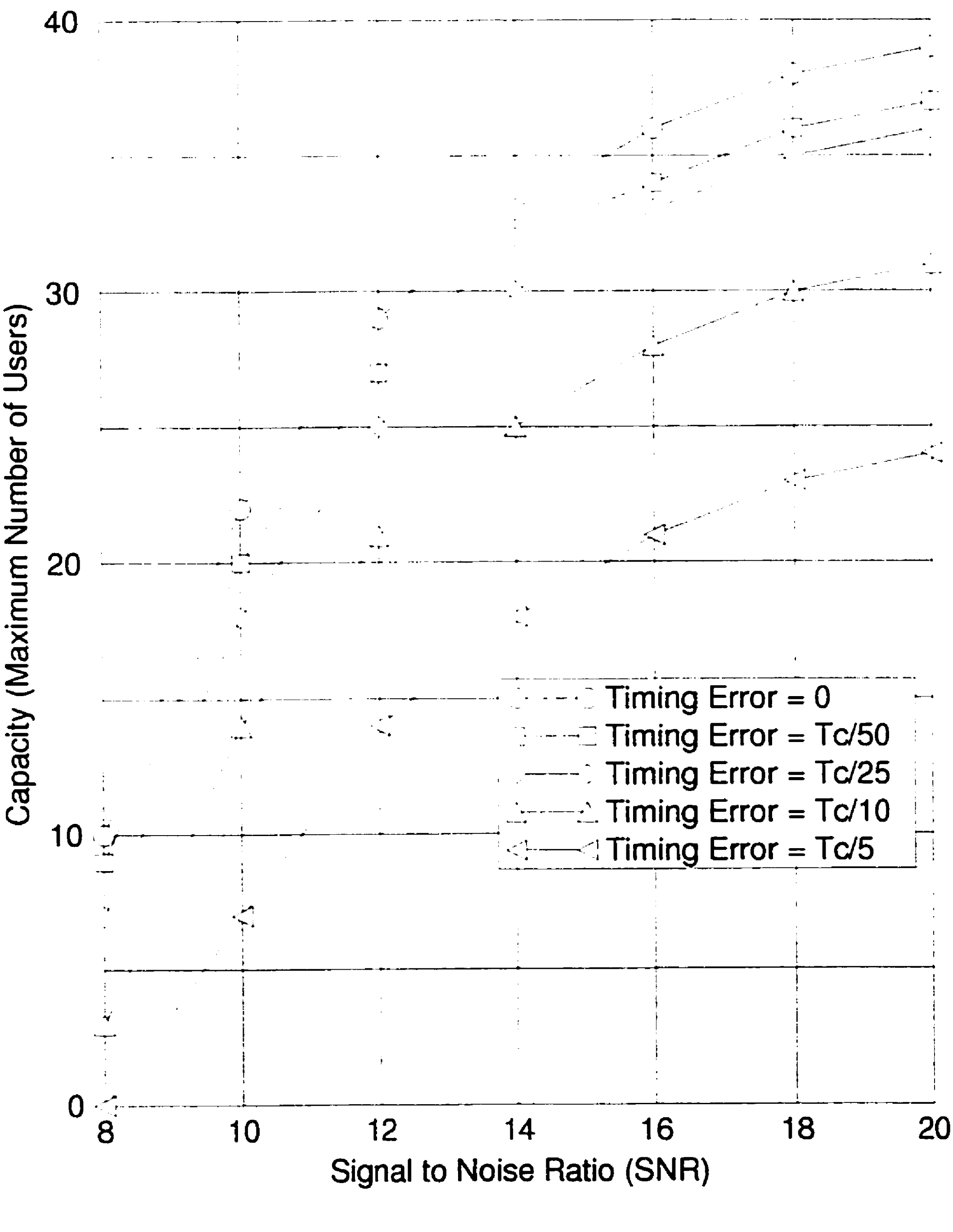

Figure 4.5: System Capacity versus SNR for the Asynchronous DS CDMA System operating at different chip timing error values. There is no carrier phase error.

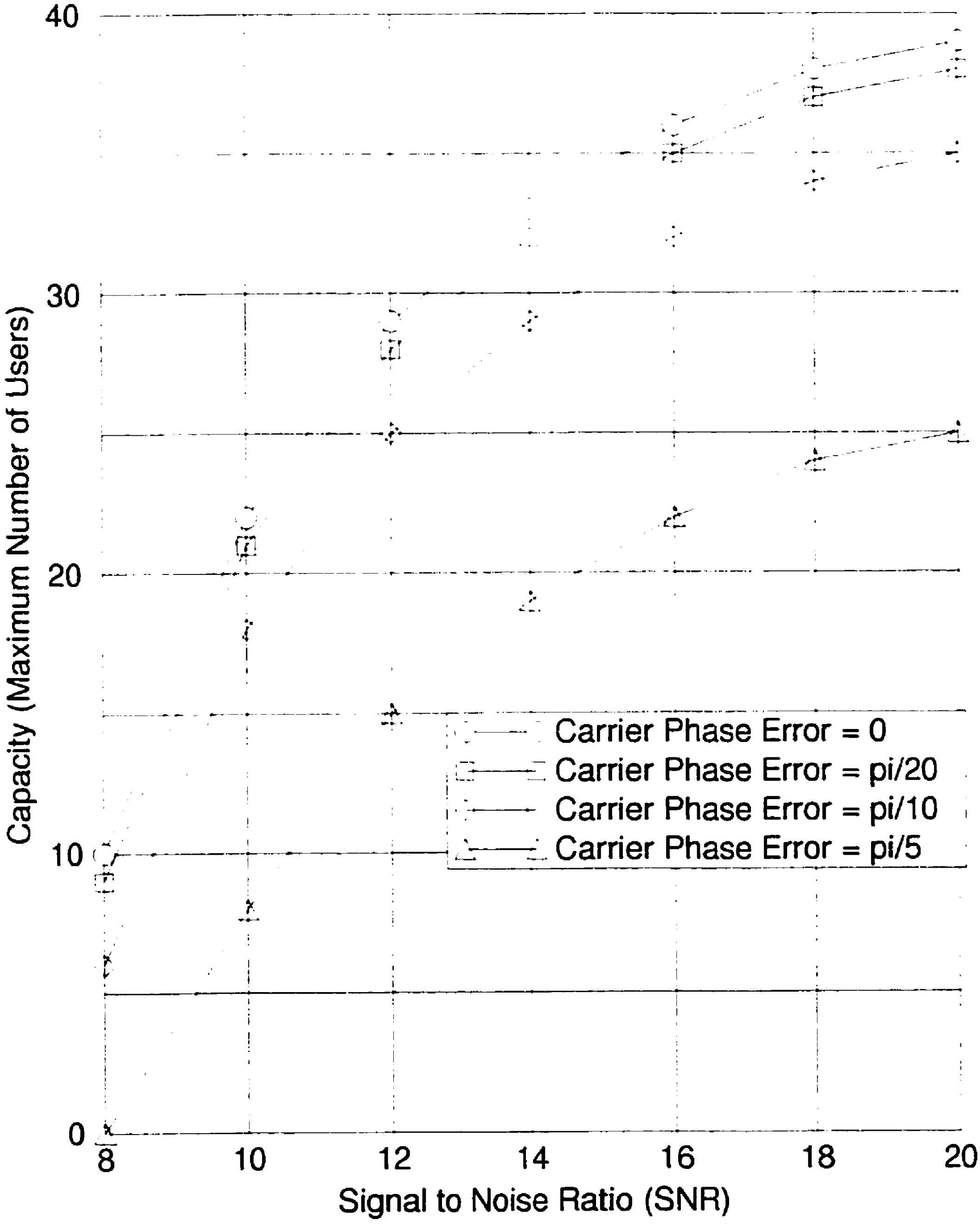

Figure 4.6: System Capacity versus SNR for the Asynchronous DS CDMA System operating at different carrier phase error values. There is no chip timing error.

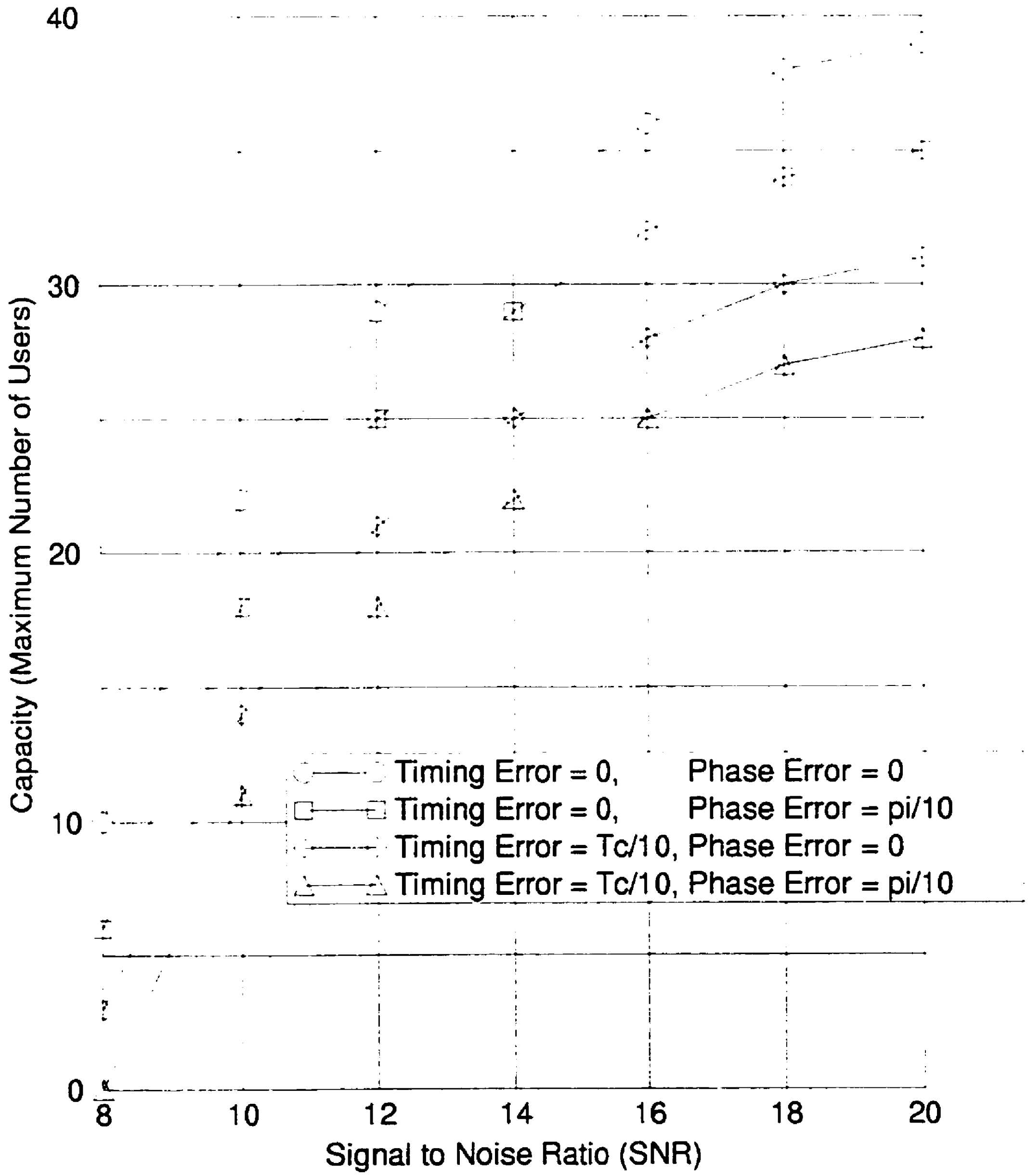

Figure 4.7: System Capacity versus SNR for the DS CDMA System operating at different levels of synchronization errors. Note that capacity losses from different types of synchronization errors are approximately additive.

$P(E)=10^{-3}$, SNR=20dB, Capacity=39		
Chip Timing Error	Capacity	Capacity Loss
$T_c/10$	31	8
$T_c/25$	36	3
$T_c/50$	37	2
$T_c/100$	38	1
Carrier Phase Error	Capacity	Capacity Loss
$\pi/5$	25	14
$\pi/10$	35	4
$\pi/20$	37	2
$\pi/50$	38	1
Combined Synch Error	Capacity	Capacity Loss
$T_c/10$, $\pi/10$	28	11
$T_c/10$, $\pi/20$	30	9
$T_c/10$, $\pi/50$	31	8

Table 4.1: Capacity Degradation with Respect to Synchronization Errors

as an effective processing gain loss. We define the effective processing gain as,

$$G_{\text{eff}} = G \cdot (1 - \frac{(\tau_1 - \hat{\tau}_1)}{T_c}) \cos(\phi_1 - \hat{\phi}_1). \tag{4.68}$$

Then, for an asynchronous system, (4.65) can be rewritten in terms of the effective processing gain as follows,

$$\begin{aligned} P(E) \;=\; & \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2 w^2/2} \sin\left(\frac{mw\sqrt{2P_1 G_{\text{eff}} T_c}}{\sqrt{N_0 T}}\right) \\ & \cdot \Phi_{D_2}\left(-\frac{mw}{\sqrt{N_0 T}}\right) \Phi_M\left(-\frac{mw}{\sqrt{N_0 T}}\right) \end{aligned} \tag{4.69}$$

The effective processing gain is approximately a linear function of the system capacity for a given level of maximum allowable error probability. This relationship is shown in Figure 4.8 for an error probability of 10^{-3}. The capacity loss due to any level of synchronization error can be found directly using this linear relationship. Using (4.68), the effective processing gain can be found for the system in question. The corresponding capacity from the graph yields the maximum number of users that can be accommodated at this level of synchronization errors. The bumps in the graph are due to the quantization that takes place when the system capacity is calculated from the bit error rate. This is because the system capacity can only take on integer values.

The nature of the system affects the system performance as well. When long PN sequences are used, synchronous transmission amongst users results in the lowest system capacity. The capacity increases as the synchronism constraint is relaxed. In quasi-synchronous systems, the receiver will have all of the transmitted signals arrive within a fixed time interval. This interval can be defined to be $[0, \lambda T_c]$ where λ characterizes the level of synchronism. When $\lambda = 0$, the system is fully synchronous whereas when $\lambda = 1$, the system is asynchronous. For a given SNR, as λ is increased from 0 towards 1, the probability of error decreases. However, for values of $\lambda \geq 0.5$,

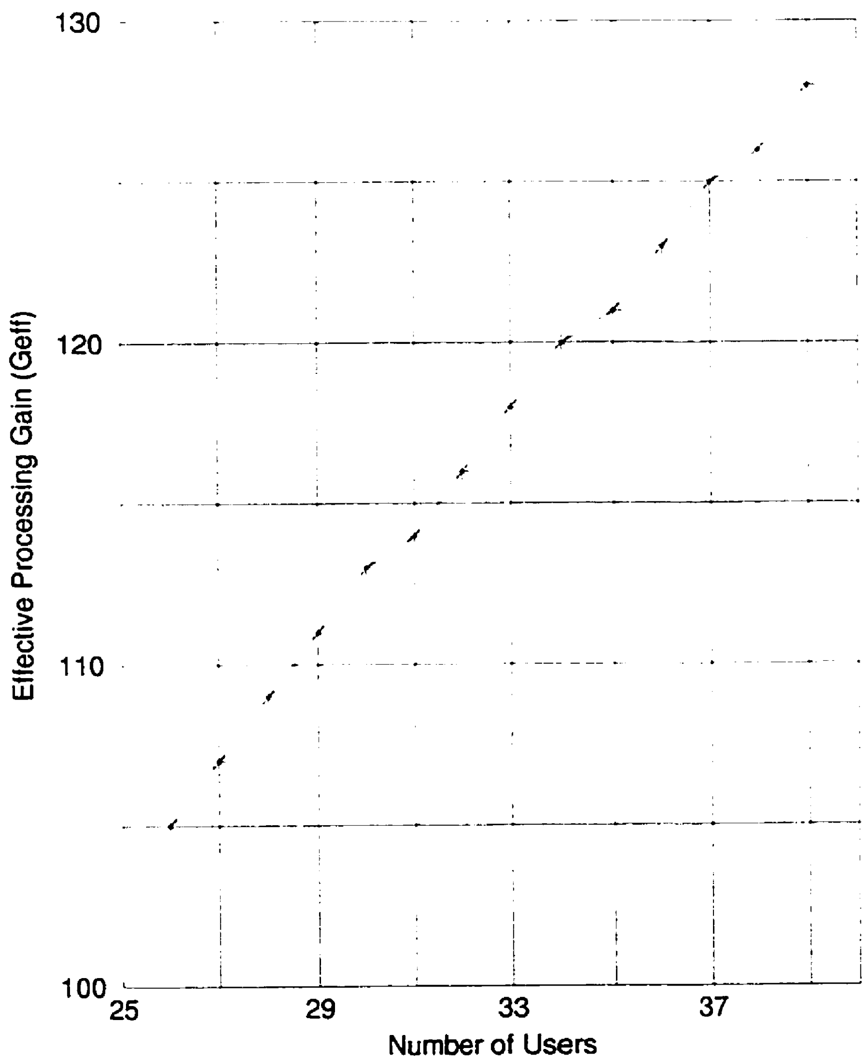

Figure 4.8: Effective Processing Gain versus capacity for the Asynchronous DS CDMA System.

the decrease in the error probability with increasing λ is too small to provide a change in the system capacity. For this reason, for such values of λ, the system capacity is identical to that of an asynchronous system. The capacity of synchronous, asynchronous and various quasi-synchronous systems for a signal to noise ratio of 20 dB are listed in a tabular form in Table 4.2 for various synchronization errors.

Whether the synchronization error is constant or random has an effect on the system capacity too. We have found using (4.65) that a constant timing error of $\tau_r = T_c/10$ results in a capacity loss of 8. Using (4.66), it can be seen that, for a random timing error τ_r that is uniformly distributed in the interval $[0, T_c/5]$ (making the mean equal to $T_c/10$), the capacity loss increases to 9. In general, for small synchronization errors, whether the error is constant or random with uniform distribution does not make any difference in the system capacity. With the increasing synchronization error, on the other hand, the random error that has a mean equal to the constant error causes more and more capacity decreases in comparison to the constant error case. A comparison between constant and random timing errors can be found in tabular form in Table 4.3 for a DS CDMA system that employs user data transmission at 9600 bit/sec and that has a processing gain of 128. The symbol signal to noise ratio is set at 20 dB.

In Tables 4.4 and 4.5, we present the loss in capacity encountered due to different levels of synchronization errors for varying values of processing gain and SNR, respectively. Note that for a fixed bandwidth system, an increase in the processing gain comes at the expense of a reduction in the information signaling rate. We observe that percentage capacity losses due to fixed synchronization errors are approximately invariant with respect to the system processing gain. We also observe that reducing the system SNR increases its volatility towards the synchronization errors. As the SNR value is reduced, the percentage loss in capacity due to a given level of synchronization error increases.

Synch Errors	**System Capacity**			
	Synch	Q-Synch ($\lambda = 0.1$)	Q-Synch ($\lambda = 0.25$)	Asynch
No Synch Errors	26	29	33	39
Chip Timing Error	Synch	Q-Synch ($\lambda = 0.1$)	Q-Synch ($\lambda = 0.25$)	Asynch
$T_c/10$	21	23	26	31
$T_c/25$	24	26	30	36
$T_c/50$	25	28	31	37
Carrier Phase Error	Synch	Q-Synch ($\lambda = 0.1$)	Q-Synch ($\lambda = 0.25$)	Asynch
$\pi/5$	17	18	21	25
$\pi/10$	23	27	30	35
$\pi/20$	25	29	32	37
Combined Synch Error	Synch	Q-Synch ($\lambda = 0.1$)	Q-Synch ($\lambda = 0.25$)	Asynch
$T_c/10, \pi/10$	19	21	24	28

Table 4.2: Performance of Synchronous, Quasi-Synchronous and Asynchronous Systems

Constant Timing Errors	Capacity Loss	**Random Timing Errors**	Capacity Loss
$T_c/50$	2	$[0, T_c/25]$	2
$T_c/25$	3	$[0, 2T_c/25]$	4
$T_c/10$	8	$[0, T_c/5]$	9
$T_c/5$	15	$[0, 2T_c/5]$	19

Table 4.3: Effects of Constant and Random Synchronization Errors on the System Capacity

4.4 *Analysis of Quadriphase Spread DS CDMA Systems in AWGN Channels*

We now study the quadriphase spread DS CDMA system modelled in Section 2.1.2. The quadriphase spread BPSK modulation scheme has in the past years enjoyed considerable attention since the current second generation wireless system standard TIA/EIA-95 uses it on the downlink channel [226]. As summarized in Chapter 2, the TIA/EIA-95 standard defines the downlink signaling scheme to consist of a signal centered on the assigned frequency, quadriphase modulated by a pair of long PN codes with an assigned time offset, biphase modulated by the encoded, interleaved and scrambled digital information signal.

4.4.1 Probability of Error Series Derivation

Recall from Chapter 2 that the k'th user in a system that uses quadriphase spreading and binary modulation has an information signal expressed by (2.2). In this case, the information bits are spread by two spreading sequences, one for the in-phase and the other for the quadrature component of the user's transmitted signal. The two

P(E)=10^{-3}, SNR=20dB, G=128, Capacity=39		
Synch Error	Capacity Loss	Perc. Capacity Loss
$T_c/10$	8	20.51%
$T_c/25$	3	7.69%
$\pi/5$	14	35.90%
$\pi/10$	4	10.26%
$T_c/10$, $\pi/10$	11	28.21%
P(E)=10^{-3}, SNR=20dB, G=256, Capacity=78		
Synch Error	Capacity Loss	Perc. Capacity Loss
$T_c/10$	16	20.51%
$T_c/25$	7	8.97%
$\pi/5$	29	37.18%
$\pi/10$	8	10.26%
$T_c/10$. $\pi/10$	22	28.21%
P(E)=10^{-3}, SNR=20dB, G=512, Capacity=157		
Synch Error	Capacity Loss	Perc. Capacity Loss
$T_c/10$	32	20.38%
$T_c/25$	15	9.55%
$\pi/5$	57	36.31%
$\pi/10$	16	10.10%
$T_c/10$. $\pi/10$	45	28.66%

Table 4.4: Effects of the Processing Gain on the Capacity Loss Encountered Due to Synchronization Errors

P(E)=10^{-3}, SNR=10dB, G=128, Capacity=22		
Synch Error	Capacity Loss	Perc. Capacity Loss
$T_c/10$	8	36.36%
$T_c/25$	1	18.18%
$\pi/5$	14	63.64%
$\pi/10$	4	18.18%
$T_c/10$, $\pi/10$	11	50%
P(E)=10^{-3}, SNR=15dB, G=128, Capacity=35		
Synch Error	Capacity Loss	Perc. Capacity Loss
$T_c/10$	8	22.86%
$T_c/25$	3	8.57%
$\pi/5$	14	40%
$\pi/10$	4	11.43%
$T_c/10$, $\pi/10$	11	31.43%
P(E)=10^{-3}, SNR=20dB, G=128, Capacity=39		
Synch Error	Capacity Loss	Perc. Capacity Loss
$T_c/10$	8	20.51%
$T_c/25$	3	7.69%
$\pi/5$	14	35.90%
$\pi/10$	4	10.26%
$T_c/10$, $\pi/10$	11	28.21%

Table 4.5: Effects of the SNR on the Capacity Loss Encountered Due to Synchronization Errors

spreading sequences are defined as (2.73) and (2.74). As in the case of the biphase spread system, with no loss of generality, we assume that $a_{1,0}$ is to be captured by the correlator receiver shown in Figure 2.5. Then,

$$P(E) = P(Z_1 > 0 | a_{1,0} = -1). \tag{4.70}$$

Recall from (2.76) that the input signal to the decision device is $Z_1 = D_1 + D_2 + M + N$, where D_1 is the desired signal, D_2 is the self interference due to chip timing errors, M is the multiple access interference and N is the AWGN term with zero mean and $2N_0T$ variance. Then, similar to the biphase spread DS CDMA system, the error probability conditioned on the random variables D_2 and M is written as,

$$\begin{aligned} P(E|D_2, M) &= P(-2\sqrt{P_1}\cos(\phi_1 - \dot{\phi})_1)G(T_c - (\tau_1 - \hat{\tau}_1)) + D_2 + M + N > 0) \\ &= Q\left[\frac{2\sqrt{P_1}\cos(\phi_1 - \dot{\phi})G(T_c - (\tau_1 - \hat{\tau}_1)) - D_2 - M}{\sqrt{2N_0T}}\right]. \end{aligned} \tag{4.71}$$

We know from the total probability theorem that the unconditional error probability is found as [166],

$$P(E) = \int_{-\infty}^{\infty} P(E|D_2, M) f_{D_2}(D_2) f_M(M) dD_2 dM. \tag{4.72}$$

We then define the $Q(x)$ in terms of its Fourier series expression as was done in the case of the biphase spread DS CDMA system in the previous section. Then, we obtain,

$$\begin{aligned} P(E) &= \sum_{m=-\infty}^{\infty} c_m e^{\frac{jmw\sqrt{2P_1}\cos(\phi_1 - \dot{\phi}_1)G(T_c - (\tau_1 - \hat{\tau}_1))}{\sqrt{2N_0T}}} \\ &\quad \cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jmw\frac{D_2 + M}{\sqrt{2N_0T}}} f_{D_2}(D_2) f_M(M) dD_2 dM \\ &= \sum_{m=-\infty}^{\infty} c_m e^{\frac{jmw\sqrt{2P_1}\cos(\phi_1 - \dot{\phi}_1)G(T_c - (\tau_1 - \hat{\tau}_1))}{\sqrt{2N_0T}}} \\ &\quad \cdot \Phi_{D_2}\left(-\frac{mw}{\sqrt{2N_0T}}\right) \cdot \Phi_M\left(-\frac{mw}{\sqrt{2N_0T}}\right). \end{aligned} \tag{4.73}$$

Towards the evaluation of (4.73), we continue with the derivation of the characteristic functions for the self interference and multiple access interference terms. We start with $\Phi_{D_2}(w)$. To this end, we first define

$$
\begin{aligned}
d_j &= \sqrt{P_1}(\cos(\phi_1 - \hat{\phi}_1)\iota_j + \sin(\phi_1 - \hat{\phi}_1)\epsilon_j)(\tau_1 - \hat{\tau}_1) \\
&\quad + \sqrt{P_1}\sin(\phi_1 - \hat{\phi}_1)\xi_j(T_c - (\tau_1 - \hat{\tau}_1))
\end{aligned} \tag{4.74}
$$

to obtain,

$$
\begin{aligned}
\Phi_{D_2}(w) &= \prod_{j=0}^{G-1} \Phi_{d_j}(w) = \left(\Phi_{d_j}(w)\right)^G \\
&= [\frac{1}{8} \cdot (1 + \cos(2w\sqrt{P_1}\cos(\phi_1 - \hat{\phi}_1)(\tau_1 - \hat{\tau}_1))) \\
&\quad \cdot (1 + \cos(2w\sqrt{P_1}\sin(\phi_1 - \hat{\tau}_1))(\tau_1 - \hat{\tau}_1))) \\
&\quad \cdot (1 + \cos(2w\sqrt{P_1}\sin(\phi_1 - \hat{\phi}_1)(T_c - (\tau_1 - \hat{\tau}_1))))]^G
\end{aligned} \tag{4.75}
$$

We then find the characteristic function of the multiple access interference. Recall from the previous section that two methods could be used for this purpose. One can either derive an exact, semi-analytical expression to find $\Phi_M(w)$, or use an approximation to derive a closed form expression. We find that for our interest, the two methods give identical results. For the quadriphase system, we derive the closed form expression here. Derivation of the semi-analytical expression is tedious and follows directly from the derivation given in the previous section for the biphase spread system. We define,

$$
\begin{aligned}
m_{k,m} &= \sqrt{P_k}\cos(\phi_k)\left((T_c - \tau_k)\tilde{\upsilon}_{k,m} + \tau_k\tilde{\gamma}_{k,m}\right) \\
&\quad + \sqrt{P_k}\sin(\phi_k)\left((T_c - \tau_k)\hat{\upsilon}_{k,m} + \tau_k\hat{\gamma}_{k,m}\right).
\end{aligned} \tag{4.76}
$$

Now, if the system is synchronous, $\lambda = 0$ and thus $\tau_k = \tau_1$, giving,

$$
\begin{aligned}
\Phi_{m_{k,m}}(w) &= \left\{\frac{1}{4} + \frac{1}{8}J_0(2w\sqrt{P_k}T_c) + \frac{1}{4}J_0(2w\sqrt{P_k}(\tau_1 - T_c)) \right. \\
&\quad \left. + \frac{1}{8}J_0(2w\sqrt{P_k}(2\tau_1 - T_c)) + \frac{1}{4}J_0(2w\sqrt{P_k}\tau_1)\right\}^2
\end{aligned} \tag{4.77}
$$

where $J_0(x)$ is the zeroth order Bessel function of the first kind. If the system is quasi-synchronous, following the procedure used for the biphase spread system we get.

$$\begin{aligned}
\Phi_{m_{k,m}}(w) = & \left[\frac{1}{4} + \frac{2\lambda+5}{16\lambda} J_0(\kappa_k) + \frac{1}{4} J_0(\lambda\kappa_k) + \frac{\lambda-1}{4\lambda} J_0((\lambda-1)\kappa_k) \right. \\
& + \frac{2\lambda-1}{16\lambda} J_0((2\lambda-1)\kappa_k) + \frac{5\pi}{32\lambda}\{J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k)\} \\
& + \frac{\pi}{8}\{J_1(\lambda\kappa_k)H_0(\lambda\kappa_k) - J_0(\lambda\kappa_k)H_1(\lambda\kappa_k)\} \\
& + \frac{\pi(\lambda-1)}{8\lambda}\Big\{J_1((\lambda-1)\kappa_k)H_0((\lambda-1)\kappa_k) \\
& - J_0((\lambda-1)\kappa_k)H_1((\lambda-1)\kappa_k)\Big\} \\
& + \frac{\pi(2\lambda-1)}{32\lambda} J_1((2\lambda-1)\kappa_k)H_0((2\lambda-1)\kappa_k) \\
& \left. - \frac{\pi(2\lambda-1)}{32\lambda} J_0((2\lambda-1)\kappa_k)H_1((2\lambda-1)\kappa_k)\right]^2 . \qquad (4.78)
\end{aligned}$$

where

$$\kappa_k = 2w\sqrt{P_k}T_c. \qquad (4.79)$$

Recall from the previous section that $J_0(x)$ and $J_1(x)$ are the zeroth and first order Bessel functions of the first kind and $H_0(x)$ and $H_1(x)$ are the zeroth and first order Struve functions, respectively. The $m_{k,m}$ in (4.78) are assumed to be iid random variables. Therefore,

$$\Phi_M(w) = \prod_{k=2}^{K}\prod_{m=0}^{G-1} \Phi_{l_{k,m}}(w) = \prod_{k=2}^{K}\left(\Phi_{l_{k,m}}(w)\right)^G . \qquad (4.80)$$

If $P_k = P_1$ (which would be the case under perfect power control), (4.80) simplifies to.

$$\Phi_M(w) = \left[\Phi_{l_{k,m}}(w)\right]^{(K-1)G} . \qquad (4.81)$$

Thus, for a synchronous system, the characteristic function of the multiple access interference can be written as.

$$\Phi_M(w) = \left\{\frac{1}{4} + \frac{1}{8}J_0(2w\sqrt{P_k}T_c) + \frac{1}{4}J_0(2w\sqrt{P_k}(\tau_1 - T_c))\right.$$

$$+\frac{1}{8}J_0(2w\sqrt{P_k}(2\tau_1 - T_c)) + \frac{1}{4}J_0(2w\sqrt{P_k\tau_1})\Big\}^{2(K-1)G} . \tag{4.82}$$

Similarly, for a quasi-synchronous system,

$$\begin{aligned}
\Phi_M(w) = & \Bigg[\frac{1}{4} + \frac{2\lambda+5}{16\lambda}J_0(\kappa_k) + \frac{1}{4}J_0(\lambda\kappa_k) + \frac{\lambda-1}{4\lambda}J_0((\lambda-1)\kappa_k) \\
& + \frac{2\lambda-1}{16\lambda}J_0((2\lambda-1)\kappa_k) + \frac{5\pi}{32\lambda}\{J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k)\} \\
& + \frac{\pi}{8}\{J_1(\lambda\kappa_k)H_0(\lambda\kappa_k) - J_0(\lambda\kappa_k)H_1(\lambda\kappa_k)\} \\
& + \frac{\pi(\lambda-1)}{8\lambda}\Big\{J_1((\lambda-1)\kappa_k)H_0((\lambda-1)\kappa_k) \\
& - J_0((\lambda-1)\kappa_k)H_1((\lambda-1)\kappa_k)\Big\} \\
& + \frac{\pi(2\lambda-1)}{32\lambda}J_1((2\lambda-1)\kappa_k)H_0((2\lambda-1)\kappa_k) \\
& - \frac{\pi(2\lambda-1)}{32\lambda}J_0((2\lambda-1)\kappa_k)H_1((2\lambda-1)\kappa_k)\Bigg]^{2G(K-1)} .
\end{aligned} \tag{4.83}$$

An asynchronous system would have $\lambda = 1$. In this case, (4.83) simplifies to,

$$\Phi_M(w) = \left[\frac{1}{4} + \frac{3}{4}J_0(\kappa_k) + \frac{5\pi}{16}(J_1(\kappa_k)H_0(\kappa_k) - J_0(\kappa_k)H_1(\kappa_k))\right]^{2G(K-1)} \tag{4.84}$$

The error probability for the quadriphase spread DS CDMA system can therefore be written as,

$$\begin{aligned}
P(E) = & \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{m=1\\ m \text{ odd}}}^{\infty}\frac{1}{m}e^{-m^2w^2/2}\sin\left(\frac{\sqrt{2P_1}mw\cos(\varphi_1 - \hat{\varphi}_1)G(T_c - (\tau_1 - \hat{\tau}_1))}{\sqrt{2N_0T}}\right) \\
& \Phi_{D_2}\left(-\frac{mw}{\sqrt{2N_0T}}\right)\cdot\Phi_M\left(-\frac{mw}{\sqrt{2N_0T}}\right)
\end{aligned} \tag{4.85}$$

where the characteristic function of D_2, $\Phi_{D_2}(w)$, is given in (4.75) and the characteristic function of M, $\Phi_M(w)$, is given in (4.82) for a fully synchronous system, in (4.83) for a quasi-synchronous system and in (4.84) for an asynchronous system. Note that both $\Phi_{D_2}(w)$ and $\Phi_M(w)$ are even functions of w since $J_0(x)$ and $H_1(x)$ are even functions and $J_1(x)$ and $H_0(x)$ are odd functions of x.

4.4.2 System Analysis

We now use (4.85) to assess the performance of the quadriphase spread DS CDMA system. As in the case of the biphase spread system, we assume a user data transmission at 9600 bits/sec and a processing gain of 128. For an SNR of 20dB and no synchronization errors, the asynchronous quadriphase spread system capacity is calculated as 39. Each of these users is guaranteed a service with an error probability less than or equal to 10^{-3}. Recall that for a biphase spread BPSK modulated system, under the same assumptions, the system capacity is 39 as well. Torrieri in [235] states that a quadriphase spread system is only slightly more immune to multiple access interference than a biphase spread system. The minor difference in the relative system performances is especially true when the number of users active in the system is small. For example, when only 18 users are present in a DS CDMA system of SNR=20dB, G=128 and a 5% observed timing error, each user experiences an error probability of $4.3608 \cdot 10^{-7}$ if quadriphase spreading is employed. The error probability for the same system is $1.9475 \cdot 10^{-6}$ if the modulation in question is biphase spreading. In both systems, however, note that every user achieves the target bit error rate. In fact they operate at a much higher service quality than required. When the number of users increases towards the system capacity, performance of the two modulation schemes converge. In fact, the difference between the error probabilities of the two systems becomes so minute that this difference does not correspond to a capacity change. The performance of the quadriphase spread and the biphase spread system are compared in Figure 4.9. Here, asynchronous transmission and perfect synchronization are assumed. In today's second generation TIA/EIA-95 system, quadriphase spreading is used in the downlink signaling. As can be deducted from Figure 4.9, the choice of quadriphase signaling over biphase spreading is due to a non-performance related issue. Quadriphase signaling produces a smaller signal peak-to-average ratio relative to biphase spreading, simplifying the design of the power amplifiers used in the transmitters.

Similar to the biphase spread system, when synchronization errors are present, they effect the quadriphase spread system capacity significantly. When timing errors are present in the system, the system capacity naturally goes down. For any level of timing error, the difference between the performance of the quadriphase spread system and the biphase spread system remains the same. That is, similar to the perfect synchronization case, the performance of the quadriphase system is only slightly better than the biphase system but this performance difference becomes smaller as the number of active users increases. Therefore, the system capacities of the biphase spread and quadriphase spread systems remain the same regardless of the level of timing error experienced. When phase errors are present in the system, however, the situation changes. As one would expect, the quadriphase spread system is more sensitive to phase errors. Thus, if the phase error is large enough, the performance of the quadriphase spread system may become worse than that of the biphase spread system. The capacity losses due to different levels of timing and phase errors are listed in Table 4.6 comparatively for biphase and quadriphase spread systems. Note that these values are for asynchronous DS CDMA systems operating at 9600 bits/sec with a processing gain of 128. The signal to noise ratio for these calculations is set to 20 dB.

These results show that, similar to the biphase spread system, the quadriphase spread system is sensitive to synchronization errors as well. For example, similar to the biphase spread system, for a quadriphase spread system, a chip timing error of 10%, will reduce the capacity by approximately 20%. Also, similar to the case of the biphase spread system, the capacity loss due to the presence of combined timing and phase errors in a quadriphase spread system is approximately the sum of the individual losses for all values of the SNR.

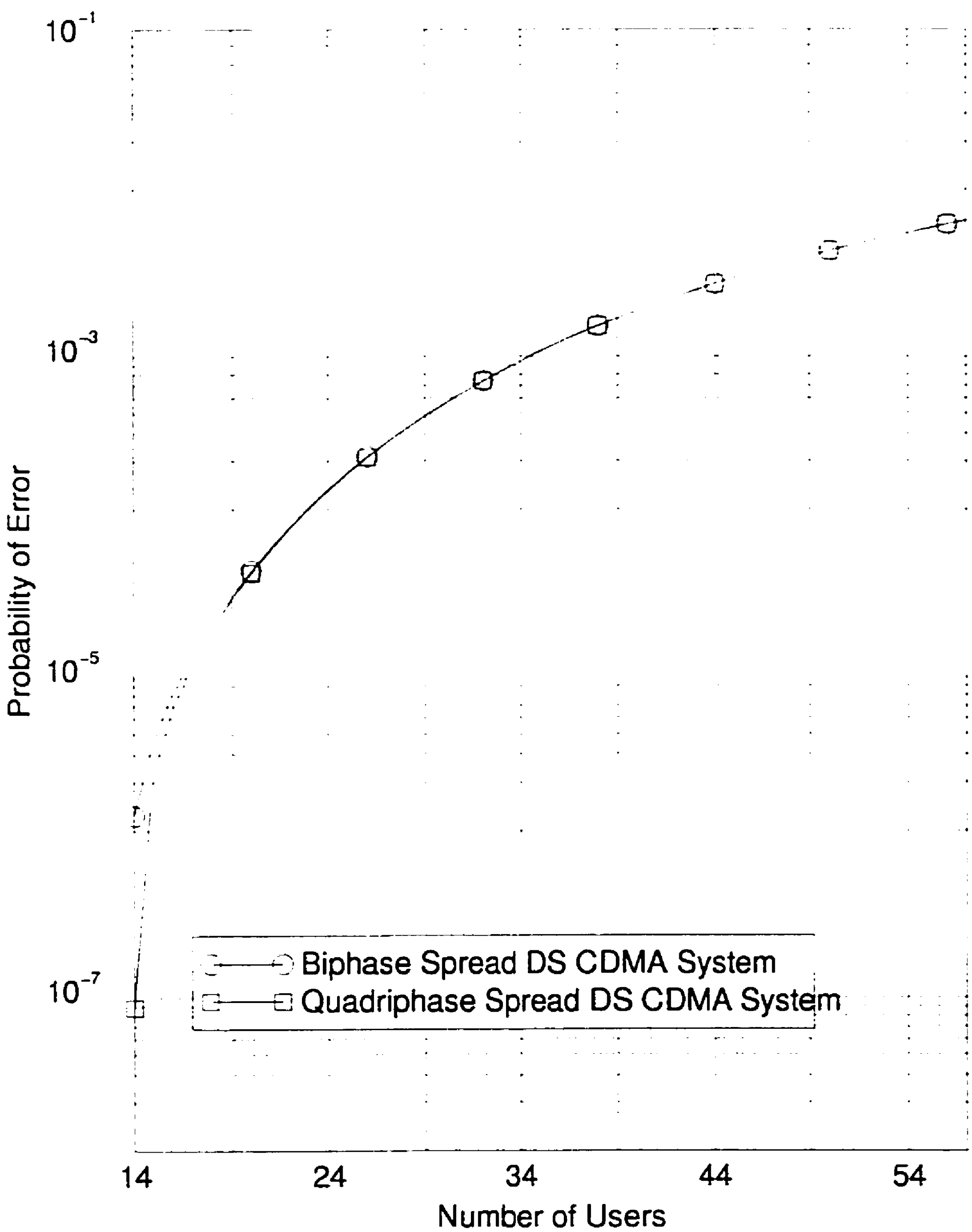

Figure 4.9: Comparison of the Probability of Error versus Number of Users for Quadriphase Spread and Biphase Spread BPSK Modulated DS CDMA Systems.

P(E)=10^{-3}, SNR=20dB, Capacity=39		
Chip Timing Error	Biphase Spread System Capacity Loss	Quadriphase Spread System Capacity Loss
$T_c/10$	8	8
$T_c/25$	3	3
$T_c/50$	2	2
$T_c/100$	1	1
Carrier Phase Error	Biphase Spread System Capacity Loss	Quadriphase Spread System Capacity Loss
$\pi/5$	14	15
$\pi/10$	4	4
$\pi/20$	2	2
$\pi/50$	1	1
Combined Synch Error	Biphase Spread System Capacity Loss	Quadriphase Spread System Capacity Loss
$T_c/10$, $\pi/5$	19	20
$T_c/10$, $\pi/10$	11	11
$T_c/10$, $\pi/20$	9	9
$T_c/10$, $\pi/50$	8	8

Table 4.6: Capacity losses due to different levels of synchronization errors for both biphase spread and quadriphase spread DS CDMA systems.

We observe that the system capacities of the biphase spread and quadriphase spread systems are in practice the same. In fact, when calculations are done to assess the relative performance of synchronous, quasi-synchronous and asynchronous quadriphase spread systems, the results obtained have consistently been identical to those obtained and presented in the previous section for the biphase spread system.

4.5 Summary

In this chapter, we derive infinite series expressions to calculate the probability of error encountered in biphase spread and quadriphase spread DS CDMA systems in AWGN channels. The series is based on a Fourier series representation of $Q(x)$, which was first derived by Beaulieu in [14]. Unlike most other approaches in the literature, the series makes no approximations on the probability density distributions of the intersymbol and multiple access interference encountered. The expressions are used also to assess the loss in system capacity when varies levels of synchronization errors are present.

Chapter 5

PERFORMANCE OF DS CDMA SYSTEMS IN MULTIPATH FADING CHANNELS

In Chapter 4, we have developed an accurate, infinite series expression for the calculation of error probabilities for DS CDMA systems in AWGN channels without making use of a Gaussian approximation for the intersymbol or multiple access interference. This expression is based on the Fourier series expression of the error function, $Q(x)$, developed by Beaulieu in [14]. In this chapter, extending on this procedure, we develop an infinite series expression for the bit error rate of the DS CDMA system in a slowly fading multipath environment presented in Chapter 2. We assume that the system uses coherent reception.

As discussed in Chapter 2, in mobile radio environments, the received signals are subjected to multipath fading which severely degrades the system performance. If the system bandwidth is larger than the coherence bandwidth of the channel, fading is frequency selective [176]. In this case, the multipath components in the received signal are resolvable with a resolution in the time delay of T_c, the chip duration. With CDMA techniques, the resolvable paths can be demodulated individually by a RAKE receiver of the form shown in Figure 2.9 which exploits the excess redundancy due to the presence of independent channel outputs from the multipaths. In a RAKE receiver, information obtained from each branch is combined in a certain way to minimize the interference and further mitigate the fading [19].

We start this chapter with the generation of the probability of error series for DS

CDMA systems in multipath fading channels. The derived series is general in the sense that it can be applied to both frequency selective and frequency nonselective channels. If additional antennas are used to create diversity or increase the degree of existing diversity, this can easily be taken into account in the series as well. Only biphase spread DS CDMA systems are studied in this chapter. Three diversity combining schemes are considered: selection diversity, equal gain combining and maximal ratio combining. We make use of the derived series for all three diversity combining schemes to study the performance of DS CDMA systems in various operating conditions.

5.1 *Probability of Error Series Derivation*

Recall from Chapter 2 that the system of interest uses binary signaling and that the k'th user's information and spreading sequences are defined as (2.2) and (2.3), respectively. The communication channel of interest has a transfer function described in (2.116). We assume that there exist L resolvable paths between the transmitter and the receiver. For each path, the associated path gain is modeled as a Rayleigh distributed random variable, $\beta_{k,l}$ which does not vary for the duration of one symbol. Also recall that without any loss of generality, one can assume the symbol $a_{1,0}$ from the information signal of the first user, $a_1(t)$ is to be captured by the RAKE receiver shown in Figure 2.9. In this case, the input signal to the decision device from the j'th path of the receiver is given by (2.150).

As in Chapter 4, suppose that $a_{1,0} = 1$ represents the binary symbol 1 and $a_{1,0} = -1$ represents the binary symbol 0. The decision device in Figure 2.9 produces the symbol 1 if the decision variable $Z > 0$ and the symbol 0 if $Z < 0$. For an L branch RAKE receiver, the decision variable Z is formed by performing diversity combining on the input signals to the decision device, Z_{1j}, from all L receiver fingers. An error occurs if $Z < 0$ when $a_{1,0} = 1$ or if $Z > 0$ when $a_{1,0} = -1$. Since $a_{1,0}$ is assumed to take on values $\{\mp 1\}$ with equal probability, the probability of error is simply equal to the probability of having $Z > 0$ when $a_{1,0} = -1$.

$$P(E) = P(Z > 0 | a_{1,0} = -1). \tag{5.1}$$

Recall from (2.150) that the input signal to the decision device from the j'th path of the receiver is $Z_{1_j} = D_{1_{ja}} + D_{1_{jk}} + I_{1_j} + M_{1_j} + N_{1_j}$ where $D_{1_{ja}}$ is the desired signal, $D_{1_{jk}}$ is the self interference due to chip timing errors, I_{1_j} is the intersymbol interference due to the multipath, M_{1_j} is the multiple access interference and N_{1_j} is the AWGN term with zero mean and $N_0 T$ variance.

Once all Z_{1_j}'s are obtained, diversity combining is performed in the receiver and the decision variable Z is formed. We consider three diversity combining schemes: selection diversity, maximal ratio diversity combining and equal gain diversity combining.

5.1.1 Selection Diversity

As discussed in Chapter 2, when selection diversity is employed, the receiver simply selects the receiver path with the highest path gain, β_{1_j}, and uses the information from this path to estimate the transmitted signal $a_1(t)$. The other paths are not used in the decision making process. In other words, the decision variable Z is equal to,

$$Z = \text{Max}_{\beta_{1_j}}\{Z_{1_j}\}. \tag{5.2}$$

Since only the j'th path is used in the decision making, the chip timing and carrier phase errors in the receiver are, τ_{1_j} and ϕ_{1_j}, respectively. When the j'th path gain is the maximum of the L gains where the individual gains are Rayleigh distributed, the probability density function of the j'th path gain will be in the form,

$$f_{\beta_{1_j}}(x) = L \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k x}{\rho_0} \exp\left(-\frac{x^2(k+1)}{2\rho_0}\right) u(x) \tag{5.3}$$

as shown in equation (5.2-7) of [87].

Following Chapter 4, the error probability conditioned on the random variables $I_{1j}, M_{1j}, \beta_{1j}$ and α_m is,

$$P(E|I_{1j}, M_{1j}, \beta_{1j}, \alpha_m) = Q\left[\frac{\sqrt{2P_1}\beta_{1j}\cos(\phi_{1j})\left((T_c - \tau_{1j})G - \tau_{1j}\sum_{m=0}^{G-1}\alpha_m\right) - I_{1j} - M_{1j}}{\sqrt{N_0 T}}\right]. \quad (5.4)$$

The random variables I_{1j} and M_{1j} all arise from different physical sources. Hence they are independent. Thus, using the total probability theorem [166], the error probability conditioned only on β_{1j} and α_m is written as,

$$P(E|\beta_{1j}, \alpha_m) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(E|I_{1j}, M_{1j}, \beta_{1j}, \alpha_m) f_{I_{1j}}(i) f_{M_{1j}}(m)\, di\, dm \quad (5.5)$$

It is clear from (2.153) and (2.154) that the probability density functions $f_{I_{1j}}(i)$ and $f_{M_{1j}}(m)$ are difficult to determine. Instead, similar to the procedure detailed in Chapter 4 for the DS CDMA systems in AWGN systems, we proceed to rewrite the conditional error probability given in (5.4) using the Fourier series expansion of $Q(x)$ given in Section 4.2. If we substitute (4.30) into (5.5), we obtain,

$$\begin{aligned} P(E|\beta_{1j}, \alpha_m) &= \sum_{m=-\infty}^{\infty} c_m e^{\frac{jm\omega\sqrt{2P_1}\beta_{1j}\cos(\phi_{1j})\left((T_c-\tau_{1j})G-\tau_{1j}\sum_{m=0}^{G-1}\alpha_m\right)}{\sqrt{N_0T}}} \\ &\quad \cdot \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-jm\omega\frac{i+m}{\sqrt{N_0T}}} f_{I_{1j}}(i) f_{M_{1j}}(m)\, di\, dm \\ &= \sum_{m=-\infty}^{\infty} c_m e^{\frac{jm\omega\sqrt{2P_1}\beta_{1j}\cos(\phi_{1j})\left((T_c-\tau_{1j})G-\tau_{1j}\sum_{m=0}^{G-1}\alpha_m\right)}{\sqrt{N_0T}}} \\ &\quad \cdot \Phi_{I_{1j}}\left(-\frac{m\omega}{\sqrt{N_0T}}\right) \cdot \Phi_{M_{1j}}\left(-\frac{m\omega}{\sqrt{N_0T}}\right) \end{aligned} \quad (5.6)$$

Thus, we need only to find the characteristic functions of I_{1j} and M_{1j}. To this end, we let,

$$i_l = \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l}) \sum_{i=0}^{G-1} [\Delta t_{1l}\tilde{\alpha}_i + (T_c - \Delta t_{1l})\underline{\alpha}_i] \quad (5.7)$$

to get

$$I_{1j} = \sum_{\substack{l=1 \\ l \neq j}}^{L} i_l. \tag{5.8}$$

Since the i_l are independent random variables,

$$\Phi_{I_{1j}}(\omega) = [\Phi_{i_l}(\omega)]^{L-1} \tag{5.9}$$

If (5.7) is studied, it is seen that the random variables β_{1l}, ϕ_{1l} and Δt_{1l} remain constant throughout the duration of G chips whereas the random variables $\tilde{\alpha}_i$ and $\underline{\alpha}_i$ vary independently from chip to chip. In this case, the characteristic function of i_l is defined as,

$$\begin{aligned}
\Phi_{i_l}(\omega) &= \mathrm{E}\left\{e^{j\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})\sum_{i=0}^{G-1}\left[\Delta t_{1l}\tilde{\alpha}_i+(T_c-\Delta t_{1l})\underline{\alpha}_i\right]}\right\} \\
&= \mathrm{E}\left\{\left[\frac{1}{2}\cos(\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c)\right.\right. \\
&\quad \left.\left. +\frac{1}{2}\cos(\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})(2\Delta t_{1l} - T_c))\right]^{G}\right\}
\end{aligned} \tag{5.10}$$

Using the binomial expansion, it is possible to rewrite (5.10) as,

$$\begin{aligned}
\Phi_{i_l}(\omega) &= \mathrm{E}\left\{\frac{1}{2^G}\sum_{p=0}^{G}\binom{G}{p}\cos^p(\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})(2\Delta t_{1l} - T_c))\right. \\
&\quad \left.\cdot\cos^{G-p}(\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c)\right\}
\end{aligned} \tag{5.11}$$

Once again, we assume that the processing gain of the system, G is even. Note that it is also possible to derive the characteristic function for odd G's. In this case, (5.11) can be rewritten as,

$$\begin{aligned}
&\Phi_{i_l}(w) = \\
&\quad \mathrm{E}\left\{\frac{1}{2^G}\sum_{p=0}^{G/2}\binom{G}{2p}\cos^{2p}\left(w\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})(T_c - 2\Delta t_{1l})\right)\right. \\
&\quad \cdot\cos^{G-2p}\left(w\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c\right)
\end{aligned}$$

$$+\frac{1}{2^G}\sum_{p=0}^{G/2-1}\binom{G}{2p+1}\cos^{2p+1}\left(w\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})(T_c-2\Delta t_{1l})\right)$$
$$\left.\cdot\cos^{G-2p-1}\left(w\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c\right)\right\} \tag{5.12}$$

It is possible to perform the expectation on Δt_{1l} using the equalities.

$$\int\cos^{2n}(x)dx = \frac{1}{2^{2n}}\binom{2n}{n}x+\frac{1}{2^{2n-1}}\sum_{k=0}^{n-1}\binom{2n}{k}\frac{\sin(2n-2k)x}{2n-2k} \tag{5.13}$$
$$\int\cos^{2n+1}(x)dx = \frac{1}{2^{2n}}\sum_{k=0}^{n}\binom{2n+1}{k}\frac{\sin(2n-2k+1)x}{2n-2k+1} \tag{5.14}$$

from page 132 of [72] where the constant of integration is omitted. Recall from Chapter 2 that Δt_{1l} can be assumed to be uniformly distributed over $[0, T_c]$. In this case, for an asynchronous system,

$$\Phi_{1l}(\omega) = \mathrm{E}\left\{\sum_{p=0}^{G/2}\binom{G}{2p}\binom{2p}{p}\frac{\cos^{G-2p}(\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c)}{2^{G+2p}}\right.$$
$$+\sum_{p=1}^{G}\binom{G}{p}\frac{\cos^{G-p}(\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c)}{2^{G+p-1}\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c}$$
$$\left.\cdot\sum_{q=0}^{\lfloor\frac{p-1}{2}\rfloor}\binom{p}{q}\frac{\sin((p-2q)\omega\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c)}{p-2q}\right\}. \tag{5.15}$$

The expectation relative to the random variables β_{1l} and ϕ_{1l} can be performed using numerical integration. From Chapter 2, we know that ϕ_{1l} are iid random variables uniformly distributed over the interval $[0, 2\pi]$. β_{1l} on the other hand, has a Rayleigh distribution given by (2.111). As in the case with the system in AWGN channel, a simple trapezoidal rule provides accurate results in a reasonably fast manner. Once the numerical integration is performed, the characteristic function of I_{1j} can simply be found using (5.9).

We now find the characteristic function of M_{1j}. To this end, we let,

$$m_{kl} = \sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\sum_{j=0}^{G-1}\left[\eta_{kj}\tau_{kl}+\kappa_{kj}(T_c-\tau_{kl})\right]. \tag{5.16}$$

to get

$$M_{1_j} = \sum_{k=2}^{K} \sum_{l=1}^{L} m_{kl}. \tag{5.17}$$

Similar to i_l in (5.8), the m_{kl} are iid random variables as well. Therefore,

$$\Phi_{M_{1_j}}(\omega) = [\Phi_{m_{kl}}(\omega)]^{(K-1)L} \tag{5.18}$$

where the characteristic function of m_{kl} is found as,

$$\begin{aligned}
\Phi_{m_{kl}}(\omega) &= \mathrm{E}\left\{ e^{j\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\sum_{j=0}^{G-1}\left[\eta_{k_j}\tau_{kl}+\kappa_{k_j}(T_c-\tau_{kl})\right]} \right\} \\
&= \mathrm{E}\Big\{ \Big[\frac{1}{2}\cos(\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})T_c) \\
&\quad + \frac{1}{2}\cos(\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})(2\tau_{kl} - T_c)) \Big]^G \Big\} \\
&= \mathrm{E}\Big\{ \frac{1}{2^G}\sum_{p=0}^{G} \binom{G}{p} \cos^p(\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})(2\tau_{kl} - T_c)) \\
&\quad \cdot \cos^{G-p}(\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})T_c) \Big\}
\end{aligned} \tag{5.19}$$

As before, the expectation on the random variable, τ_{kl} can be performed analytically. Using (5.13) and (5.14) we get,

$$\begin{aligned}
\Phi_{m_{kl}}(\omega) &= \mathrm{E}\left\{ \sum_{p=0}^{G/2} \binom{G}{2p}\binom{2p}{p} \frac{\cos^{G-2p}(\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})T_c)}{2^{G+2p}} \right. \\
&\quad + \sum_{p=1}^{G} \binom{G}{p} \frac{\cos^{G-p}(\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})T_c)}{2^{G+p-1}\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})T_c} \\
&\quad \left. \cdot \sum_{q=0}^{\lfloor\frac{p-1}{2}\rfloor} \binom{p}{q} \frac{\sin((p-2q)\omega\sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})T_c)}{p-2q} \right\}.
\end{aligned} \tag{5.20}$$

The expectation relative to the random variables β_{kl} and ϕ_{kl} in equation (5.6) can be performed using numerical integration. Note that the characteristic functions of i_l and m_{kl} become identical if $P_1 = P_k$. Once again, trapezoidal rule can be used to perform these integrations.

An alternative way to find the characteristic functions of I_{1_j} and M_{1_j} would be to make some independence assumptions as explained in Chapter 4 at the expense of some degradation in the accuracy of the technique. If one assumes that the products $\beta_{1l}\cos(\phi_{1l})[\Delta t_{1l}\tilde{\alpha}_i + (T_c - \Delta t_{1l})\underline{\alpha}_i]$ and $\beta_{kl}\cos(\phi_{kl})\,[\eta_{kj}|\tau_{kl}| + \kappa_{kj}(T_c - |\tau_{kl}|)]$ vary independently from chip to chip, it is possible to find closed form expressions for the characteristic functions of I_{1j} and M_{1j} and these expressions would be significantly faster to compute. Under these assumptions we first define,

$$i_{li} = \sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})\,[\Delta t_{1l}\tilde{\alpha}_i + (T_c - \Delta t_{1l})\underline{\alpha}_i] \tag{5.21}$$

to get

$$I_{1j} = \sum_{\substack{l=1 \\ l\neq j}}^{L} \sum_{i=0}^{G-1} i_{li}. \tag{5.22}$$

and since i_{li} are assumed to be iid, this would result in,

$$\Phi_{I_{1j}}(\omega) = [\Phi_{i_{li}}(\omega)]^{G(L-1)}. \tag{5.23}$$

Thus, we find the characteristic function of i_{li}. We first perform the expectation over the random variables $\tilde{\alpha}$ and $\underline{\alpha}$ to get,

$$\begin{aligned} \Phi_{i_{li}}(\omega) = \mathrm{E}\Big\{ & \frac{1}{4}e^{-jw\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c} \\ & + \frac{1}{4}e^{jw\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})T_c} \\ & + \frac{1}{4}e^{-jw\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})(2\Delta t_{1l}-T_c)} \\ & + \frac{1}{4}e^{jw\sqrt{2P_1}\beta_{1l}\cos(\phi_{1l})(2\Delta t_{1l}-T_c)} \Big\}. \end{aligned} \tag{5.24}$$

Next, using the following equation from (9.1.21) on page 360 of [4],

$$J_0(z) = \frac{1}{2\pi}\int_0^{2\pi} e^{jz\cos(\theta)}d\theta \tag{5.25}$$

we perform the expectation on ϕ_{1l} to get,

$$\Phi_{i_{li}}(\omega) = \mathrm{E}\{\frac{1}{2}J_0(\omega\sqrt{2P_1}\beta_{1l}T_c) + \frac{1}{2}J_0(\omega\sqrt{2P_1}\beta_{1l}(2\Delta t_{1l} - T_c))\}. \tag{5.26}$$

We then use the equality.

$$\int_0^a J_0(x)dx = 2\sum_{k=0}^{\infty} J_{2k+1}(a) \tag{5.27}$$

from (6.511.3) on page 666 of [72] to perform the expectation over Δt_{1l}. We get.

$$\begin{aligned}\Phi_{i_{l1}}(\omega) &= E\Big\{\frac{1}{2}J_0(\omega\sqrt{2P_1}\beta_{1l}T_c) \\ &+\frac{2}{\omega\sqrt{2P_1}\beta_{1l}T_c}\sum_{k=0}^{\infty} J_{2k+1}(\omega\sqrt{2P_1}\beta_{1l}T_c)\Big\}.\end{aligned} \tag{5.28}$$

Next, we perform the expectation over β_{1l} using the equality (6.631) on page 716 of [72] given by.

$$\int_0^{\infty} e^{-a^2t^2}t^{\mu-1}J_v(bt)dt = \frac{\Gamma(\frac{1}{2}v+\frac{1}{2}\mu)(\frac{1}{2}\frac{b}{a})^v}{2a^{\mu}\Gamma(v+1)}M(\frac{1}{2}v+\frac{1}{2}\mu.v+1.-\frac{b^2}{4a^2}) \tag{5.29}$$

where $M(a.b.z)$ is the confluent hypergeometric function and is defined as.

$$M(a.b.z) = \sum_{k=0}^{\infty}\frac{(a)_k z^k}{(b)_k k!} \tag{5.30}$$

where $(a)_0 = 1$ and $(a)_n = a(a+1)\ldots(a+n-1)$ [4]. Similarly, $\Gamma(n)$ is the gamma function and is defined as.

$$\Gamma(n+1) = n! \tag{5.31}$$

for integer values of n. After performing the integration, we obtain.

$$\begin{aligned}\Phi_{i_{l1}}(\omega) &= \frac{1}{2}e^{-\omega^2P_1\rho_0T_c^2} \\ &+\frac{1}{2}\sum_{n=0}^{\infty}\frac{n!(\omega\sqrt{P_1\rho_0}T_c)^{2n}}{(2n+1)!}M(n+1.2(n+1).-\omega^2P_1\rho_0T_c^2).\end{aligned} \tag{5.32}$$

Note that (5.32) is in its closed form and does not require numerical integration. Once (5.32) is computed, the characteristic function of I_{1j} can easily be found using (5.23).

We continue with the derivation of the characteristic function for the multiple access interference term, M_{1j}. For this purpose we define.

$$m_{klj} = \sqrt{2P_k}\beta_{kl}\cos(\phi_{kl})\left[\eta_{kj}\tau_{kl} + \kappa_{kj}(T_c-\tau_{kl})\right]. \tag{5.33}$$

From (2.135),

$$M_{1j} = \sum_{k=2}^{K} \sum_{l=1}^{L} \sum_{j=0}^{G-1} m_{klj} \tag{5.34}$$

Once again, using the equalities (5.25), (5.27) and (5.29) successively, we get,

$$\begin{aligned}\Phi_{m_{klj}}(\omega) &= \mathrm{E}\left\{\frac{1}{2}J_0(\omega\sqrt{2P_k}\beta_{kl}T_c) + \frac{1}{2}J_0(\omega\sqrt{2P_k}\beta_{kl}(2\tau_{kl} - T_c))\right\} \\ &= \frac{1}{2}e^{-\omega^2 P_k \rho_0 T_c^2} \\ &\quad + \frac{1}{2}\sum_{n=0}^{\infty} \frac{n!(\omega\sqrt{P_k\rho_0}T_c)^{2n}}{(2n+1)!} M(n+1, 2(n+1), -\omega^2 P_k \rho_0 T_c^2)\end{aligned} \tag{5.35}$$

and since m_{klj} are iid random variables, from (5.34),

$$\Phi_{M_{1j}}(\omega) = \prod_{k=2}^{K} \prod_{l=1}^{L} \prod_{j=0}^{G-1} \Phi_{m_{klj}}(\omega) \tag{5.36}$$

Having found the expressions for the characteristic functions of I_{1j} and M_{1j}, we can now find the infinite series for the error probability. Using (4.31) and (5.6),

$$\begin{aligned}P(E|\beta_{1j}, \alpha_m) &= \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m\,odd}}^{\infty} \frac{1}{m} e^{-m^2\omega^2/2} \\ &\quad \cdot \sin\left(\frac{m\omega\sqrt{2P_1}\beta_{1j}\cos(\phi_{1j})\left((T_c - \tau_{1j})G - \tau_{1j}\sum_{m=0}^{G-1}\alpha_m\right)}{\sqrt{N_0 T}}\right) \\ &\quad \cdot \Phi_I(-\frac{m\omega}{\sqrt{N_0 T}}) \cdot \Phi_M(-\frac{m\omega}{\sqrt{N_0 T}})\end{aligned} \tag{5.37}$$

where the characteristic functions of I_{1j} and M_{1j} can be found using either (5.9) and (5.18) or (5.23) and (5.36). We need to integrate $P(E)$ over the distributions of β_{1j} and $\alpha_m, m = 0, \ldots, G-1$ to get the unconditional error probability expression. Recall that when selection diversity is employed β_{1j} has a probability density function of the form given in (5.3). α_m, on the other hand, are iid random variables that take on values $\{\mp 1\}$ randomly. We define,

$$\alpha = \sum_{m=0}^{G-1} \alpha_m. \tag{5.38}$$

Then, α is a random variable with binomial distribution. Since β_{1j} and α are independent random variables,

$$P(E) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(E|\beta_{1j}, \alpha) f_{\beta_{1j}}(y) f_{\alpha}(x)\, dy\, dx \tag{5.39}$$

By using equality (3.952.1) on page 495 of [72] given by,

$$\int_0^{\infty} x e^{-p^2x^2} \sin(ax) dx = \frac{a\sqrt{\pi}}{4p^2} e^{-a^2/4p^2}, \tag{5.40}$$

the equality (3.323.2) on page 307 of [72] given by,

$$\int_{-\infty}^{\infty} e^{p^2x^2 \mp qx} dx = e^{q^2/4p^2} \frac{\sqrt{\pi}}{p}, \quad p > 0, \tag{5.41}$$

and the equality (3.462.6) on page 338 of [72] given by,

$$\int_{-\infty}^{\infty} x e^{-px^2+2qx} dx = \frac{q}{p}\sqrt{\frac{\pi}{p}} e^{q^2/p}, \quad p > 0 \tag{5.42}$$

successively and performing some substitutions we get,

$$\begin{aligned} P(E) &= \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2\omega^2/2} L \sum_{k=0}^{L-1} \binom{L-1}{k} \frac{(-1)^k C_{mk}}{k+1} \sqrt{\frac{\pi}{D_{mk}^2 + 1/(2G)}} \\ &\cdot \left(1 - \frac{D_{mk}^2}{D_{mk}^2 + 1/(2G)}\right) \exp\left\{ \frac{C_{mk}^2 D_{mk}^2}{D_{mk}^2 + 1/(2G)} - C_{mk}^2 \right\} \frac{1}{\sqrt{2G}} \\ &\cdot \Phi_{I_{1j}}\left(-\frac{m\omega}{\sqrt{N_0 T}}\right) \cdot \Phi_{M_{1j}}\left(-\frac{m\omega}{\sqrt{N_0 T}}\right) \end{aligned} \tag{5.43}$$

where

$$C_{mk} = \frac{m\omega\sqrt{2P_1}\cos(\phi_{1j})(T_c - \tau_{1j})G}{2\sqrt{N_0 T}\left(\frac{k+1}{2\rho_0}\right)^{1/2}}, \tag{5.44}$$

$$D_{mk} = \frac{m\omega\sqrt{2P_1}\cos(\phi_{1j})\tau_{1j}}{2\sqrt{N_0 T}\left(\frac{k+1}{2\rho_0}\right)^{1/2}}. \tag{5.45}$$

We use both methods of finding the characteristic functions for the interference terms to compute the system capacities at different chip timing and carrier phase errors. System performance as a function of the number of active users is graphed for both methods in Figure 5.1 for systems at various synchronization error levels. Here, systems with 2 multipaths and 20 dB SNR are considered. As can be seen from Figure 5.1, the probability of error calculated from the two methods is slightly different; the independence assumption results in slightly optimistic values. The difference, however, is never large enough to grant a discrepancy in the system capacity calculated using these methods. We find that when an error probability of 10^{-3} is desired, the two methods give exactly same value for the system capacity at all synchronization error levels.

5.1.2 Maximal Ratio Combining

When maximal ratio combining is employed, the receiver, having perfect knowledge of the individual path gains, weights each path with its corresponding path gain and then sums these weighted terms. It is this sum that is used in the decision making. Then,

$$\begin{aligned} Z &= \sum_{j=1}^{L} \beta_{1j} Z_{1j} \\ &= \sum_{j=1}^{L} \beta_{1j}\{D_{1j} + I_{1j} + M_{1j} + N_{1j}\} \\ &= D + I + M + N \end{aligned} \quad (5.46)$$

where D is the sum of the desired signal terms from all branches, I is the intersymbol interference, M is the multiple access interference and N is the AWGN term with variance equal to,

$$\sigma_N^2 = 2N_0 T \rho_0 L\{1 + (L-1)\frac{\rho_0^2 \pi}{4}\} \quad (5.47)$$

since $E\{\beta_{1j}^2\} = 2\rho_0$ and $E\{\beta_{1j}\} = \rho_0\sqrt{2\pi\rho_0}/2$. Since all of the branches are used in the decision making, each branch in the receiver structure of Figure 2.9 has the potential to have synchronization errors. Thus the chip timing errors are de-

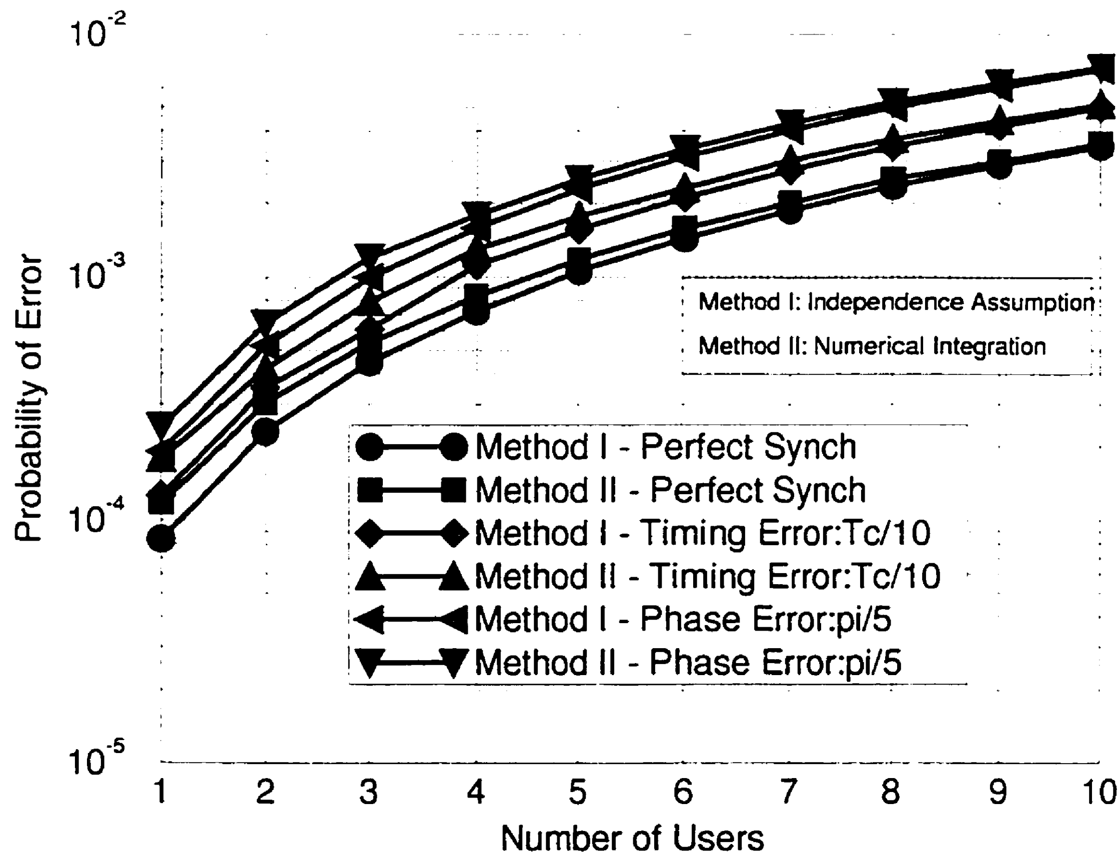

Figure 5.1: Comparison of the two methods in evaluating the system capacity for a system employing selection diversity on 2 paths and operating at 20dB SNR.

fined as, $\tau_{11}, \tau_{12}, \ldots, \tau_{1L}$, and correspondingly, the carrier phase errors are defined as, $\phi_{11}, \phi_{12}, \ldots, \phi_{1L}$.

Following the same procedure outlined for the selection diversity receiver, one can write the error probability for maximal ratio combining conditioned on the L path gains and the Gaussian random variable α as,

$$
\begin{aligned}
P(E|\beta_{11}, \ldots, \beta_{1L}, \alpha) \quad = \quad & \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2\omega^2/2} \\
& \cdot \sin \left[\frac{m\omega\sqrt{2P_1}\sum_{j=1}^{L} \beta_{1j}^2 \cos(\phi_{1j})[G(T_c - \tau_{1j}) - \alpha\tau_{1j}]}{\sigma_N} \right] \\
& \Phi_I \left(-\frac{m\omega}{\sigma_N} \right) \Phi_M \left(-\frac{m\omega}{\sigma_N} \right). \qquad (5.48)
\end{aligned}
$$

Once again, we only need to find the characteristic functions of I and M to find the error probability expression. These functions can be found either semi-analytically or using the independence assumption discussed in the previous section. Using the equations (9.1.18) on page 360 of [4] and (6.629) on page 716 of [72], we obtain,

$$
\begin{aligned}
\Phi_I(\omega) \quad = \quad & \left[\frac{1}{2} e^{-\omega^2 P_1 \rho_0 T_c^2 (\sum_{j=1}^{L} \beta_{1j})^2} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{n!(\omega\sqrt{P_1\rho_0}T_c(\sum_{j=1}^{L} \beta_{1j}))^{2n}}{(2n+1)!} \right. \\
& \left. \cdot M(n+1, 2(n+1), -\omega^2 P_1 \rho_0 T_c^2 (\sum_{j=1}^{L} \beta_{1j})^2) \right]^{G(L-1)} . \qquad (5.49)
\end{aligned}
$$

and

$$
\begin{aligned}
\Phi_M(\omega) \quad = \quad & \left[\frac{1}{2} e^{-\omega^2 P_k \rho_0 T_c^2 (\sum_{j=1}^{L} \beta_{1j})^2} + \frac{1}{2} \sum_{n=0}^{\infty} \frac{n!(\omega\sqrt{P_k\rho_0}T_c(\sum_{j=1}^{L} \beta_{1j}))^{2n}}{(2n+1)!} \right. \\
& \left. \cdot M(n+1, 2(n+1), -\omega^2 P_k \rho_0 T_c^2 (\sum_{j=1}^{L} \beta_{1j})^2) \right]^{GL(K-1)} . \qquad (5.50)
\end{aligned}
$$

It is possible to uncondition (5.48) from the random variable α analytically. The error probability conditioned only on the individual path gains is given by,

$$P(E|\beta_{11}, \ldots, \beta_{1L})$$

$$
\begin{aligned}
= & \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2\omega^2/2} \exp(-m^2\omega^2 P_1 G(\sum_{j=1}^{L} \beta_{1j}^2 \cos(\phi_{1j})\tau_{1j})^2/\sigma_N^2) \\
& \cdot \sin\left[\frac{m\omega\sqrt{2P_1}\sum_{j=1}^{L}\beta_{1j}^2\cos(\phi_{1j})G(T_c-\tau_{1j})}{\sigma_N}\right] \\
& \cdot \Phi_I\left(-\frac{m\omega}{\sigma_N}\right)\cdot\Phi_M\left(-\frac{m\omega}{\sigma_N}\right). \qquad (5.51)
\end{aligned}
$$

Unconditioning (5.51) from the individual path gains requires numerical integration. We use the simple trapezoidal rule to perform this integration.

5.1.3 Equal Gain Combining

When equal gain combining is employed, the receiver simply sums each path term and uses this sum in the decision making. Then,

$$
\begin{aligned}
Z &= \sum_{j=1}^{L} Z_{1j} \\
&= \sum_{j=1}^{L} D_{1j} + I_{1j} + M_{1j} + N_{1j} \\
&= D + I + M + N \qquad (5.52)
\end{aligned}
$$

where D is the sum of the desired signal terms from all branches, I is the intersymbol interference, M is the multiple access interference and N is the AWGN term with variance equal to,

$$\sigma_N^2 = N_0 T L. \qquad (5.53)$$

Similar to the maximal ratio combining case, when equal gain combining is employed, it is clear from (5.52) that all of the L paths are used in the decision making process. Therefore, the chip timing errors are defined as, $\tau_{11}, \tau_{12}, \ldots, \tau_{1L}$ and similarly, the carrier phase errors are defined as, $\phi_{11}, \phi_{12}, \ldots, \phi_{1L}$.

Following the same procedure outlined for the selection diversity receiver one can write the error probability for equal gain combining conditioned on the L path gains and α as,

$$\begin{aligned} P(E|\beta_{11},\ldots,\beta_{1L},\alpha) &= \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{m=1\\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2\omega^2/2} \\ &\cdot \sin\left[\frac{m\omega\sqrt{2P_1}\sum_{j=1}^{L}\beta_{1j}\cos(\phi_{1j})[G(T_c-\tau_{1j})-\alpha\tau_{1j}]}{\sigma_N}\right] \\ &\cdot \Phi_I\left(-\frac{m\omega}{\sigma_N}\right)\cdot\Phi_M\left(-\frac{m\omega}{\sigma_N}\right). \end{aligned} \tag{5.54}$$

The characteristic functions are found the same way as in the two previous cases as,

$$\begin{aligned} \Phi_I(\omega) &= [\frac{1}{2}e^{-\omega^2 P_1\rho_0 T_c^2} + \frac{1}{2}\sum_{n=0}^{\infty}\frac{n!(\omega\sqrt{P_1\rho_0}T_c)^{2n}}{(2n+1)!} \\ &\cdot M(n+1, 2(n+1), -\omega^2 P_1\rho_0 T_c^2)]^{G(L-1)}. \end{aligned} \tag{5.55}$$

and

$$\begin{aligned} \Phi_M(\omega) &= [\frac{1}{2}e^{-\omega^2 P_k\rho_0 T_c^2} + \frac{1}{2}\sum_{n=0}^{\infty}\frac{n!(\omega\sqrt{P_k\rho_0}T_c)^{2n}}{(2n+1)!} \\ &\cdot M(n+1, 2(n+1), -\omega^2 P_k\rho_0 T_c^2)]^{GL(K-1)}. \end{aligned} \tag{5.56}$$

Similar to the maximal ratio case, it is possible to uncondition the error probability expression from the variable α analytically,

$$\begin{aligned} P(E|\beta_{1j}) &= \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{m=1\\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2\omega^2/2}\exp(-m^2\omega^2 P_1 G(\sum_{j=1}^{L}\beta_{1j}\cos(\phi_{1j})\tau_{1j})^2/\sigma_N^2) \\ &\cdot \sin\left[\frac{m\omega\sqrt{2P_1}\sum_{j=1}^{L}\beta_{1j}\cos(\phi_{1j})G(T_c-\tau_{1j})}{\sigma_N}\right] \\ &\cdot \Phi_I\left(-\frac{m\omega}{\sigma_N}\right)\Phi_M\left(-\frac{m\omega}{\sigma_N}\right) \end{aligned} \tag{5.57}$$

However, as before, unconditioning (5.57) from the individual path gain variables requires numerical integration. Once again, we use the trapezoidal rule for this purpose.

5.2 System Analysis

For the three different diversity combining schemes considered in this thesis the probability of error, as can be observed in (5.43), (5.51) and (5.57), is a function of the received signal to noise ratio, SNR=$2\rho_0 P_1 T/N_0$, the number of users present in the system, K, the number of diversity paths, L, the synchronization errors, τ_{1_j} and ϕ_{1_j} and the processing gain, G. Thus, it is possible to find the system capacity for different values of SNR, number of diversity paths, synchronization errors and processing gain. When G, SNR, L, τ_{1_j} and ϕ_{1_j} are fixed, we can evaluate the error probability of the system for an increasing number of users. The capacity of the system is simply the maximum number of users that will still yield an error probability below a certain threshold.

We consider a user data transmission at 9600 bits/sec that requires an error probability $\leq 10^{-3}$ and system bandwidths of 1.25 MHz, 5MHz and 10MHz. The 1.25MHz system has a processing gain of 128, the 5MHz system has a processing gain of 512 and the 10MHz system has a processing gain of 1024. We assume that the maximum multipath delay spread is in the range of 25 to 200 nanoseconds [89]. Thus, we conclude that the 1.25MHz channel has only one, the 5MHz channel has two and the 10MHz system has three resolvable paths. We assume a slowly fading channel.

For all three diversity combining schemes, multipath fading affects the system performance dramatically. The 1.25MHz system at 20dB SNR has a capacity of zero if no coding and diversity is employed and no voice activity factor is taken into account. The same system is shown to have a capacity of 39 users in an AWGN environment in Chapter 4. As stated previously, the 1.25MHz system has no inherent diversity through multipath; other means such as the use of multiple antennas at the receiver are to be employed to achieve a nonzero capacity. When an artificial diversity of 2 is achieved, the system has a capacity of 4 if selection diversity is employed, 9 if equal gain combining is employed and 13 if maximal ratio combining is employed.

The wideband systems of 5MHz and 10MHz, on the other hand, both have resolvable multipath branches of 2 and 3, respectively. If all of these branches are utilized by means of selection diversity at the receiver, the 5MHz system has a capacity of 17 and the 10MHz system has a capacity of 58, respectively.

Synchronization errors affect the system capacity significantly. For example, for the 1.25MHz system with 2 artificial multipaths, a chip timing error of 10% will reduce capacity of the selection diversity system to 3, the capacity of the equal gain combining system to 7 and the capacity of the maximal ratio combining system to 10, an approximately 20% reduction in the system capacity in all cases. This percentage loss in the system capacity is about the same as the percentage loss of the same system in the AWGN environment discussed in Chapter 4. Here, we assume that for maximal ratio and equal gain combining systems, all of the receiver branches suffer from the same level of synchronization errors, i.e., $\tau_{11} = \tau_{12} = \ldots = \tau_{1L}$ and $\phi_{11} = \phi_{12} = \ldots = \phi_{1L}$. From (5.43), (5.51) and (5.57), it can be seen that the synchronization errors effect the system performance by potentially reducing the energy of the desired signal component of the received signal and by introducing self interference. As in the case of the AWGN channel, we have numerically found that the self interference, in comparison to the signal energy reduction is negligibly small. The reduction in the desired signal energy level can alternatively be interpreted as an effective processing gain loss. Then, the effective processing gain is approximately a linear function of the system capacity for a given level of maximum allowable error probability.

System error probabilities as a function of SNR are plotted for the three diversity combining schemes when 10 users are present in the system and perfect synchronization is achieved in Figure 5.2. The graph confirms the well-known conclusion that, for the same operation point, maximal ratio combining provides the user with the best performance. Selection diversity, on the other hand, consistently under-performs the other two schemes. Equal gain combining has a performance that is in between those of selection diversity and maximal ratio combining at all times.

For the selection diversity receiver, the capacity losses for various levels of syn-

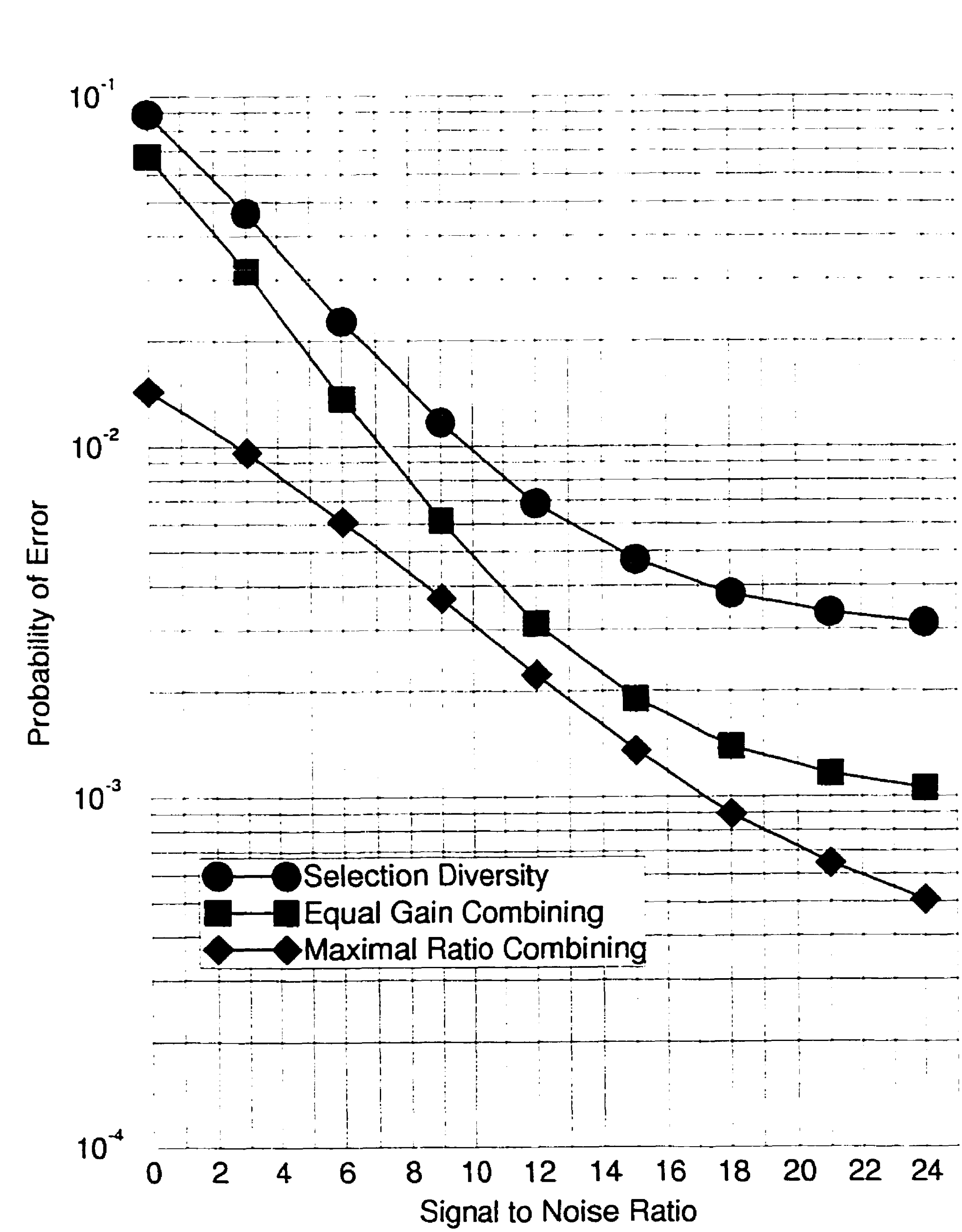

Figure 5.2: Comparison of the diversity combining schemes for the 1.25MHz system at 20dB when there are no synchronization errors.

G=128 (BW=1.25MHz), P(E)=10^{-3}, SNR=20dB		
Synchronization Errors	Capacity	Percentage Capacity Loss wrt Perfect Synchronization Case
L=1		
perf. synch.	0	
L=2		
perf. synch.	4	
$T_c/10$	3	25%
$T_c/25$	4	0%
$T_c/50$	4	0%
$\pi/5$	3	25%
$\pi/10$	4	0%
$\pi/20$	4	0%
$T_c/10, \pi/10$	3	25%

Table 5.1: Capacity losses due to different levels of synchronization errors for the 1.25MHz system with L=1, 2 at 20 dB when selection diversity is employed.

chronization errors are listed in tabular form in Tables 5.1 and 5.2 for the 1.25MHz system, in Tables 5.3 and 5.4 for the 5MHz system and in Table 5.5 for the 10MHz system, respectively, all at 20dB SNR. From these tables, it can be seen that the percentage capacity loss due to a certain level of synchronization error are comparable in all three systems independent of the number of diversity branches considered. The occasional deviations from this conclusion are due to the quantization inherent in the process of finding the system capacity from the bit error rate. This is because, the system capacity can only take on integer values.

G=128 (BW=1.25MHz), P(E)=10^{-3}, SNR=20dB		
Synchronization Errors	Capacity	Percentage Capacity Loss wrt Perfect Synchronization Case
L=3		
perf. synch.	7	
$T_c/10$	6	14.29%
$T_c/25$	6	14.29%
$T_c/50$	7	0%
$\pi/5$	4	42.86%
$\pi/10$	6	14.29%
$\pi/20$	7	0%
$T_c/10, \pi/10$	5	28.57%
L=5		
perf. synch.	9	
$T_c/10$	7	22.22%
$T_c/25$	8	11.11%
$T_c/50$	8	11.11%
$\pi/5$	5	44.44%
$\pi/10$	8	11.11%
$\pi/20$	8	11.11%
$T_c/10, \pi/10$	6	33.33%

Table 5.2: Capacity losses due to different levels of synchronization errors for the 1.25MHz system with L=3, 5 at 20 dB when selection diversity is employed.

G=512 (BW=5MHz), P(E)=10^{-3}, SNR=20dB		
Synchronization Errors	Capacity	Percentage Capacity Loss wrt Perfect Synchronization Case
L=2		
perf. synch.	17	
$T_c/10$	13	23.53%
$T_c/25$	16	5.88%
$T_c/50$	17	0%
$\pi/5$	10	41.18%
$\pi/10$	15	11.76%
$\pi/20$	17	0%
$T_c/10, \pi/10$	12	29.41%
L=4		
perf. synch.	33	
$T_c/10$	27	18.18%
$T_c/25$	30	9.09%
$T_c/50$	32	3.03%
$\pi/5$	21	36.36%
$\pi/10$	30	9.09%
$\pi/20$	32	3.03%
$T_c/10, \pi/10$	24	27.27%

Table 5.3: Capacity losses due to different levels of synchronization errors for the 5MHz system with L=2, 4 at 20dB when selection diversity is employed.

G=512 (BW=5MHz), P(E)=10^{-3}, SNR=20dB		
Synchronization Errors	Capacity	Percentage Capacity Loss wrt Perfect Synchronization Case
L=6		
perf. synch.	34	
$T_c/10$	27	20.59%
$T_c/25$	31	8.82%
$T_c/50$	33	2.94%
$\pi/5$	22	35.29%
$\pi/10$	31	8.82%
$\pi/20$	33	2.94%
$T_c/10, \pi/10$	25	26.47%

Table 5.4: Capacity losses due to different levels of synchronization errors for the 5MHz system with L=6 at 20dB when selection diversity is employed.

G=1024 (BW=10MHz), P(E)=10^{-3}, SNR=20dB		
Synchronization Errors	Capacity	Percentage Capacity Loss wrt Perfect Synchronization Case
L=3		
perf. synch.	58	
$T_c/10$	46	20.69%
$T_c/25$	53	8.62%
$T_c/50$	56	3.45%
$\pi/5$	36	37.93%
$\pi/10$	52	10.34%
$\pi/20$	57	1.72%
$T_c/10, \pi/10$	41	29.31%
L=6		
perf. synch.	69	
$T_c/10$	55	20.29%
$T_c/25$	63	8.69%
$T_c/50$	66	4.35%
$\pi/5$	44	36.23%
$\pi/10$	62	10.14%
$\pi/20$	67	2.89%
$T_c/10, \pi/10$	49	28.99%

Table 5.5: Capacity losses due to different levels of synchronization errors for the 10MHz system at 20dB when selection diversity is employed.

G=128 (BW=1.25MHz), P(E)=10^{-3}, SNR=20dB, L=2			
	System Capacity		
Synchronization Errors	Selection Diversity	Equal Gain Combining	Maximal Ratio Combining
perf. synch.	1	9	13
$T_c/10$	3	7	10
$T_c/25$	4	8	12
$\pi/5$	3	5	7
$\pi/10$	4	8	11
$T_c/10, \pi/10$	3	6	9

Table 5.6: Capacity losses due to different levels of synchronization errors for the 1.25 system at 20dB when selection diversity, equal gain combining and maximal ratio combining is employed.

In Tables 5.6 and 5.7, the capacity losses of the three diversity combining schemes are compared for various chip timing and carrier phase error values when the system bandwidth is 1.25MHz and 5MHz, respectively. It is seen once again that the percentage capacity losses due to a certain level of synchronization error are comparable for the three diversity schemes. Thus, it can be concluded that all three diversity combining schemes are equally sensitive to synchronization errors.

In a practical system, both chip timing and carrier phase errors will be present. If we define the capacity loss of a system as the difference between the capacity when there are no synchronization errors and the capacity when synchronization errors are present, it can be seen from Tables 5.6-5.7 that the capacity loss from the presence of both chip timing and carrier phase errors is approximately the sum of individual losses for all values of SNR for all systems considered.

G=512 (BW=5MHz), P(E)=10^{-3}, SNR=20dB, L=2			
	System Capacity		
Synchronization Errors	Selection Diversity	Equal Gain Combining	Maximal Ratio Combining
perf. synch.	17	35	50
$T_c/10$	13	28	39
$T_c/25$	16	32	46
$\pi/5$	10	21	28
$\pi/10$	15	33	43
$T_c/10, \pi/10$	12	24	35

Table 5.7: Capacity losses due to different levels of synchronization errors for the 5MHz system at 20dB when selection diversity, equal gain combining and maximal ratio combining is employed.

Error probability as a function of the number of active users present in the system is plotted in Figures 5.3, 5.4 and 5.5 for the 1.25 MHz, 5 MHz and 10 MHz systems, respectively when selection diversity is employed. The graphs show that as the number of users increase, the performance of systems with different number of diversity paths converge. However, when low error probabilities are required, as is the case for reliable transmission of voice, data and video signals, having multiple diversity branches increase the system performance that was reduced by fading. However, diversity combining on its own, is not sufficient to gain back all of the capacity that is lost due to fading. Additional means such as coding, interleaving and/or antenna diversity are necessary to further increase the capacity. It is seen from Figures 5.4 and 5.5 that when the multipath branches inherent in the 5MHz and 10MHz systems are not utilized (i.e. when L=1 for the 5 MHz system and when L=1,2 for the 10 MHz system), the performance of these systems become almost as poor as the 1.25MHz system that does not employ any diversity.

5.3 Summary

An infinite series for the probability of error of a BPSK modulated DS CDMA system with nonzero chip timing and carrier phase errors in a slowly fading, multipath environment is derived in this chapter. We assume that the receiver is a coherent RAKE receiver employing selection diversity, equal gain combining or maximal ratio combining. The derived probability of error expression is used to compare the relative performances of the three diversity combining schemes considered. Effects of non-zero synchronization errors on the performance of the diversity combining schemes are also investigated.

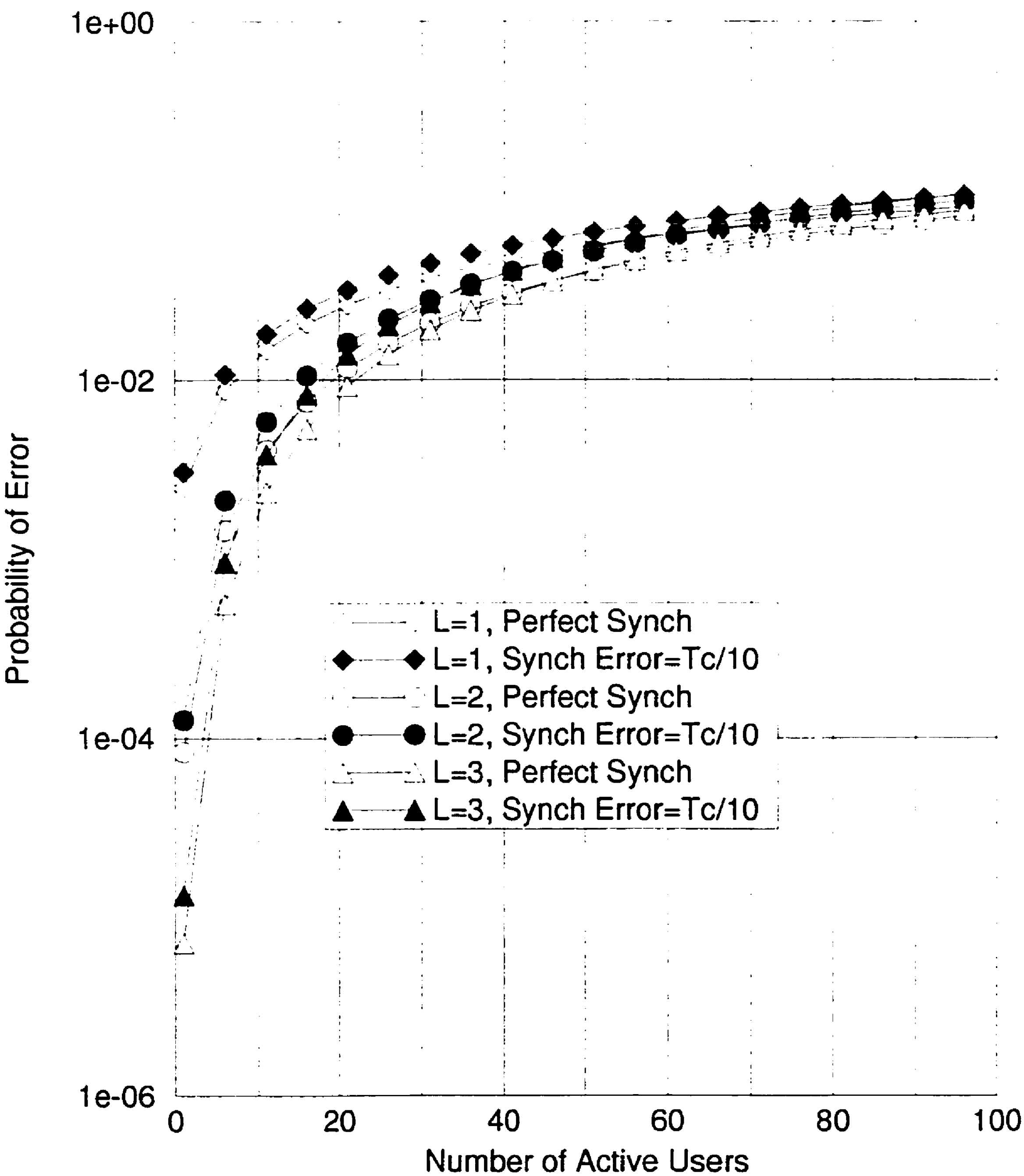

Figure 5.3: Probability of error versus the number of active users for the 1.25MHz system using various number of diversity branches when there are no synchronization errors and when there is a timing error of $T_c/10$ (Selection Diversity).

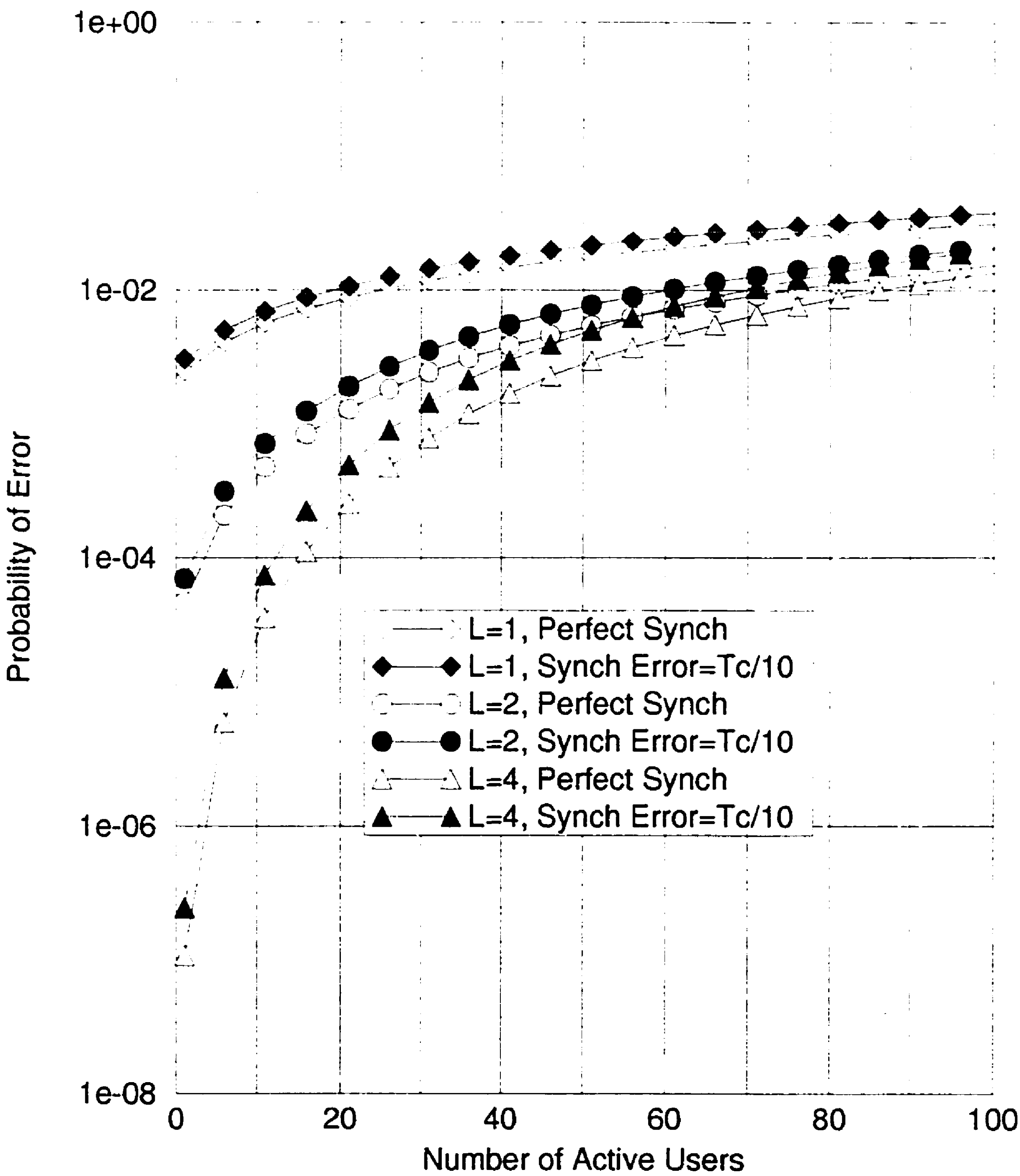

Figure 5.4: Probability of error versus the number of active users for the 5MHz system using various number of diversity branches when there are no synchronization errors and when there is a timing error of $T_c/10$ (Selection Diversity).

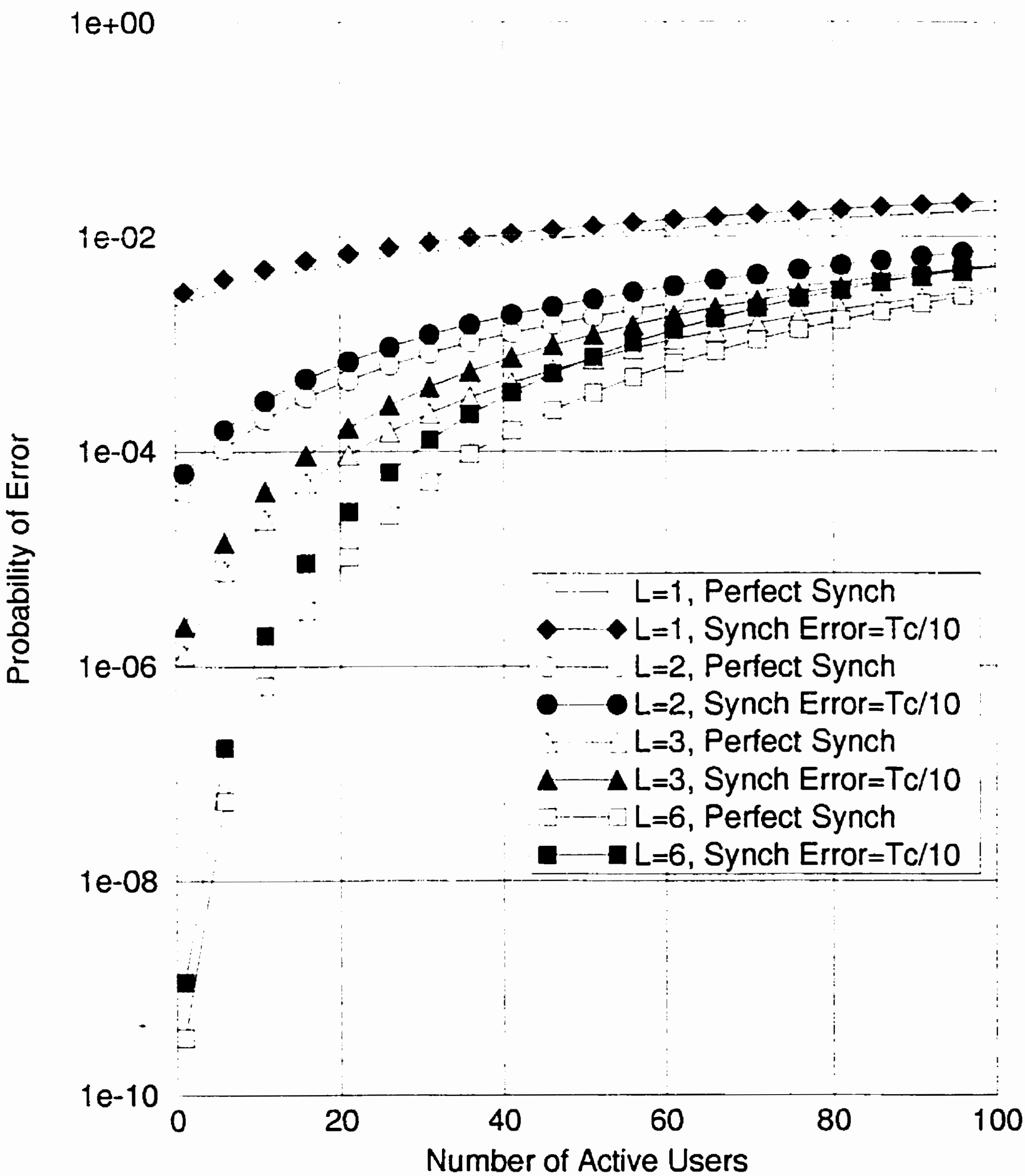

Figure 5.5: Probability of error versus the number of active users for the 10MHz system using various number of diversity branches when there are no synchronization errors and when there is a timing error of $T_c/10$ (Selection Diversity).

Chapter 6

ANALYSIS OF UNBALANCED DS CDMA SYSTEMS

As discussed in Chapter 3. there have been many papers on the calculation of error probabilities for DS CDMA systems. Throughout the years. one particularly popular approximation has been the Standard Gaussian Approximation (SGA). As explained in detail before. in SGA. only the desired user's signal is considered and all other users which are simultaneously using the channel are treated as interfering additive Gaussian noise having uniform power spectrum over the band of interest. The SGA is based on the central limit theorem which states that the sum of a large number of mutually independent random variables tends towards the normal distribution provided certain constraints are satisfied[1]. Thus. the SGA assumes that the interference terms are mutually independent and that there exist a large number of them. However. we have shown in Chapter 2 that the multiple access interference terms in a DS CDMA system are independent only when conditioned on a set of specific operating conditions of every user. Thus. the SGA is not always accurate enough. This observation led to the proposal of another central limit theorem based approximation. namely. the Improved Gaussian Approximation (IGA) [156]. Here. the multiple access interference terms are conditioned on the particular operating conditions of each user and the central limit theorem is invoked to find the conditional error probability. The total probability theorem is then used to numerically find the unconditional error probability. The IGA is naturally more accurate than the SGA. However in [156]. evaluation of the expressions used to describe the IGA

[1]The Central Limit Theorem is described in Section 3.1.1.

technique requires significant computational time and complexity. This is overcome by Holtzman by using an accurate Taylor series based approximation in [80]. Both the SGA and the IGA have been discussed in detail in Chapter 3.

Even though the IGA provides a more accurate estimation of the bit error rate than the SGA, ultimately both approximations are based on the central limit theorem and thus they both fail for cases where the central limit theorem is not applicable. It is well known that the central limit theorem requires a large number of random variables where no single variable or a group of variables can dominate. In practice, one may think of scenarios where the DS CDMA system will not satisfy this constraint. A scarcely populated system, one or a small group of interferers dominating the interference term or a system where services that require different signaling rates and associated error probabilities are examples of scenarios one might encounter in practice where the validity of the central limit theorem is questionable.

The Fourier series based expression derived in this thesis does not rely on a Gaussian approximation for the multiple access interference. Therefore, even in scenarios such as the ones listed above, the expression is still valid. In this chapter the probability of error expressions for the above scenarios are obtained using the derived Fourier series expression as well as the SGA and the IGA, since these are the most popular analysis techniques in use. Since the Fourier series error probability expression makes no approximations on the distribution of the multiple access and intersymbol interference terms, it is used as a tool to assess the level of accuracy of the central limit based approximations are in determining the error probabilities.

6.1 Analysis of a Scarcely Populated DS CDMA System

We first consider the biphase spread, asynchronous DS CDMA system in additive white Gaussian noise (AWGN) channel described in Chapter 2. We investigate the case where the system is scarcely populated since one of the underlying requirements for the central limit theorem to hold is to have a large number of users contributing to the multiple access interference. We consider a system that provides user data transmission at 9600 bits/sec at a system bandwidth of 1.2288 MHz. Thus, the processing gain is 128. A signal to noise ratio ($SNR = E_b/N_0 = P_1T/N_0$) of 20 dB is considered. We assume that the system provides perfect power control so that $P_1 = P_2 = \ldots = P_K$.

If SGA is used to find the error probability of such a system, from (3.14), one can write,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{1}{\frac{N_0}{2PT} + \frac{1}{3G}(K-1)}}\right] \tag{6.1}$$

Similarly, using IGA, from (3.26), the error probability of the system can be written as,

$$\begin{aligned} P(E)_{IGA} &= \frac{2}{3}Q\left(\frac{\sqrt{2}PT}{\sqrt{N_0T+\mu_\nu}}\right) + \frac{1}{6}Q\left(\frac{\sqrt{2}PT}{\sqrt{N_0T+\mu_\nu+\sqrt{3}\sigma_\nu}}\right) \\ &\quad +\frac{1}{6}Q\left(\frac{\sqrt{2}PT}{\sqrt{N_0T+\mu_\nu-\sqrt{3}\sigma_\nu}}\right) \end{aligned} \tag{6.2}$$

where

$$\mu_\nu = \frac{2G}{3}T_c^2(K-1)P \tag{6.3}$$

and

$$\sigma_\nu^2 = T_c^4P^2\left[\frac{23G^2+18G-18}{90}(K-1) + \frac{G-1}{9}(K-2)(K-1)\right]. \tag{6.4}$$

respectively.

When the error probability is calculated using the Fourier series based scheme, (4.65) can be written for the system in question as,

$$P(E)_{FS} = \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{m=1 \\ m\text{odd}}}^{\infty}\frac{1}{m}e^{-m^2w^2/2}\sin\left(mw\sqrt{\frac{2PT}{N_0}}\right)\Phi_M\left(-\frac{mw}{\sqrt{N_0T}}\right) \tag{6.5}$$

where the characteristic function of the multiple access interference, M is given as,

$$\Phi_M(w) = (\Phi_{m_k}(w))^{K-1} \tag{6.6}$$

since $\Phi_{m_2}(w) = \Phi_{m_3}(w) = \ldots = \Phi_{m_k}(w) = \ldots = \Phi_{m_K}(w)$ for this case. $\Phi_{m_k}(w)$ is given in (4.49).

We calculate the probability of error for the system in consideration using all of these three techniques for various number of active users. The results are shown in Figure 6.1 and Table 6.1. Here and below, sometimes, error rates are given to more than practical accuracy. This is done to provide an aid to those readers who wish to test the numerical accuracy of their implementation of the Fourier Series method for computing CDMA error rates. From the graph and the table, it can be observed that when the number of active users in the system is small, the SGA becomes overly optimistic in approximating the error probabilities the users experience. For example, for a system with only 13 active users, the SGA predicts the error probability to be 7.512318×10^{-8} when in fact the true error probability calculated using the Fourier series scheme is 5.533682×10^{-7}. The IGA on the other hand, gives reasonably accurate results even when the active number of users is small. For the above example where 13 users are active in the DS CDMA system, IGA predicts the error probability to be 5.019415×10^{-7}. As seen from the graph and the table, as the number of active users increase, as one would predict, the SGA starts giving more and more accurate results.

It is seen from Figure 6.1 and Table 6.1 that when the DS CDMA system is scarcely populated, the SGA is not a reliable scheme to find the error probabilities. The IGA on the other hand, seems to be acceptably accurate even when the number of users is very small.

We repeat the error probability calculations for the same DS CDMA system, this time transmitting 4800 bits/sec and operating at a processing gain of 256. The

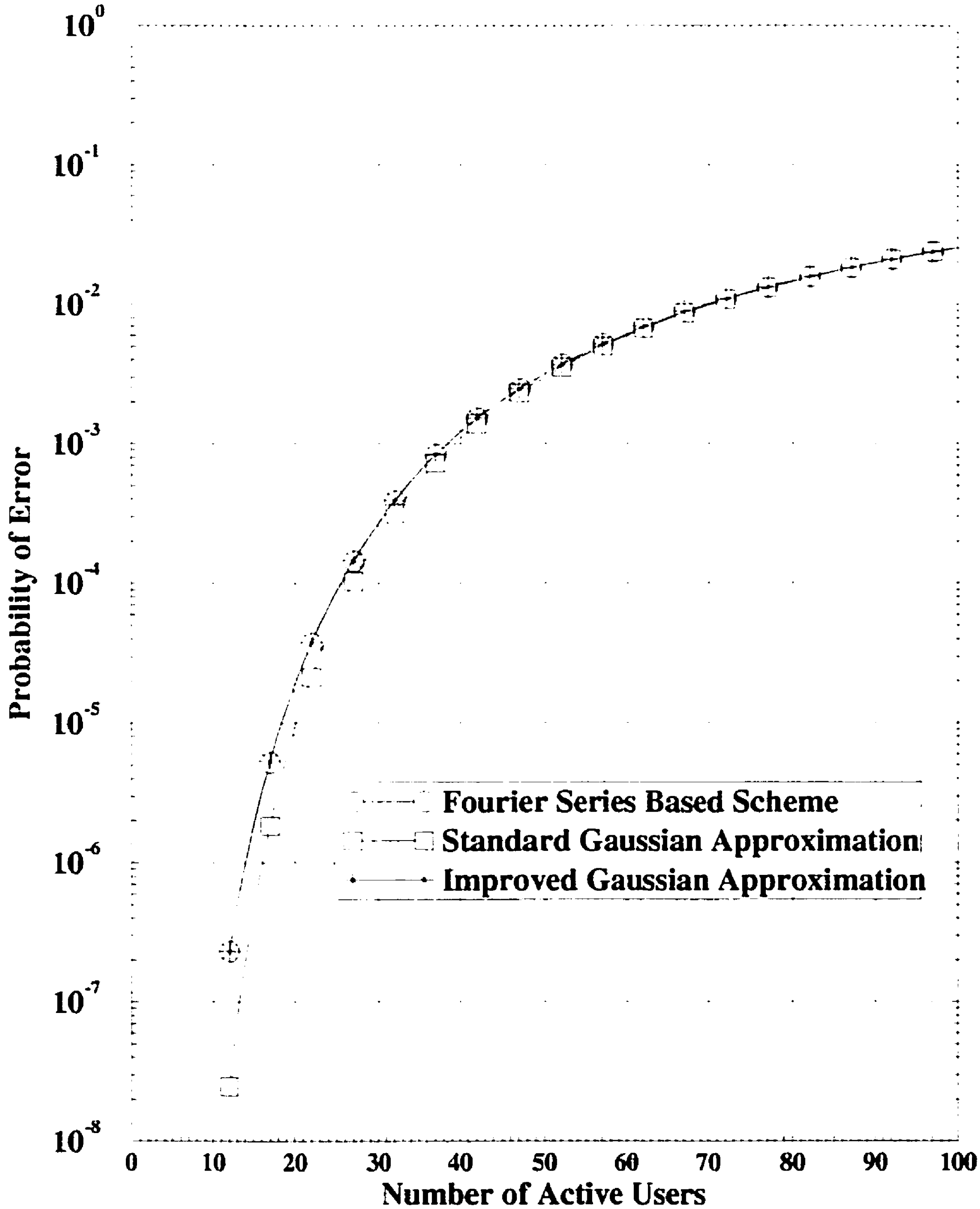

Figure 6.1: Error Probabilities for the DS CDMA System in an AWGN channel when $G = 128$ and SNR = 20 dB.

Number of Users	$P(E)_{FS}$	$P(E)_{SGA}$	$P(E)_{IGA}$
13	5.533682×10^{-7}	7.512318×10^{-8}	5.019415×10^{-7}
14	1.063543×10^{-7}	1.955999×10^{-7}	1.000582×10^{-6}
15	1.964392×10^{-6}	4.524349×10^{-7}	1.854171×10^{-6}
16	3.356542×10^{-6}	9.492888×10^{-7}	3.232929×10^{-6}
20	1.946503×10^{-5}	9.163780×10^{-6}	1.905882×10^{-5}
40	1.228632×10^{-3}	1.094339×10^{-3}	1.227311×10^{-3}
60	6.218714×10^{-3}	6.025505×10^{-3}	6.218683×10^{-3}

Table 6.1: Error Probabilities for the Scarcely Populated DS CDMA System

results are given in Figure 6.2. It is seen that at this higher processing gain, the optimism of the SGA in assessing the error probabilities at low values of number of users is accentuated. The IGA still gives acceptably accurate results but for small number of users, the discrepancy between the actual bit error rate and the value obtained from IGA is larger than that observed in the system with the processing gain of 128. Therefore, we conclude that the accuracy of a central limit theorem based approximation decreases with increasing processing gain.

6.2 Analysis of a DS CDMA System with Dominant Interferers

We now consider the same biphase spread, asynchronous DS CDMA system in AWGN channel where there is no perfect power control. Lack of power control may generate situations where one or more users' signals dominate the multiple access interference term for at least a short amount of time. As stated earlier, this is a direct violation of one of the key constraints of the central limit theorem.

Now consider that in the system of interest, one of the interferers dominate the

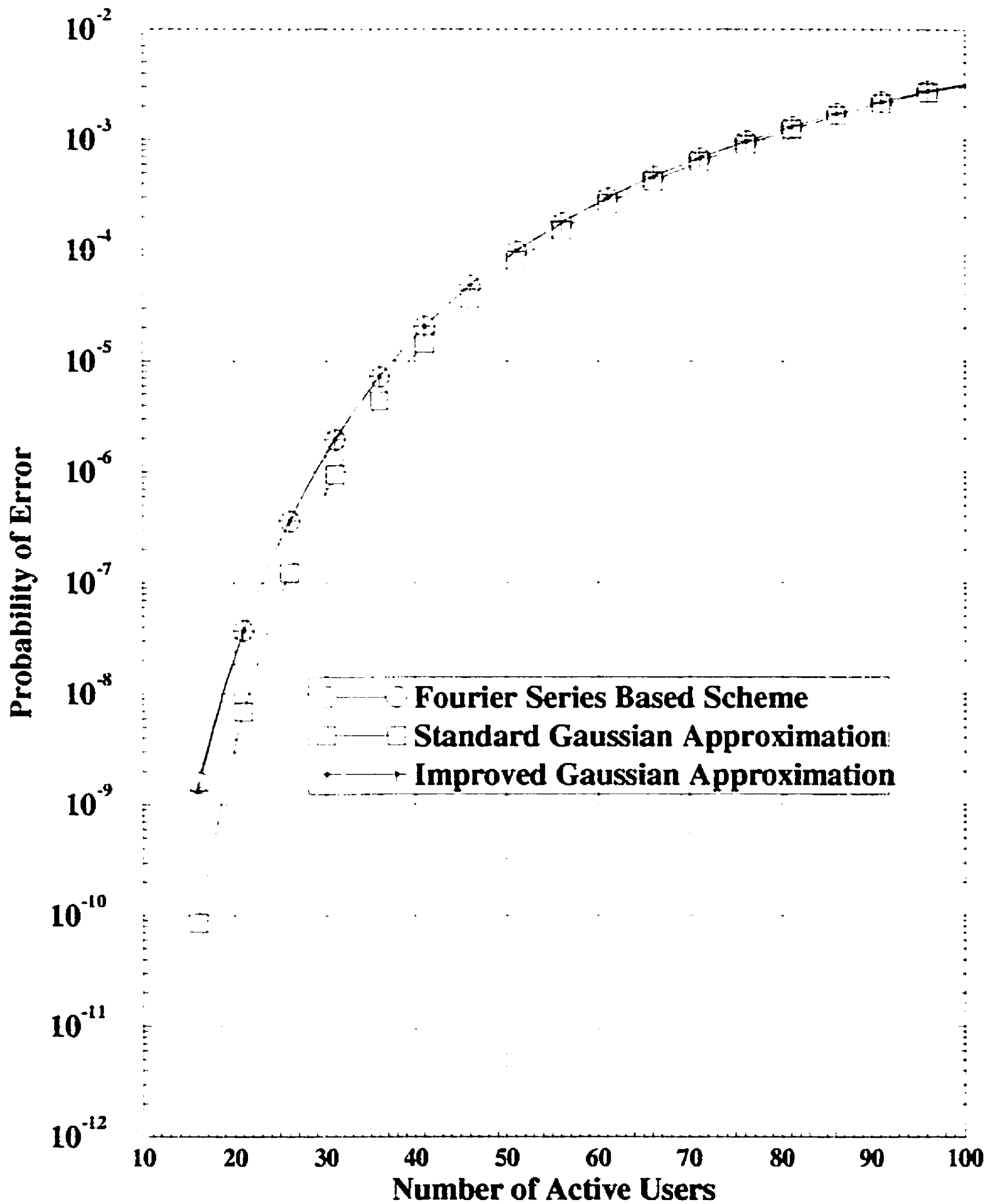

Figure 6.2: Error Probabilities for the DS CDMA System in an AWGN channel when $G = 256$ and SNR = 20dB.

multiple access interference, i.e., $P_1 = P_3 = P_4 = \ldots = P_K = P$ and $P_2 = \alpha \cdot P$, where α is a constant. In this case the error probability equation for the SGA in (6.1) can be modified as,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{1}{\frac{N_0}{2P_1T} + \frac{1}{3G}(K - 2 + \alpha)}}\right]. \tag{6.7}$$

Similarly the IGA error probability equation can be written as,

$$\begin{aligned} P(E)_{IGA} &= \frac{2}{3}Q\left(\frac{\sqrt{2PT}}{\sqrt{N_0T + \mu_\nu}}\right) + \frac{1}{6}Q\left(\frac{\sqrt{2PT}}{\sqrt{N_0T + \mu_\nu + \sqrt{3}\sigma_\nu}}\right) \\ &+ \frac{1}{6}Q\left(\frac{\sqrt{2PT}}{\sqrt{N_0T + \mu_\nu - \sqrt{3}\sigma_\nu}}\right). \end{aligned} \tag{6.8}$$

where

$$\mu_\nu = \frac{2G}{3}T_c^2(K - 2 + \alpha)P. \tag{6.9}$$

$$\begin{aligned} \sigma_\nu^2 &= T_c^4P^2\left[\frac{23G^2 + 18G - 18}{90}(K - 2 + \alpha^2)\right. \\ &\left. + \frac{G-1}{9}\left[(K-2)(K-3+\alpha)\right]\right]. \end{aligned} \tag{6.10}$$

The Fourier series based scheme on the other hand, still uses the equation given in (6.5) with the only difference that now, the characteristic function of the multiple access interference is given by,

$$\Phi_M(w) = \Phi_{m_2}(w)\left(\Phi_{m_k}(w)\right)^{K-2}, \qquad k \neq 2 \tag{6.11}$$

since the terms m_2 and m_k, $k \neq 2$ in (4.49) are now different since $P_2 = \alpha \cdot P_k$.

We calculate the probability of error for the system in consideration using the three techniques for various number of active users and different values of α. We first consider α to be 20. The results are given in Figure 6.3 and Table 6.2. From the graph and the table, it can be seen that when there is a dominant interferer present in the system, the SGA gives overly optimistic results even for moderately large

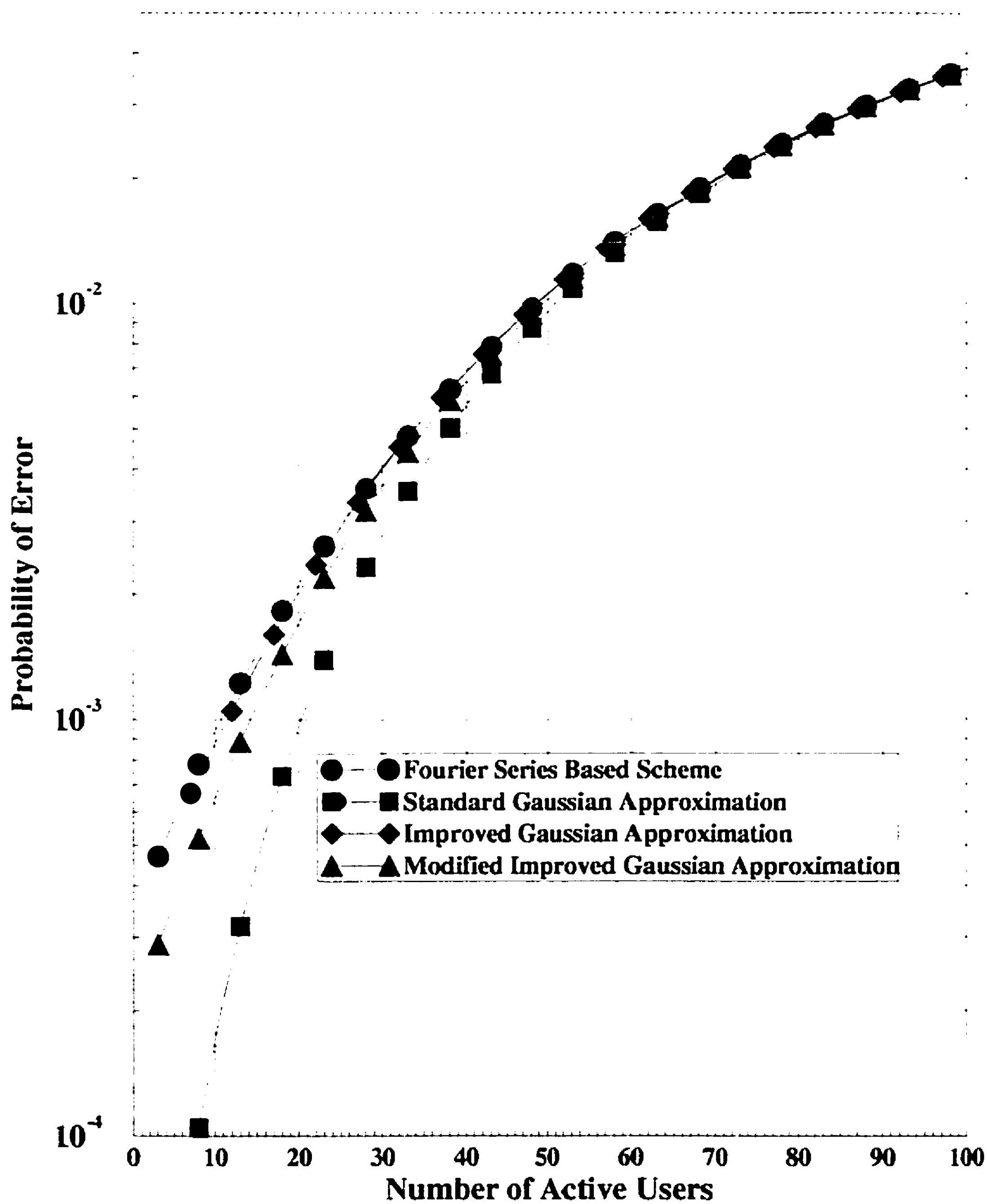

Figure 6.3: Error Probabilities for the DS CDMA System with a Dominant Interferer when $\alpha = 20, G = 128$ and SNR = 20dB.

Number of Users	$P(E)_{FS}$	$P(E)_{SGA}$	$P(E)_{IGA}$
4	5.215312×10^{-4}	3.078866×10^{-5}	4.738602×10^{-4}
5	5.791056×10^{-4}	4.328040×10^{-5}	5.297259×10^{-4}
6	6.415167×10^{-4}	5.930007×10^{-5}	5.903322×10^{-4}
7	7.088237×10^{-4}	7.941675×10^{-5}	6.655761×10^{-4}
8	7.811884×10^{-4}	1.042115×10^{-4}	7.306202×10^{-4}
9	8.587992×10^{-4}	1.342688×10^{-4}	8.010851×10^{-4}
10	9.418506×10^{-4}	1.701682×10^{-4}	8.773308×10^{-4}

Table 6.2: Error Probabilities for a DS CDMA System with a Dominant Interferer ($\alpha = 20$).

number of users. On the other hand, when there are dominant interferers, the IGA analysis becomes problematic. For small number of users (1-6 users when $\alpha = 20$), the inequality given in (3.37), which maps the region of operation for the IGA, is not satisfied. Due to this reason, the IGA as described in [80] cannot be used when there are few number of users and when one of them is dominant. If one lowers the value of the h described in (3.24) from $\sqrt{3\sigma^2}$ to $\sqrt{2\sigma^2}$, we observe that the inequality of (3.37) holds even in the 1-6 user region. For $h = \sqrt{2}\sigma$, (3.26) can be rewritten as,

$$P(E)_{IGA} = \frac{1}{2}Q\left(\frac{\sqrt{2PT}}{\sqrt{N_0T + \mu_\nu}}\right) + \frac{1}{4}Q\left(\frac{\sqrt{2PT}}{\sqrt{N_0T + \mu_\nu + \sqrt{2}\sigma_\nu}}\right) + \frac{1}{4}Q\left(\frac{\sqrt{2PT}}{\sqrt{N_0T + \mu_\nu - \sqrt{2}\sigma_\nu}}\right) \quad (6.12)$$

Recall however from Chapter 3 that as the value of h is decreased, the IGA performance approaches to that of the SGA. In Figure 6.3 we plot the error probabilities calculated using both the original IGA (in its region of operation) and the modified IGA as well as the SGA and the Fourier series based method. It is clearly seen here that for small to moderate number of users, both the SGA and the modified IGA produce optimistic results. Within its operational region, the original IGA gives accurate results. As the number of users increase, once again, all central limit based approximations produce more and more accurate results. The error probabilities for the system in question as calculated by the SGA, IGA and the Fourier Series based methods are given in tabular form for increasing number of active users in Table 6.2. Note that here, for 1-6 users the modified IGA with $h = \sqrt{2}\sigma$ is used and for 7-10 users, the IGA with $h = \sqrt{3}\sigma$ is used.

It is seen from Figure 6.3 and Table 6.2, when the DS CDMA system has a dominant interferer, the SGA is not a reliable scheme to find the error probabilities even when there is a moderately large number of users active in the system. The IGA as described in [80] on the other hand, cannot be used in such systems when the number of users is small. For large number of users, the IGA produces relatively accurate results. For the range where the IGA is not applicable, a modified IGA can be used to get estimates of the error probability. The modification, however, comes at the expense of a decreased accuracy.

We repeat the error probability calculations for the same DS CDMA system, this time with a dominant interferer of $\alpha = 50$. The results can be found in Figure 6.4. As can be seen from the figure, when the dominant interferer is stronger, both SGA and IGA become less accurate. As the strength of the dominant interferer is increased, the minimum number of active users required to get an estimate from the original IGA becomes larger (for $\alpha = 50$, a minimum of 17 users is required).

6.3 Analysis of a Multi-Service DS CDMA System

As discussed in Chapters 1 and 2, third generation (3G) wireless systems are in the last stages of their development. Globally, DS CDMA has emerged as the promi-

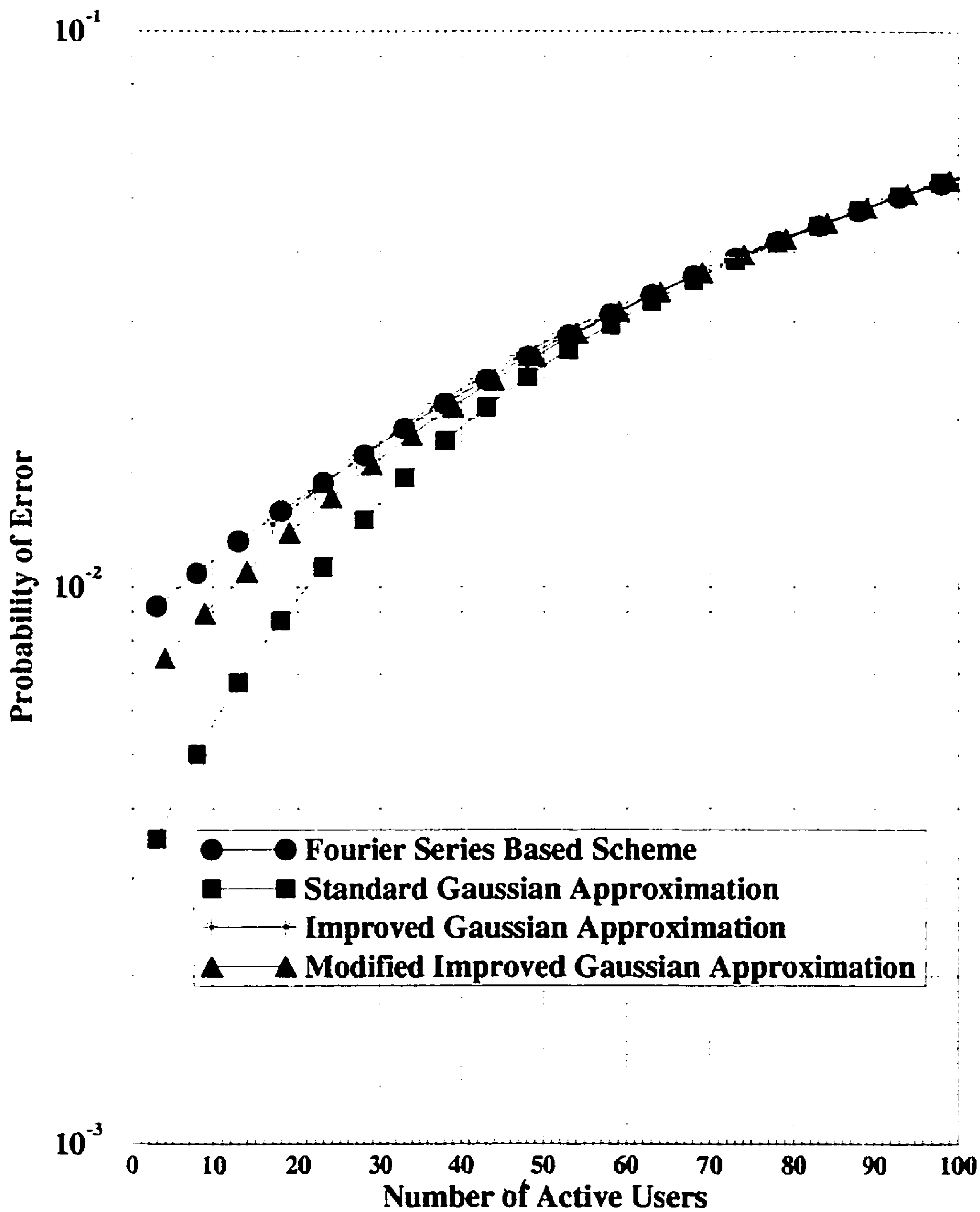

Figure 6.4: Error Probabilities for the DS CDMA System with a Dominant Interferer when $\alpha = 50, G = 128$ and SNR = 20dB.

nent multiple access scheme in 3G. Present second generation systems are primarily designed for low, fixed rate voice and data. However, the emerging 3G systems will support a wide range of services which may require different source rates and probability of error requirements. New approaches are necessary to accommodate the performance needs of mixed service users. Provision of a multitude of possible data rates in DS CDMA systems is possible by employing one of two possible methods [224]:

- *Variable Spreading:* Here, a single spreading sequence is assigned to each user regardless of the source rate. Since the signaling bandwidth is kept constant and is shared by all users in DS CDMA systems, the processing gain for each user will depend on the associated source rate in such a way that the product of the source rate and the corresponding processing gain will always be equal to the system bandwidth. Therefore, as the source rates increase, the observed processing gains decrease.

  Consider two baseband signals at rates r_1 and $r_2 = k \times r_1$, respectively. If the system bandwidth is BW, the corresponding processing gains are, $G_1 = BW/r_1$ and $G_2 = BW/r_2 = G_1/k$, respectively. If the error probability requirements are the same, then the power control scheme at the receiver sets the power level of the second signal, r_2 to k times that of the first signal, r_1 [83]. This ensures the signal to multiple access interference ratios of the two signals after despreading and filtering to be approximately the same.

- *Multi-Code Transmission:* Here, a basic transmission rate, r_b and its associated processing gain, G_b is established as the basis for the signaling bandwidth, BW. If a user needs M times the basic source rate, it converts its signal stream, serial to parallel, to M streams, each at the basic transmission rate, and spreads each stream with a different spreading sequence. In other words, the user is given M code channels to transmit at M times the basic transmission rate. Multi-code transmission is used in TIA/EIA-95-B to provide higher user data rates [182].

We now investigate an asynchronous DS CDMA system where two types of services are supplied using variable spreading. We assume that the first service requires a signaling rate of 9600 bits/sec and the information bits are spread to a bandwidth of 1.2288 MHz by a processing gain of $G_1 = 128$. Similarly, the second service requires a signaling rate of 192000 bits/sec and the information bits are spread to the same 1.2288 MHz bandwidth by a processing gain of $G_2 = G_1/20$. We assume that there are K_1 users using the first service and K_2 users using the second service. In this case, from (2.5), assuming perfect power control we can write the total received signal at the receiver of Figure 2.2 as,

$$\begin{aligned} r(t) &= \sum_{k=1}^{K_1} \sqrt{2P} a_k(t-\tau_k) b_k(t-\tau_k) \cos(w_c t + \phi_k) \\ &+ \sum_{k=1}^{K_2} \sqrt{40P} \tilde{a}_k(t-\tilde{\tau}_k) \tilde{b}_k(t-\tilde{\tau}_k) \cos(w_c t + \tilde{\phi}_k) + n(t) \end{aligned} \tag{6.13}$$

where, $a_k(t), \tilde{a}_k(t)$ are the information sequences of the k'th service one user and k'th service two user, respectively. Similarly, $b_k(t), \tilde{b}_k(t)$ are the spreading sequences of the k'th service one user and k'th service two user, respectively. τ_k, ϕ_k are the time and phase offsets for the k'th service one user, and $\tilde{\tau}_k, \tilde{\phi}_k$ are the time and phase offsets for the k'th service two user.

We are interested in the impact of service two users on the bit error rates of service one users. Without any loss of generality, we assume that the signal from the first service one user, $a_1(t)$ is to be captured. We also assume that correlator receivers are utilized and perfect synchronization of time and phase as well as perfect power control are maintained. We consider the data symbol interval of $[0, T_1]$ in the analysis, where $T_1 = 1/9600$ seconds. In this case, from (2.6), the input to the decision device for the receiver of Figure 2.2 is.

$$
\begin{aligned}
Z_1 &= 2\int_0^{T_1} b_1(t) r(t)\cos(w_c t)\, dt \\
&= \sqrt{2P}T_1 a_{1,0} + \sum_{k=2}^{K_1} \sqrt{2P}\cos(\phi_k)\int_0^{T_1} a_k(t-\tau_k) b_k(t-\tau_k) b_1(t)\, dt \\
&\quad + \sum_{k=1}^{K_2} \sqrt{10P}\cos(\tilde{\phi}_k)\int_0^{T_1} \tilde{a}_k(t-\tilde{\tau}_k)\tilde{b}_k(t-\tilde{\tau}_k) b_1(t)\, dt \\
&\quad + 2\int_0^{T_1} n(t) b_1(t)\cos(w_c t)\, dt \\
&= D + M_1 + M_2 + N
\end{aligned}
\tag{6.14}
$$

where D is the desired signal, M_1 is the multiple access interference from service one users, M_2 is the multiple access interference from service two users and N is the channel noise. Following (2.20) and (2.70), one can write,

$$
D_1 = \sqrt{2P}T_1 a_{1,0} \tag{6.15}
$$

$$
\begin{aligned}
M_1 &= \sum_{k=2}^{K_1} \sqrt{2P}\cos(\phi_k) \sum_{j=0}^{G_1-1} \left[v_{k,j}(T_c-\tau_k) + \gamma_{k,j}\tau_k\right] \\
&= \sum_{k=2}^{K_1} m_{1k}
\end{aligned}
\tag{6.16}
$$

$$
\begin{aligned}
M_2 &= \sum_{k=1}^{K_2} \sqrt{40P}\cos(\tilde{\phi}_k) \sum_{j=0}^{G_1-1} \left[\tilde{v}_{k,j}(T_c-\tilde{\tau}_k) + \tilde{\gamma}_{k,j}\tilde{\tau}_k\right] \\
&= \sum_{k=1}^{K_2} m_{2k}
\end{aligned}
\tag{6.17}
$$

The random variable N is zero mean, $N_0 T$ variance Gaussian distributed. It can clearly be observed from (6.17) that as far as the bit error rate analysis is concerned, the variable spread multi-service system is identical to the system with dominant interferers. Relative to the service one users, service two users dominate the multiple access interference that a server one user observes. The larger the data rate for the service two user is, the more dominating the service two users will be as multiple access interferers. This is because, as the data rate is increased within a fixed bandwidth, the observed processing gain decreases. To maintain the quality of service, the subscriber transmit powers need to be increases proportionally to compensate for the loss in the processing gain. In general, using (6.1), the observed error probability equation for the SGA for a service one user can be written as,

$$P(E)_{SGA} = Q\left[\sqrt{\frac{1}{\frac{N_0}{2PT_1} + \frac{K_1-1}{3G_1} + \frac{kK_2}{3G_1}}}\right] \tag{6.18}$$

where k is the ratio of the service two information rate to that of service one.

Similarly the IGA error probability equation for one of the service one users can be written as,

$$\begin{aligned} P(E)_{IGA} &= \frac{2}{3}Q\left(\frac{\sqrt{2P}T}{\sqrt{N_0T + \mu_\nu}}\right) + \frac{1}{6}Q\left(\frac{\sqrt{2P}T}{\sqrt{N_0T + \mu_\nu + \sqrt{3\sigma_\nu^2}}}\right) \\ &\quad + \frac{1}{6}Q\left(\frac{\sqrt{2P}T}{\sqrt{N_0T + \mu_\nu - \sqrt{3\sigma_\nu^2}}}\right) \end{aligned} \tag{6.19}$$

where

$$\mu_\nu = \frac{2}{3}PT_c^2G(K_1 + kK_2 - 1) \tag{6.20}$$

$$\begin{aligned} \sigma_\nu^2 &= P^2T_c^4(K_1 + k^2K_2 - 1)\frac{23G^2 + 18G - 18}{90} \\ &\quad P^2T_c^4\left((K_1 - 1)(K_1 + 2kK_2 - 2) + k^2K_2(K_2 - 1)\right)\frac{G-1}{9} \end{aligned} \tag{6.21}$$

When the error probability is calculated using the Fourier series based scheme, the equation given in (6.5) can still be used. However, the characteristic function of M is now calculated using the equation,

$$\begin{aligned} \Phi_M(w) &= \Phi_{M_1}(w)\Phi_{M_2}(w) \\ &= \left(\Phi_{m_{1k}}(w)\right)^{K_1-1}\left(\Phi_{m_{2k}}(w)\right)^{K_2} \end{aligned} \tag{6.22}$$

since the terms m_{1k} and m_{2k} in (4.49) are now different since the received signal power of a service two user is k times that of a user one service user.

Number of Voice Users	$P(E)_{FS}$	$P(E)_{SGA}$	$P(E)_{IGA}$
3	5.215312×10^{-4}	3.078866×10^{-5}	4.738602×10^{-4}
4	5.791056×10^{-4}	4.328040×10^{-5}	5.297259×10^{-4}
6	7.088237×10^{-4}	7.941675×10^{-5}	6.655761×10^{-4}
9	9.418506×10^{-4}	1.701682×10^{-4}	8.773308×10^{-4}

Table 6.3: Error Probabilities for a Multi-Service DS CDMA System with a single service two user.

We calculate error probabilities for the variable rate multi-service DS CDMA system with one service two user and many service one users using all of the above mentioned schemes. The results are shown in Figure 6.5 and Table 6.3. It is clear from the graph and the table that the SGA gives overly optimistic results even for moderately large number of users. IGA, on the other hand, gives respectably accurate results in its region of operation. When the IGA is modified to get error probability results for small number of users, as before, the performance of the modified IGA moves towards that of the SGA. The error probabilities for the system in question as calculated by the SGA, IGA and the Fourier Series based method are given in tabular form for increasing number of active users in Table 6.3.

Use of multi-code transmission to provide the variety of signaling rates necessary in 3G does not cause any of the central limit theorem conditions to fail. On the contrary, since the high data rate users transmit their signals using k parallel code channels all at the basic spreading level, their impact on the low data rate users is identical to the impact of k separate low data rate users. Thus, the presence of high data rate users in the system merely increases the effective number of low data rate interferers, thereby making both SGA and IGA respectable approximations in bit error rate estimation. It should be noted here that use of multi-code or variable

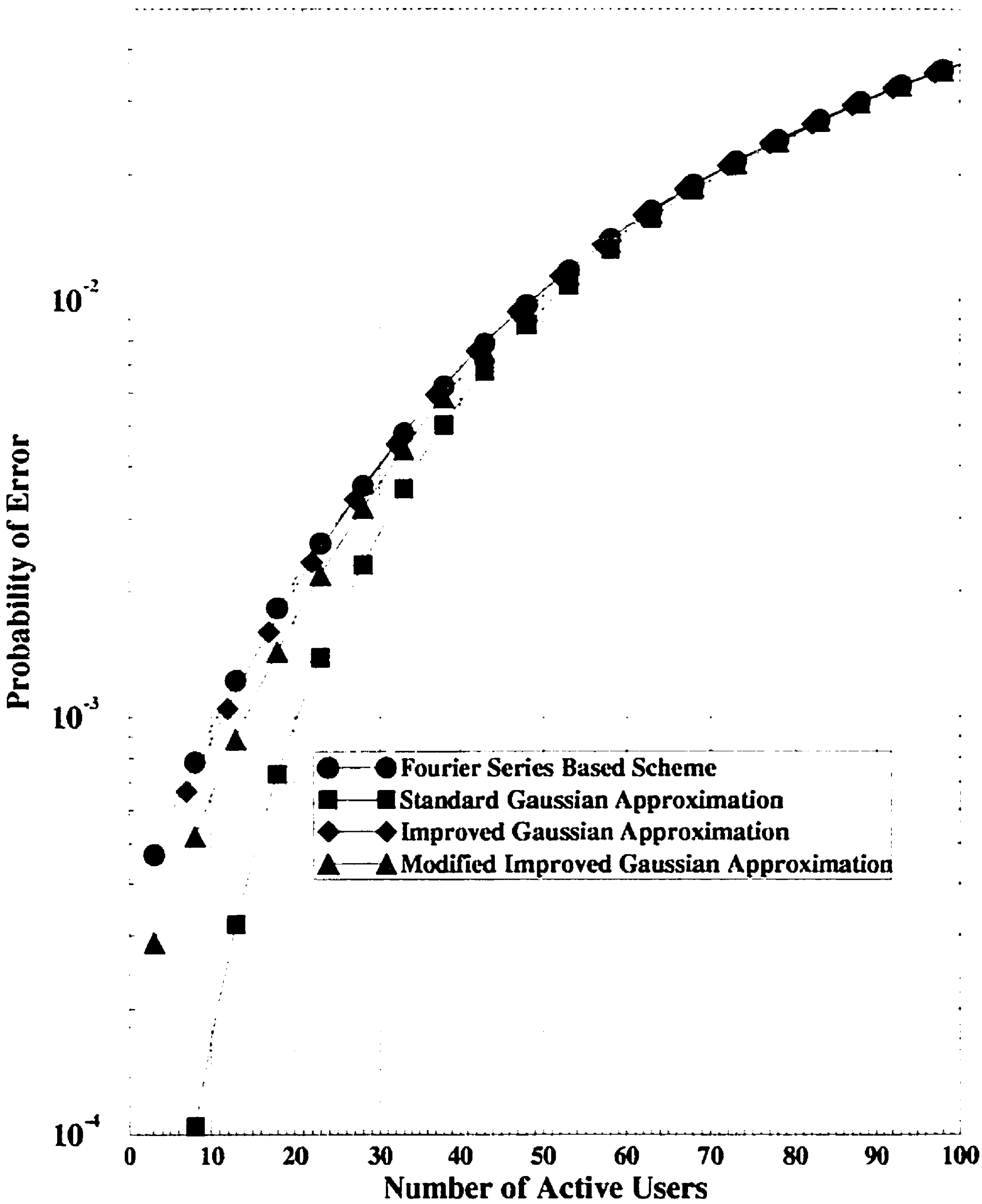

Figure 6.5: Error Probabilities for the multi-service DS CDMA System with a single service two interferer at SNR=20dB.

spreading to achieve the higher data rates results in approximately the same system performance [224]. Thus, the choice of one scheme over the other has to be based on non-performance related issues.

6.4 Analysis of a DS CDMA System in a Frequency Non-Selective Fading Channel

Next, we consider the flat Rayleigh fading multipath channel. In this case, the path gain variable β_k for each user described in (2.110) is a Rayleigh random variable with the distribution,

$$f_\beta(x) = \frac{x}{\rho_0} e^{-x^2/2\rho_0} u(x) \tag{6.23}$$

with $E\{\beta\} = \sqrt{\rho_0\pi/2}$, $\sigma_\beta^2 = (2-\pi/2)\rho_0$ and $E\{\beta^2\} = 2\rho_0$. We assume that $P_1 = P_2 = \ldots = P_K = P$. To find the error probabilities for this case, the equations given in (3.19), (3.26) and (5.37) have to be averaged over the distributions of the path gains.

When SGA is used to find the error probability for the DS CDMA system in a flat Rayleigh fading channel, (3.14) becomes,

$$\begin{aligned} P(E)_{SGA} &= E_{\beta_1}\left\{Q\left[\frac{\sqrt{2P}\beta_1 T}{\sqrt{N_0 T + \frac{2G}{3}T_c^2 P\rho_0(K-1)}}\right]\right\} \\ &= \frac{1}{\rho_0}\int_0^\infty Q\left[\frac{\sqrt{2P}\beta_1 T}{\sqrt{N_0 T + \frac{2G}{3}T_c^2 P\rho_0(K-1)}}\right]\beta_1 e^{-\beta_1^2/(2\rho_0)}\, d\beta_1 \end{aligned} \tag{6.24}$$

The above integral can be solved using equality (6.287) in [72] on page 649.

$$\int_0^\infty (1-\Phi(Bx))e^{-Ax^2}x\, dx = \frac{1}{2A}\left(1 - \frac{B}{\sqrt{A+B^2}}\right) \tag{6.25}$$

where $\Phi(x)$ is the Exponential-Integral function defined by,

$$\Phi(x) = \frac{2}{\pi}\int_0^x e^{-t^2}\, dt \tag{6.26}$$

resulting in,

$$Q(x) = \frac{1}{2}\left(1 - \Phi\left(\frac{x}{\sqrt{2}}\right)\right). \tag{6.27}$$

Using (6.25), one can rewrite (6.24) as,

$$\begin{aligned} P(E)_{SGA} &= \frac{1}{2\rho_0}\int_0^{\infty}\left(1 - \Phi\left(\frac{\sqrt{P_1}\beta_1 T}{\sqrt{N_0 T + \frac{2g}{3}T_c^2 P\rho_0(K-1)}}\right)\right)\beta_1 e^{-\beta_1^2/(2\rho_0)}\, d\beta_1 \\ &= \frac{1}{2}\left(1 - \sqrt{\frac{1}{1 + \frac{N_0}{2P_1\rho_0 T} + \frac{2}{3}\frac{K-1}{G}}}\right) \end{aligned} \tag{6.28}$$

Similarly, in a fading environment, the IGA error probability equation of (3.26) becomes,

$$\begin{aligned} P(E)_{IGA} &= \mathrm{E}_{\beta_1}\left\{\frac{2}{3}Q\left(\frac{\sqrt{2P}\beta_1 T}{\sqrt{N_0 T + \mu_\nu}}\right) + \frac{1}{6}Q\left(\frac{\sqrt{2P}\beta_1 T}{\sqrt{N_0 T + \mu_\nu + \sqrt{3}\sigma_\nu}}\right)\right. \\ &\quad \left. + \frac{1}{6}Q\left(\frac{\sqrt{2P}\beta_1 T}{\sqrt{N_0 T + \mu_\nu - \sqrt{3}\sigma_\nu}}\right)\right\} \\ &= \frac{1}{3}\left(1 - \sqrt{\frac{1}{1 + \frac{N_0}{2P\rho_0 T} + \frac{\mu_\nu}{2P\rho_0 T^2}}}\right) + \frac{1}{12}\left(1 - \sqrt{\frac{1}{1 + \frac{N_0}{2P\rho_0 T} + \frac{\mu_\nu + \sqrt{3}\sigma_\nu}{2P\rho_0 T^2}}}\right) \\ &\quad + \frac{1}{12}\left(1 - \sqrt{\frac{1}{1 + \frac{N_0}{2P\rho_0 T} + \frac{\mu_\nu - \sqrt{3}\sigma_\nu}{2P\rho_0 T^2}}}\right) \end{aligned} \tag{6.29}$$

where

$$\mu_\nu = \frac{4}{3}T_c^2 G\rho_0(K-1)P \tag{6.30}$$

$$\begin{aligned} \sigma_\nu^2 &= 8P^2\rho_0^2 T_c^4\left(\frac{23G^2 + 18G - 18}{90}\right)(K-1) \\ &\quad + 4P^2\rho_0^2 T_c^4\left(\frac{G-1}{9}\right)(K-1)(K-2) \end{aligned} \tag{6.31}$$

A Fourier series expression is already derived in Chapter 5 to calculate the error probabilities for DS CDMA systems in multipath fading environments. For an asynchronous DS CDMA system with no synchronization errors, the series for the error probability conditioned on the distributions on the path gains is given in (5.37). To find the system error probability here, we need to average over the individual path gains. Note here that we assume the fading experienced by different users to be independent from one another. In this case,

$$P(E) = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{1}{m} e^{-m^2 w^2/2} \mathrm{E}_{\beta_1} \left\{ \sin\left(\frac{mw\sqrt{2P\beta_1 T}}{\sqrt{N_0 T}} \right) \right\}$$
$$\cdot \mathrm{E}_{\beta_k} \left\{ \Phi_M \left(-\frac{mw}{\sqrt{N_0 T}} \right) \right\} \tag{6.32}$$

where

$$\Phi_M(\omega) = \prod_{k=2}^{K} \Phi_{m_k}(\omega) \tag{6.33}$$

The characteristic function of m_k can be found by averaging equation (4.62) over the distributions of β_k numerically. Once again, a simple trapezoidal rule is sufficient to obtain accurate results in a reasonably fast manner. Averaging over the distribution of β_1 can be performed analytically. We use the following equality from page 495, equation (3.952.1) of [72] for this purpose,

$$\int_0^{\infty} \sin(ax) e^{-p^2 x^2} x \, dx = \frac{a\sqrt{\pi}}{4p^3} e^{-a^2/(4p^2)}. \tag{6.34}$$

In this case, (6.32) becomes,

$$P(E) = \frac{1}{2} - 2 \sum_{\substack{m=1 \\ m \text{ odd}}}^{\infty} \frac{w\sqrt{P_1 \rho_0 T}}{\sqrt{N_0}} \cdot e^{-m^2 w^2 \left(\frac{1}{2} + \frac{P_1 \rho_0 T}{N_0} \right)} \cdot \mathrm{E}_{\beta_k} \left\{ \Phi_M \left(-\frac{mw}{\sqrt{N_0 T}} \right) \right\} \tag{6.35}$$

We calculate the probability of error for the system in consideration using all of these three techniques for various number of active users. The results are shown in Figure 6.6. This graph shows that, for the calculation of error probabilities for DS CDMA systems in multipath fading channels the central limit theorem based schemes provide reasonably accurate results. However, unlike the previous scenarios, as the number of users increase, the SGA or the IGA do not converge to the true error probability value in this case, but rather maintain their error margins. The IGA, as before, provides better results than SGA but it still overestimates the system performance slightly even for large number of users.

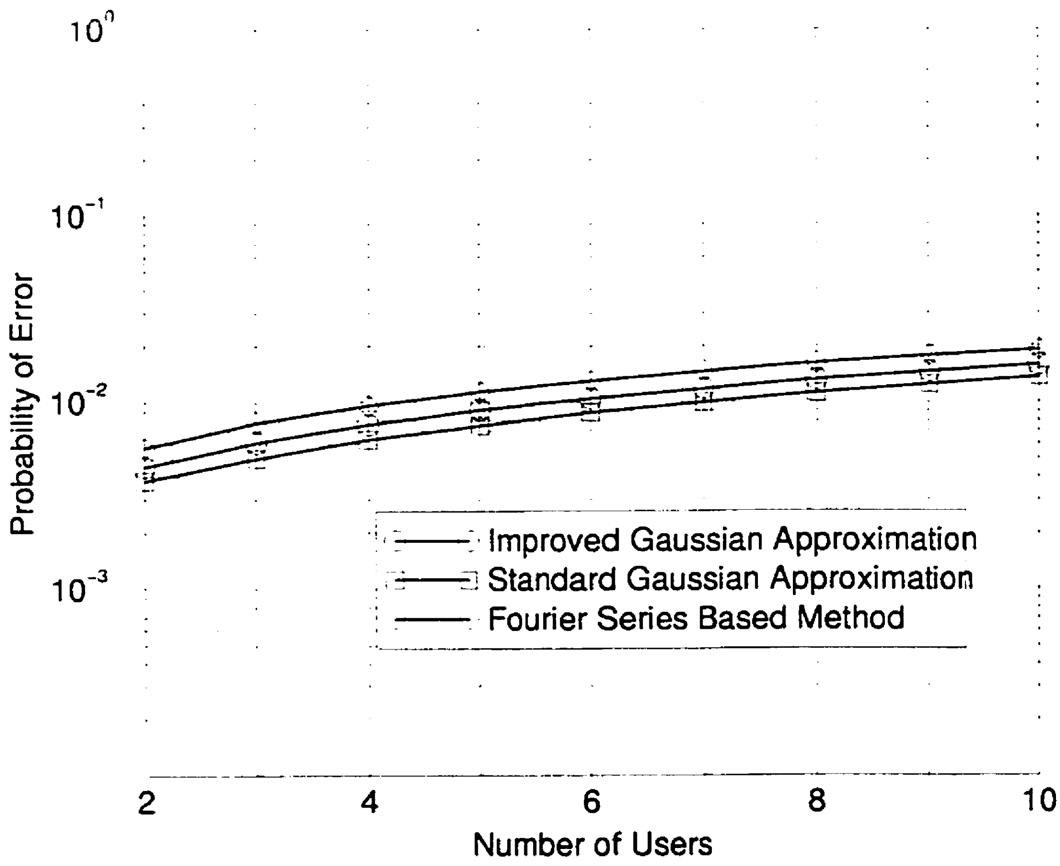

Figure 6.6: Error Probabilities for the DS CDMA System in a Multipath Fading Channel

6.5 Conclusions

In this chapter we calculate error probabilities for various DS CDMA systems using the standard Gaussian approximation, improved Gaussian approximation and the Fourier series based schemes. The scenarios are selected such that conditions for the validity of the central limit theorem are questionable. A scarcely populated system, a system with dominant interferers, a multi-service system with uneven number of subscribers to the offered services and a system in a fading channel are considered. It is seen that the SGA in all of these cases yields optimistic results, especially when the number of users active in the system is small. The IGA on the other hand, gives accurate results for a scarcely populated system, a system with a dominant interferer and a multi-service system. However, in the dominant interferer and multi-service cases, if the strength of the dominant interferer is significantly large, the IGA does not work at all. A modification can be made to overcome this problem at the expense of a reduced accuracy. For systems in fading channels neither of the Gaussian approximations seem to ever converge to the true error probability. For all of these cases, the Fourier series based scheme is used to calculate the true error probabilities.

Chapter 7

CONCLUSIONS

At a time where there is an ever growing move towards wireless communications. it is important to provide the literature with a rigorous. accurate performance analysis tool for DS CDMA systems since most third generation systems will be based on this multiple access scheme. In this light. this thesis is concerned with providing an analysis scheme to calculate the error probabilities of DS CDMA systems in additive white Gaussian noise (AWGN) and multipath fading channels with diversity combining when correlator receivers are used to coherently receive the signal. Unlike the popular approximations used for performance analysis. the scheme developed here is valid for all types of systems at all operating points.

7.1 Summary

The main objective of this thesis is to develop semi-analytical error probability expressions for DS CDMA systems without approximating the distributions of the multiple access interference and the intersymbol interference. Developing an analysis scheme for DS CDMA is important for two aspects. First. it provides a better understanding of such systems and second. such an analysis scheme could be used to find just how accurate the popular approximations that are currently in use to estimate the system performance are and whether they have any limitations.

The error probability expressions derived throughout the thesis are based on a Fourier series approximation of the error function. $Q(x)$ first developed by Beaulieu in [14, 15]. Computation of the error probabilities using the derived expressions is fast in all cases.

Once the error probability expressions are derived, we evaluate the performance of biphase and quadriphase spread, coherent DS CDMA systems in AWGN and multipath fading channels.

In AWGN channels, we observe that the quadriphase spreading system provides a slightly better error probability performance than that of the biphase spread system. The difference in the performance however, is never large enough to grant a capacity gain for the quadriphase spread system. We investigate the impact of non-zero chip timing and carrier phase synchronization errors on the capacity of the biphase and quadriphase spread systems. We observe that both systems are quite sensitive to such errors. In fact, we observe that a 10% error in chip timing corresponds to approximately a 20% capacity loss for both systems. We realize that the synchronization errors effect the system performance by potentially reducing the energy of the desired signal component of the received signal and by introducing self interference. We find that the effects of the self interference, in comparison to the desired signal energy reduction, is negligibly small. Furthermore, we find that the reduction in the desired signal energy level can alternatively be interpreted as an effective processing gain loss where the effective processing gain is approximately a linear function of the system capacity for a given level of maximum allowable error probability. If we define the capacity loss of a system as the difference between the capacity when there are no synchronization errors and the capacity when synchronization errors are present, our calculations show that the capacity loss from the presence of both chip timing and carrier phase errors is approximately the sum of individual losses for all values of signal to noise ratio.

We consider various DS CDMA schemes: synchronous, quasi-synchronous and asynchronous. When long PN sequences are used for spreading the information sequences, we find that synchronous transmission amongst users results in the lowest system capacity. The capacity increases as the synchronism constraint is relaxed. Asynchronous systems have the best performance when long PN codes are used.

In multipath fading channels, we investigate system capacities when selection diversity, equal gain combining and maximal ratio combining are employed at the receiver. We consider a number of signaling bandwidths. For all three diversity combining schemes, we find that multipath fading affects the system performance dramatically. If a system does not have inherent diversity to exploit, the system capacity drops dramatically relative to its performance in AWGN channels. Diversity combining is beneficial. We find that our simulations reiterate the well-known fact that maximal ratio combining performs best and selection diversity performs worst.

We find that synchronization errors effect the system capacity significantly in multipath fading channels as well. The percentage loss in the system capacity due to a given level of chip timing or carrier phase errors is approximately the same as the percentage loss of the same system in the AWGN environment. Analogous to the AWGN channel case, we observe that the synchronization errors effect the system performance by potentially reducing the energy of the desired signal component of the received signal and by introducing self interference. As in the case of the AWGN channel, we have numerically found that the self interference, in comparison to the signal energy reduction is negligibly small. The reduction in the desired signal energy level can alternatively be interpreted as an effective processing gain loss. Then, the effective processing gain is approximately a linear function of the system capacity for a given level of maximum allowable error probability.

We also calculate error probabilities for various DS CDMA systems using the standard Gaussian approximation, the improved Gaussian approximation and the Fourier series based expression we derived in this thesis. The scenarios are selected such that conditions for the validity of the central limit theorem are questionable.

We investigate a scarcely populated system, a system with dominant interferers, a multi-service system with uneven number of subscribers to the offered services and and system in a frequency non-selective Rayleigh fading channel. We observe that the SGA in all of these cases yields optimistic results, especially when the number of users active in the system is small. The IGA on the other hand, gives accurate results for a scarcely populated system, a system with a dominant interferer and a multi-service system. However, in the dominant interferer and multi-service cases, if the strength of the dominant interferer is significantly large, or if the signaling rates between the different services are large, the IGA does not work at all. A modification can be made to overcome this problem at the expense of a reduced accuracy. For systems in fading channels neither of the Gaussian approximations seem to converge to the true error probability.

We conclude that the error probability expressions derived throughout the thesis for various DS CDMA systems are powerful tools. Since the derivations do not rely on approximating the multiple access interference or the intersymbol interference, we can use them at all operating points, even in cases where the Central Limit Theorem does not apply. Despite the lack of an underlying approximation, calculation of error probabilities is fast. Thus, the expressions form a sound alternative to many other techniques that have appeared in the literature that are either not as accurate or have significantly more computational complexity.

7.2 Suggestions for Further Research

- The DS CDMA based systems used in practice are significantly more involved than the systems that are considered in this thesis. No coding or interleaving blocks have been considered here. One possible extension would be to investigate the possibility of incorporating the error correction coding and interleaving effects into the analysis.

- Only correlator receivers are considered in this thesis. It is possible that expressions can also be developed for the iterative interference cancelling receiver explained in depth in [240]. It is anticipated that as the iterations progress in this receiver and more and more of the interference terms are cancelled out, the number of remaining terms will become too small for the CLT based approximations to accurately assess the system performance.

- Only Rayleigh fading is considered in this thesis. One can attempt to get results for Ricean and Nakagami fading using the same technique.

- Effects of long term fading which is usually modeled as a log-normally distributed random variable may be incorporated into the analysis.

- The application of the method developed herein should be extended to multi-cell DS CDMA systems.

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